Applications of Stochastic Simulations in Inventory Management Optimization: Cost Accounting Perspectives

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Inventory Management in cost accounting is important since materials or merchandise costs account for a large portion of total costs of manufacturing and merchandising companies. An effective and efficient system of inventory management can hence be critical for controlling business costs and to improve profitability. This paper focuses on applications of nonlinear optimization and stochastic models to study various issues in inventory management, especially from cost accounting perspectives. The applications illustrated in this paper can be fully implemented in real world inventory management situations using a spreadsheet software tool, and can also be integrated into the cost accounting curriculum.

INTRODUCTION

Inventory management is an important subject in business operations since materials or merchandise costs account for a large portion of the total costs of manufacturing and merchandising (Horngren 2015). Excess inventory may cause waste of financial resources and increase costs while a low level of inventory may have high risks of losing customers and causing stock-out costs (Emmelhainz et al., 1991; Fitzsimons, 2000; Anderson et al., 2006; McKinnon et al., 2007). Several theories and methods have been introduced with the aim of minimizing costs of financing and inventory related costs. Researchers from both the operations management and the cost accounting fields have proposed theoretical models to deal with inventory related problems (Williams B.D. et al, 2008, Ziukov, S., 2015, Oyedokun, G.E., 2017). These models consider discrete or continuous, deterministic or stochastic, and linear or nonlinear cases. Some models also consider various resource limitations or constraints being applied to businesses.

Economic Order Quantity (EOQ) models are quite popular due to their simplicity. Deterministic models can be particularly easy to apply. However, given complex inventory management conditions, many factors have uncertainties. Scenario and sensitivity analysis is often applied with changing key input variables. However, when there are many input variables, it can be very challenging to investigate scenarios. Very limited information of output variables can be generated through scenario or sensitivity analysis (Mietzner, D., 2005). Therefore, management can be misled or make suboptimal decisions.

Historical records or predictions of these key input variables may suggest stochastic models with certain distributions. Simply using expected or average values for input variables, but ignoring statistical distributions of input variables, is myopic. Since statistical distributions contain rich information and consider all possible input values, incorporating these distributions into a model is necessary. Deterministic models using only expected values for input variables may be converted to stochastic models with observed or predicted distributions for input variables (Berman,O. et al, 1999; Sobel M. et al , 2001; Davies, 2014). In business planning and budgeting tasks, stochastic models with Monte Carlo simulations allow managers to gain insights into business operations and assess risks due to unexpected conditions. Monte Carlo simulations focus on iterations that use a large number of trials and repeats the process many times based on the distributions of input variables (Thomopoulos, 2013). As a result, distributions of outcome variables are produced. Monte Carlo simulations have been widely used in business research and practice (Boyle, P.P. et al, 1997; Kelliher, C.F. et al, 2000; Kwaka, Y.H. et al, 2007, Mohan, A.K. et al, 2016). Recently, Big Data platforms and analysis tools are being integrated into applications of stochastic simulations, and that can impact business research practices greatly. However, cost accounting education is still very traditional and may need more topics and case studies dedicated to these topics.

Many firms have implemented advanced inventory management systems such as Just-in-time (JIT), Materials Requirements Planning (MRP), and Enterprise Resource Planning (ERP). On the other hand, small or medium size companies with only a few number of products and very limited inventory management budget cannot afford to implement advanced systems. In fact, many small companies are still managing inventories very inefficiently. This paper focuses on Economic Order Quantity models with a general overview of deterministic models and related theories. Some applications can be used in these small and medium companies with inefficient inventory management systems. Both one-product inventory and multiple-product inventory cases are covered. Stochastic models with Monte Carlo simulations are then introduced to demonstrate how to assess risks. The methods and tools described in this paper can be used to study real business problems. The research can also be extended to cases of multiple products in multiple warehouses and activity-based costing systems. With more and more Big Data platforms and analysis tools being integrated into accounting practices, applications of stochastics simulations can also be extended to more complex inventory management and analysis cases.

OPTIMIZATION AND LINEAR PROGRAMMING METHODS

Inventory management studies in this paper focus on how to minimize materials costs or maximize profits, especially under complex conditions. Many inventory problems can hence be formulated as optimization problems. Accountants can simply apply well developed computer technology tools to find optimal solutions although solving optimization problems can be mathematically challenging in theory (Vasek, C., 1983; Nocedal, J. et al, 2006; Dennis F. T., 2008). Let us first begin with a few definitions.

Definition 1. A linear model is a function that can be expressed in the following form:

$$f(x) = c_1 x_1 + c_2 x_2 + \dots + c_n x_n, \text{ where } \quad x = (x_1, x_2, \dots, x_n),\ x \in D,\ D \subseteq R^n \tag{1}$$

Definition 2. A nonlinear model is a function that cannot be expressed in the form of (1).

Definition 3. A linear inequality is an expression that can be expressed in the following form:

$$f(x) \le b, \text{ where } \ f(x) \text{ is linear, } \ x = (x_1, x_2, \dots, x_n),\ x \in D, D \subseteq R^n, b \in R\ . \tag{2}$$

Definition 4. A nonlinear inequality is an expression that cannot be expressed in the form of (2).

Definition 5. An unconstrained objective function is a linear or nonlinear function that needs to be maximized or minimized on its domain, such that

$$\max_{x \in D} / \min f(x), \text{ where } x = (x_1, x_2, ..., x_n), x \in D, D \subseteq R^n . \tag{3}$$

Definition 6. A constrained objective function is a linear or nonlinear function that needs to be maximized or minimized on its domain with a set of constraints, such that

$$\begin{aligned} &\max_{x \in D} / \min f(x) \\ &\text{subject to } g_i(x) \le b_i, \text{where } x = (x_1, x_2, ..., x_n), x \subseteq R^n, i = 1, ..., m, b_i \in R \end{aligned} . \tag{4}$$

Definition 7. x^* is said to be an absolute maximizer (minimizer) to the constrained problem in Equation (4), $f(x)$ if $f(x^*) \ge f(x)$ ($f(x^*) \le f(x)$) for any $x \in D$, where $x = (x_1, x_2, ..., x_n), D \subseteq R^n$ and all constraints are satisfied.

The management of manufacturing and merchandising companies need to keep their inventories at healthy levels. Companies must hold sufficient inventories of products or materials since stock-out or back-order can be very harmful to businesses. In the meantime, companies also want to minimize costs of capital without building up levels inventory that are too high. Inventory management theory considers six categories of costs associated with inventory, including purchasing costs, ordering costs, carrying costs, stockout costs, costs of quality, and shrinkage costs (Horngren, C.T. et al, 2017). With information technology and Big Data tools, companies can gather and analyze inventory information in greater detail than ever before. Manufacturing companies normally have more complicated inventory conditions than merchandising companies do. In this paper, we will only study inventory problems in merchandising companies.

Depending on the business objectives, inventory problems can be formulated in many ways. In order to find optimal solutions to these problems, it always requires solving either maximization or minimization problems. In most cost accounting textbooks, only one product in one business unit is considered in the minimization of total inventory costs. However, this paper focuses on more complex situations, including multiple products, deterministic models, stochastic models, and the minimization of total costs.

Theorem 1. If D is closed and $f(x)$ is a continuous function, then $f(x)$ has an absolute maximum value at $x = x^*$ such that $f(x^*) \ge f(x)$ for all $x \in D \subset R^n$.

The proof needs advanced topics in mathematics and is beyond the context of cost accounting. Readers who are interested can find related topics in advanced calculus or real analysis textbooks.

One-Product Inventory Management

In this paper, we only briefly review this well studied case. A case with one product in one warehouse can be formulated as a simple equation in one variable. Many studies show how to derive an optimal solution. Inventory costs are normally introduced as a sum of purchasing costs, ordering costs, carrying costs, stockout costs, costs of quality, and shrinkage costs. The economic-order-quantity (EOQ) decision model is the one that has been applied to determine the optimal quantity of inventory to order. The simplest EOQ model considers only ordering and carrying costs.

Theorem 2. $f(x)$ is a continuous function defined over an internal $I \subset R$, c is the critical value of $f(x)$. If $f''(c)$ is positive, then $f(c)$ is the absolute minimum value or in other words, $f(x)$ has the absolute minimum value at c.

Relevant costs included in the EOQ model include ordering costs and carrying costs. Annual costs is defined as

$$F = \frac{DP}{Q} + \frac{QC}{2} \tag{5}$$

where D is annual demand in units, Q is the size of each order, P is ordering cost per purchase order, and C is carrying cost of one unit in stock for the time period used of D.

Theorem 3. The cost function as defined in (5) has an absolute minimum at $Q = \sqrt{\frac{2DP}{C}}$, where D, P, and C are known constants, and $Q > 0$.

Proof: The critical values of function F as defined in (5) can be determined in the following steps. The derivative of the cost function with respect to Q is

$\frac{\partial F}{\partial Q} = -\frac{DP}{Q^2} + \frac{C}{2}$. To find critical values, $F' = -\frac{DP}{Q^2} + \frac{C}{2} = 0$.

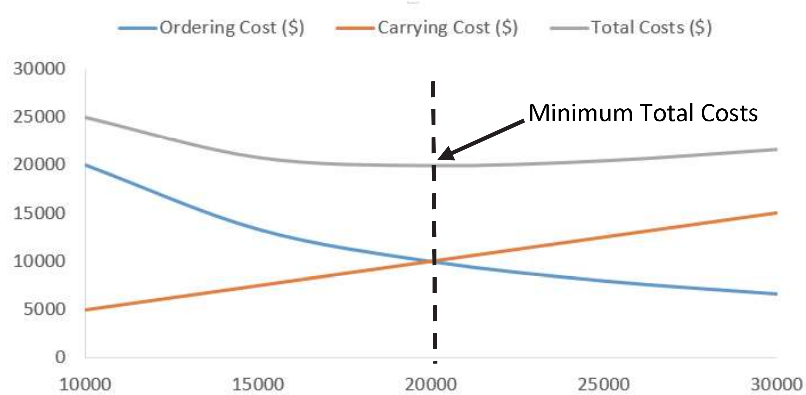
Solve for Q, $-\frac{DP}{Q^2} + \frac{C}{2} = 0$, $\frac{DP}{Q^2} = \frac{C}{2}$. Hence, $Q^2 = \frac{2DP}{C}$. Finally, $Q = \sqrt{\frac{2DP}{C}}$

The second derivative test then can be applied, note that $\frac{\partial^2 F}{\partial Q^2} = 2\frac{DP}{Q^3} > 0$

Therefore, the second derivative test shows that the cost function F has an absolute minimum at $Q = \sqrt{\frac{2DP}{C}}$.

The following graphic example can show how the optimal solution is located. In this example, assume that D=1,000,000 units; P=\$200, C = \$1, $Q = \sqrt{\frac{2DP}{C}}$ = 20,000. In the graph, the ordering cost equation is $\frac{DP}{Q} = \frac{200{,}000{,}000}{Q}$ and carrying cost equation is $\frac{QC}{2} = Q\frac{1}{2} = 0.5Q$. The sum is total costs which has the absolute minimum of \$ 20,000 at $Q = 20{,}000$.

FIGURE 1
CARRYING, ORDERING, AND TOTAL COSTS, AND ECONOMIC ORDER QUANTITY



This simplest EOQ model has been well studied and widely applied. However, the limitation of this model is that it can only be applied to one product in one inventory warehouse.

Multiple-Product Inventory Management

Not many cases or contents about multiple-product inventory management can be found in typical cost or managerial accounting instructional materials. However, in practice, many companies do have to deal with ordering multiple products for one distribution center or retailing store.

For instance, assume that a merchandising company has products A_1, A_2, A_3.., and A_n. Assume that the simplest EOQ model is applied, in which the ordering costs and carrying costs are considered. Then total costs can be formulated as

$$F = \frac{D_1 P_1}{Q_1} + \frac{Q_1 C_1}{2} + \frac{D_2 P_2}{Q_2} + \frac{Q_2 C_2}{2} + \ldots + \frac{D_n P_n}{Q_n} + \frac{Q_n C_n}{2} = \sum_{i=1}^{n} \left(\frac{D_i P_i}{Q_i} + \frac{Q_i C_i}{2}\right)$$

where D_i, P_i, Q_i and C_i are defined as demand, ordering cost, order number, and carrying cost for the *i*th product.

Theorem 4. $f(x)$ is a continuously real-valued function defined over an internal *I*, where $x \in I \subset R^n$. c is the critical value of $f(x)$. If $H(x)$ is the Hessian matrix of $f(x)$. If $H(c)$ is positive definite, then $f(c)$ is the absolute minimum value.

Proof. The proof to this theorem can be found in many multivariate calculus textbooks and might be beyond this topic. However, this theorem can be very useful in applications of optimization methods in inventory management.

Theorem 5. $H(x)$ is an $n \times n$ real-valued matrix. If all the eigenvalues of $H(x)$ are positive, then $H(x)$ is positive definite.

Proof. This proof requires knowledge in advanced linear algebra. It might be beyond this subject. Readers are encouraged to review optimization theories.

Theorem 6. The cost function as defined

$$F = \frac{D_1 P_1}{Q_1} + \frac{Q_1 C_1}{2} + \frac{D_2 P_2}{Q_2} + \frac{Q_2 C_2}{2} + \ldots + \frac{D_n P_n}{Q_n} + \frac{Q_n C_n}{2} = \sum_{i=1}^{n} \left(\frac{D_i P_i}{Q_i} + \frac{Q_i C_i}{2}\right),$$

where D_i, P_i, Q_i and C_i are defined as demand, ordering cost, order number, and carrying cost for the *i*th product. Assume that D_i, P_i and C_i are known constants. Then F has an absolute minimum at $Q_i = \sqrt{\frac{2 D_i P_i}{C_i}}$ for the *i*th product, $i = 1,2,\ldots n$.

Proof. F is a multivariate function defined over $Q = \begin{bmatrix} Q_1 \\ Q_2 \\ \ldots \\ Q_{n-1} \\ Q_n \end{bmatrix} \in I \subset R^n$. In order to find critical values of F, the first order derivative must be determined:

$$\frac{\partial F}{\partial Q}=\begin{bmatrix}\frac{\partial F}{\partial Q_1}\\ \frac{\partial F}{\partial Q_2}\\ \dots\\ \frac{\partial F}{\partial Q_{n-1}}\\ \frac{\partial F}{\partial Q_n}\end{bmatrix}=\begin{bmatrix}-\frac{D_1P_1}{Q_1^2}+\frac{C_1}{2}\\ -\frac{D_2P_2}{Q_2^2}+\frac{C_2}{2}\\ \dots\\ -\frac{D_{n-1}P_{n-1}}{Q_{n-1}^2}+\frac{C_{n-1}}{2}\\ -\frac{D_nP_n}{Q_n^2}+\frac{C_n}{2}\end{bmatrix}$$

. Then an equation is set up as:

$$\frac{\partial F}{\partial Q}=\begin{bmatrix}\frac{\partial F}{\partial Q_1}\\ \frac{\partial F}{\partial Q_2}\\ \dots\\ \frac{\partial F}{\partial Q_{n-1}}\\ \frac{\partial F}{\partial Q_n}\end{bmatrix}=\begin{bmatrix}-\frac{D_1P_1}{Q_1^2}+\frac{C_1}{2}\\ -\frac{D_2P_2}{Q_2^2}+\frac{C_2}{2}\\ \dots\\ -\frac{D_{n-1}P_{n-1}}{Q_{n-1}^2}+\frac{C_{n-1}}{2}\\ -\frac{D_nP_n}{Q_n^2}+\frac{C_n}{2}\end{bmatrix}=0$$

Hence, the only critical value for F is

$$Q=\begin{bmatrix}\sqrt{\frac{2D_1P_1}{C_1}}\\ \sqrt{\frac{2D_2P_2}{C_2}}\\ \dots\\ -\sqrt{\frac{2D_{n-1}P_{n-1}}{C_{n-1}}}\\ \sqrt{\frac{2D_nP_n}{C_n}}\end{bmatrix}=0$$

. In the meantime, the Hessian matrix is determined as the following,

$$H=\frac{\partial^2 F}{\partial Q^2}=\begin{bmatrix}\frac{\partial^2 F}{\partial Q_1^2} & \frac{\partial^2 F}{\partial Q_1\partial Q_2} & \dots & \frac{\partial^2 F}{\partial Q_1\partial Q_{n-1}} & \frac{\partial^2 F}{\partial Q_1\partial Q_n}\\ \frac{\partial^2 F}{\partial Q_2\partial Q_1} & \dots & \dots & \dots & \frac{\partial^2 F}{\partial Q_2\partial Q_n}\\ \dots & \dots & \dots & \dots & \dots\\ \frac{\partial^2 F}{\partial Q_{n-1}\partial Q_1} & \dots & \dots & \dots & \frac{\partial^2 F}{\partial Q_{n-1}\partial Q_n}\\ \frac{\partial^2 F}{\partial Q_n\partial Q_1} & \frac{\partial^2 F}{\partial Q_n\partial Q_2} & \dots & \frac{\partial^2 F}{\partial Q_n\partial Q_{n-1}} & \frac{\partial^2 F}{\partial Q_n^2}\end{bmatrix}=\begin{bmatrix}2\frac{D_1P_1}{Q_1^3} & 0 & \dots & 0 & 0\\ 0 & 2\frac{D_2P_2}{Q_2^3} & \dots & 0 & 0\\ \dots & 0 & \dots & 0 & \dots\\ 0 & 0 & \dots & 2\frac{D_{n-1}P_{n-1}}{Q_{n-1}^3} & 0\\ 0 & 0 & \dots & 0 & 2\frac{D_nP_n}{Q_n^3}\end{bmatrix}$$

This Hessian matrix H is a diagonal matrix, and all the diagonal entries are also its eigenvalues. These eigenvalues are all positive, and therefore H is positive definite. Theorem 4 applies and F has an absolute minimum at $Q_i=\sqrt{\frac{2D_iP_i}{C_i}}$ for the *i*th product, $i=1,2,..n$.

The above mathematical models and analyses provide a theoretical foundation about how to determine the economic-order-quantity EOQ for both single-product and multiple-product cases. Interested readers can also review and study related mathematics theories (Nocedal, J., 2006; Stewart, J., 2013).

Deterministic and Stochastic Models

The above studies assume that the market demand, carrying costs, and order costs are known and deterministic in models and analysis. However, in practice, these inputs of variables or parameters in EOQ models are normally expected values or predicted values. Applying these deterministic models will also give you estimates that might be close to the actual operating outcomes, but sometimes they will be very different. In both business applications and accounting education, scenario and sensitive analysis is an important topic, and we can study these issues by examining the effects of these input values on outcomes. The limitation of scenario and sensitive analysis is that it gives very limited information. With complex business operations, different and advanced simulation tools are needed to provide detailed analysis results to gain deep insights into businesses.

The method of stochastic simulations is a useful tool as is scenario and sensitive analysis. Normally, a deterministic model is derived first and then converted to a stochastic model. The inputs will be probabilistic distributions, and the process of stochastic simulations is then applied. A very time-consuming but important tool can facilitate this process—Monte Carlo simulations. Different from the sensitive and scenario analysis that tries a limited number of input variable values, Monte Carlo simulations use a large-scale number of trials and repeats the process many times based upon the distributions of input variables. When Monte Carlo simulations are applied, users can analyze the scenarios with lots of details, such as distributions, and gain deep insights into the model.

For instance, in the case of inventory management, the EOQ model, $Q=\sqrt{\frac{2DP}{C}}$, where D is annual demand in units, Q is the size of each order, P is ordering cost per purchase order, and C is carrying cost of one unit in stock for the time period used of D, is the derived deterministic model. Stochastic simulations can be studied by using distributions for these input variables. For example, P and C are fixed parameters, $P=\$200$, $C=\$1$. Then the deterministic model can be further simplified as $Q=\sqrt{\frac{2DP}{C}}=20\sqrt{D}$. Depending on D's distribution, Q can have the corresponding distribution outcome.

Applying Monte Carlo simulations provides a possibility to understand output distributions and assess risks in the businesses overall. The following numerical examples with graphic demonstrations can show how Monte Carlo simulations can be applied.

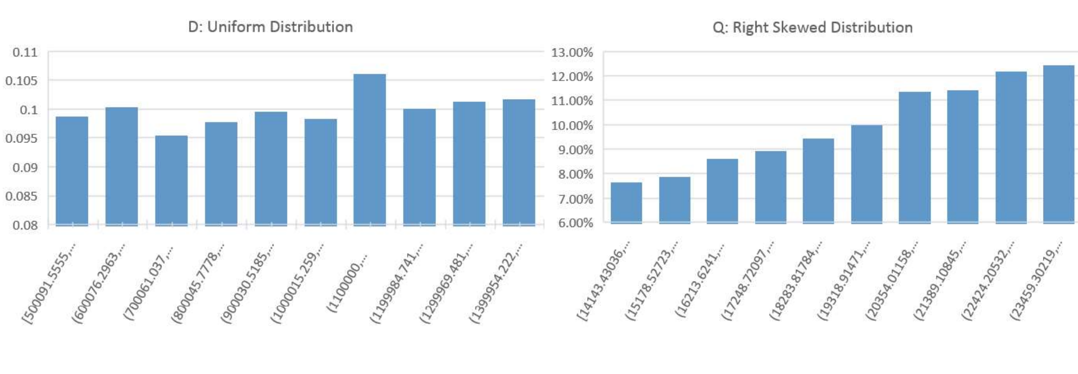
1) Assume that D is a uniform distribution on [500,000, 1,500,000] with pdf $f(d)=\frac{1}{1{,}000{,}000}$ and cdf $F(d)=\frac{d-500{,}000}{1{,}000{,}000}$.

In this case, the expected value of D is

$$E(d)=\int_{500{,}000}^{1{,}500{,}000} df(d)dd=\int_{500{,}000}^{1{,}500{,}000}\frac{d}{1{,}000{,}000}dd=\frac{d^2}{2{,}000{,}000}\Big|_{500{,}000}^{1{,}500{,}000}=1{,}000{,}000$$

Hence, the deterministic outcome of EOQ $Q=20\sqrt{D}=20{,}000$ using the expected value of *D.* After 10,000 simulations are performed, distributions can be illustrated for both input and output variables. In Figure 2, the top graph shows the input variable distribution of D, which is a uniform distribution. The bottom graph shows the output variable distribution of Q which is the right skewed distribution.

FIGURE 2
D: UNIFORM DISTRIBUTION AND Q: RIGHT SKEWED DISTRIBUTION

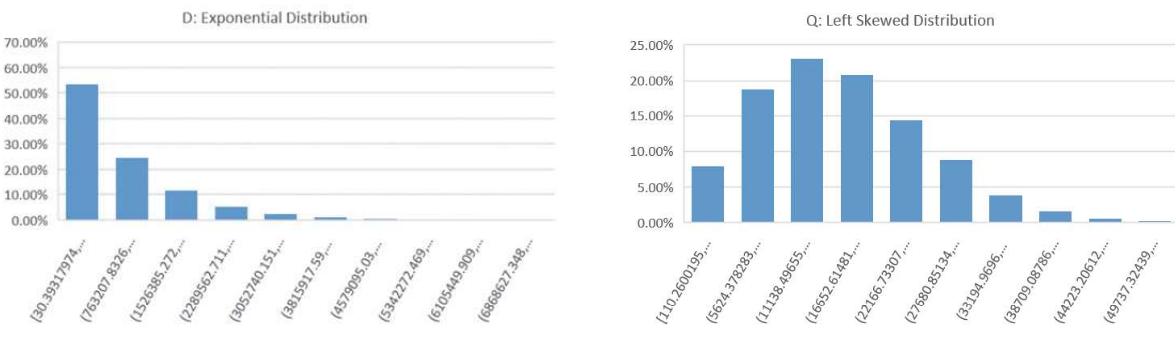


2) Assume that D is an exponential distribution with pdf $f(d) = 1{,}000{,}000e^{-1{,}000{,}000d}$ and cdf $F(d) = 1 - e^{-1{,}000{,}000d}$. In this case, the expected value of D is

$E(d) = \int_0^\infty df(d)dx = \int_0^\infty d1{,}000{,}000e^{-1{,}000{,}000d}dd = 1{,}000{,}000$. Hence the deterministic outcome of EOQ $Q = 20\sqrt{D} = 20{,}000$ using the expected value of D.

Similarly, 10,000 simulations are performed, and distributions can be illustrated for both input and output variables. In Figure 3, the top graph shows the input variable distribution of D, which is an exponential distribution. The bottom graph shows the output variable distribution of Q, which is the left skewed distribution.

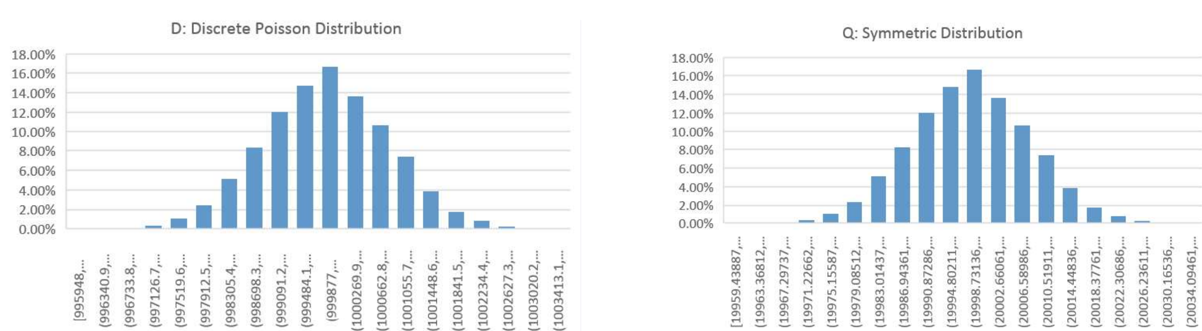
FIGURE 3
D: EXPONENTIAL DISTRIBUTION AND Q: LEFT SKEWED DISTRIBUTION



3) Assume that D is a discrete Poisson distribution with pdf $P(D = d) = \frac{\lambda^d e^{-1{,}000{,}000}}{d!}$. The expected value of D is $E(d) = \sum_{d=0} \frac{\lambda^d e^{-1{,}000{,}000}}{d!} = 1{,}000{,}000$. Hence the deterministic outcome of EOQ $Q = 20\sqrt{D} = 20{,}000$ using the expected value of D.

10,000 simulations are performed. In Figure 4, the top graph shows the input variable distribution of D which is a Poisson distribution; the bottom graph shows the output variable distribution of Q, which is a symmetric distribution.

FIGURE 4
D: DISCRETE POISSON DISTRIBUTION AND Q: SYMMETRIC DISTRIBUTION

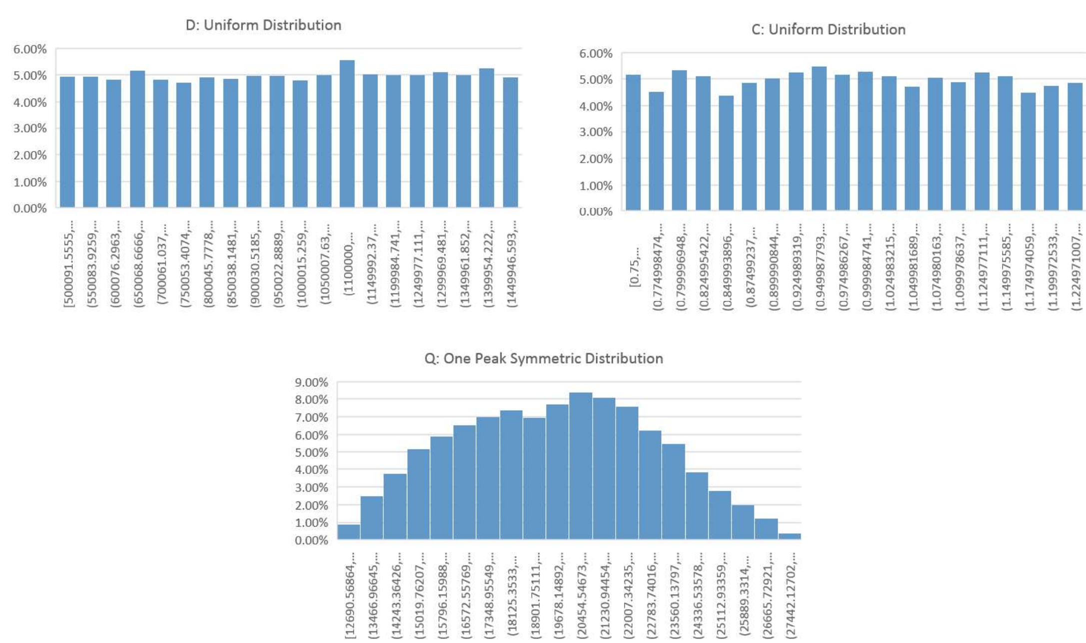


The above cases are simplified so that only one input is a random variable. However, in practice, it is possible that the original EOQ number $Q=\sqrt{\frac{2DP}{C}}$ has more than one input that is a random variable. Therefore, it can be particularly challenging to use scenario or sensitive analysis to study the EOQ outcomes. However, Monte Carlo simulations can be applied to study these complex stochastic models effectively and efficiently. The following numerical examples with graphic demonstrations can show how Monte Carlo simulations can be applied.

4) Assume that D is a uniform distribution on [500,000, 1,500,000] with pdf $f(d)=\frac{1}{1{,}000{,}000}$ and cdf $F(d)=\frac{d-500{,}000}{1{,}000{,}000}$. As shown above, the expected value of D is 1,000,000. Also assume that P is constant, where P= \$200 and C is a uniform distribution on [0.75, 1.25] with pdf $f(c)=\frac{1}{0.5}$ and cdf $F(c)=\frac{c-0.75}{0.5}$. The expected value of C is \$1. Normally D and C are independent random variables. Hence the deterministic outcome of EOQ $Q=\sqrt{\frac{2DP}{C}}=20{,}000$ using the expected value of D and C.

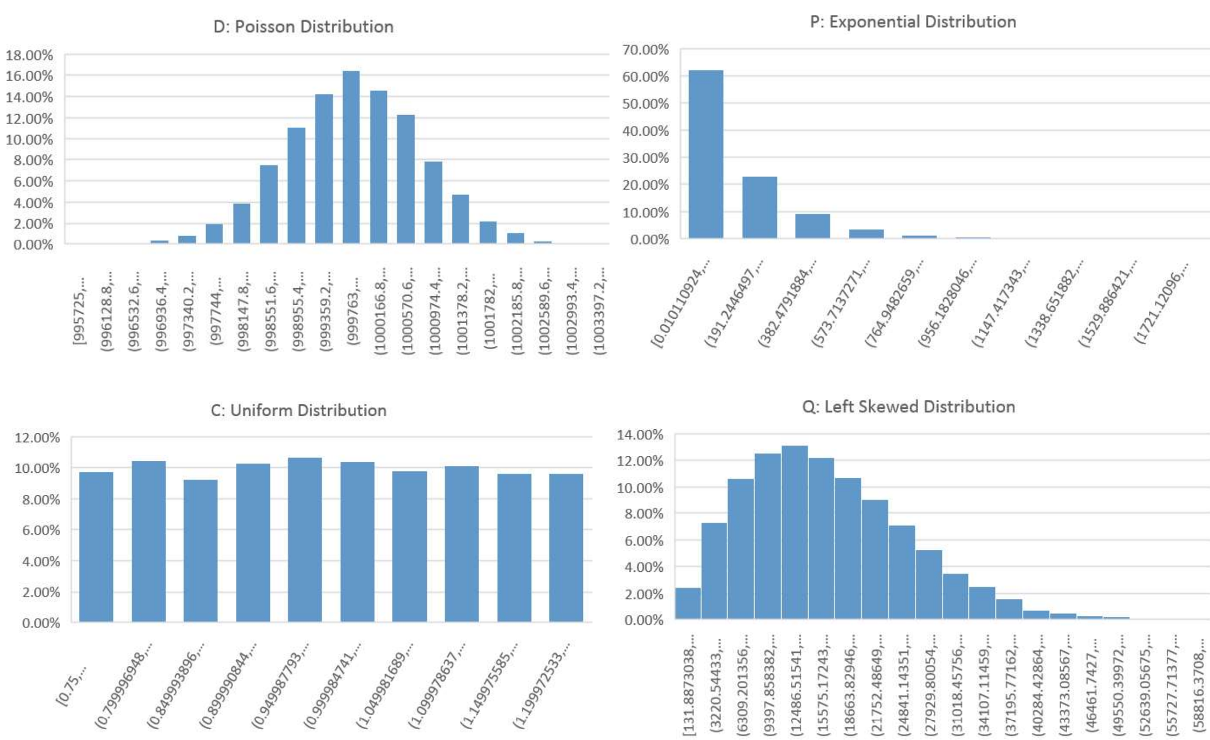
10,000 simulations are performed. In Figure 5, the top graph shows the input variable distribution of D which is a uniform variable distribution; the middle graph shows the input variable distribution of C which is a uniform variable; the right graph shows the output variable distribution of Q which is a symmetric distribution.

FIGURE 5
D: UNIFORM DISTRIBUTION, C: UNIFORM DISTRIBUTION AND
Q: ONE PEAK SYMMETRIC DISTRIBUTION



5) Assume that D is a discrete Poisson distribution with pdf $P(D=d)=\dfrac{\lambda^d e^{-1,000,000}}{d!}$. The expected value of D is $E(d)=\sum_{d=0}\dfrac{\lambda^d e^{-1,000,000}}{d!}=1,000,000$. Assume that P is an exponential distribution with pdf $f(p)=200e^{-200p}$ and cdf $F(p)=1-e^{-200p}$. The expected value of P is \$200. Assume that C is a uniform distribution on [0.75, 1.25] with pdf $f(c)=\dfrac{1}{0.5}$ and cdf $F(c)=\dfrac{c-0.75}{0.5}$. The expected value of C is \$1. Hence the deterministic outcome of EOQ $Q=\sqrt{\dfrac{2DP}{C}}=20{,}000$ using the expected value of D, P and C.

FIGURE 6
ILLUSTRATION OF D: POISSON DISTRIBUTION, P: EXPONENTIAL DISTRIBUTION, C: UNIFORM DISTRIBUTION AND Q: ONE PEAK SYMMETRIC DISTRIBUTION



When 10,000 simulations are performed, outcomes of EOQ Q will show the summary, and its graph will suggest a certain distribution. In particular, in Figure 6, the top left graph shows the input variable distribution of D that is a Poisson random variable; the top right graph shows the input variable distribution of P that is an exponential random variable; the bottom left graph shows the input variable distribution of C that is a uniform random variable; the bottom right graph shows the output variable distribution of Q that has a left skewed distribution.

For some distributions for D, it might be possible to derive the analytical results for Q.

Theorem 7. If X is a uniform random variable of (0,1) and $Y=\sqrt{X}$, then Y is a Beta function with Beta (2,1).

Proof. Since X is a uniform random variable, then pdf $f_X(x)=1$ and cdf $F_X(x)=x$. For $Y=\sqrt{X}$, the cdf of Y can be derived as $F_Y(y)=P(Y\le y)=P(\sqrt{X}\le y)=P(X\le y^2)=F_X(y^2)=y^2$ for $0<y<1$. Therefore, the pdf of Y is $f_Y(y)=F_Y'(y)=2y$.
Recall that Beta (2,1) has the pdf $f_X(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}=\frac{\Gamma(3)}{\Gamma(2)\Gamma(1)}x^{2-1}(1-x)^{1-1}=\frac{2!}{1!0!}x=2x$ for $0<x<1$. Note that $\Gamma(n)=(n-1)!$. Therefore, Y is a Beta(2,1) function.

Theorem 8. In the EOQ model, $Q=\sqrt{\frac{2DP}{C}}$ with fixed P and C. If D is a uniform random variable over [500000, 1500000], P= \$200 and C = \$1, then Q is a Beta random variable.

Proof. Since D is a uniform random variable, then pdf $f_D(d)=\frac{1}{1000000}$ and cdf $F_D(d)=\frac{d-500{,}000}{1{,}000{,}000}$. For $Q=20\sqrt{D}$ and $500{,}000<D<1{,}500{,}000$, the cdf of Q can be derived as

$$F_Q(q)=P(Q\le q)=P(20\sqrt{D}\le q)=P(D\le\frac{q^2}{400})=F_D(\frac{q^2}{400})=\frac{\frac{q^2}{400}-500{,}000}{1{,}000{,}000}$$

for $10000\sqrt{2}<q<10000\sqrt{6}$.

Therefore, the pdf of Q is $f_Q(q)=F_Q'(q)=\frac{q}{200{,}000{,}000}$ for $10000\sqrt{2}<q<10000\sqrt{6}$. Note that Theorem 7's argument also applies to show that Q is a Beta (2,1) variable.

The case 1) is exactly the application of Theorem 8. In addition, Figure 2 also demonstrates that Q is a right-skewed distribution and linear, and that is a feature of Beta (2, 1).

APPLICATIONS OF MONTE CARLO SIMULATIONS IN INVENTORY MANAGEMENT EXAMPLES

Med-Device Company Background

Med-Device Company is budgeting its next year's sales and inventories of two major products, M1 disposable medical gloves and M2 medical injection needles. The company is a retailing company that buys a large quantity of these products from suppliers and then sells them to local hospitals and medical centers. The demand for these medical products is very stable in recent years. Med-Device has studied their sales and demands and collected data for further uses. Med-Device used to over-order products to prevent stock-out and had a high-level buildup of inventories. However, Med-Device concluded its high waste of resources caused by poor cost control and inventory management. As a result, Med-Device decides to optimize its inventory management and adopts EOQ Models.

Economic Order Quantity Models

Med-Device considers only carrying costs and order costs in its model. For instance, Table 1 has a summary of these costs for products as well as annual demand of these products in the local region.

TABLE 1
SUMMARY OF COSTS IN EOQ MODEL

	M1-Disposable Medical Gloves	M2-Medical Injection Needles
Annual Demand	1,000,000 pairs	50,000 units
Ordering Cost/order	$100	$250
Carrying Cost/each	$.50	$1

Med-Device uses the deterministic models:

$Q_1 = \sqrt{\frac{2D_1P_1}{C_1}}$ for M1 products ; $Q_2 = \sqrt{\frac{2D_2P_2}{C_2}}$ for M2 products.

The calculations are straightforward. Table 2 has a summary of EOQ numbers and minimum costs.

TABLE 2
SUMMARY OF EOQ NUMBERS AND MINIMUM COSTS FOR M1 AND M2 PRODUCTS

	M1-Disposable Medical Gloves	M2-Medical Injection Needles
Annual Demand	1,000,000 pairs	50,000 units
Ordering Cost/order	$100	$250
Carrying Cost/each	$.50	$1
EOQ Quantity	20,000	5,000
Minimum Costs	$10,000	$5,000

Med-Device uses the expected value of each input variable in the formula from its records with some necessary adjustments. These results will give Med-Device some inputs and ideas how to budget its orders of products economically and to plan its inventory management. This analysis can help Med-Device in its operating and financial budgeting processes.

TABLE 3
SUMMARY OF INPUT VARIABLES WITH DISTRIBUTIONS

	M1-Disposable Medical Gloves	
	Expected Value	Distribution
Annual Demand	1,000,000 pairs	Poisson Distribution of 1,000,000
Ordering Cost/order	$100	Constant
Carrying Cost/each	$.50	Constant
EOQ Quantity	20,000	N.A
Minimum Costs	$10,000	N.A
	M2-Medical Injection Needles	
	Expected Value	Distribution
Annual Demand	50,000 units	Poisson Distribution of 50,000
Ordering Cost/order	$250	Uniform Distribution [$225 $275]
Carrying Cost/each	$1	Uniform Distribution [$.75 $1.25]
EOQ Quantity	5,000	N.A
Minimum Costs	$5,000	N.A

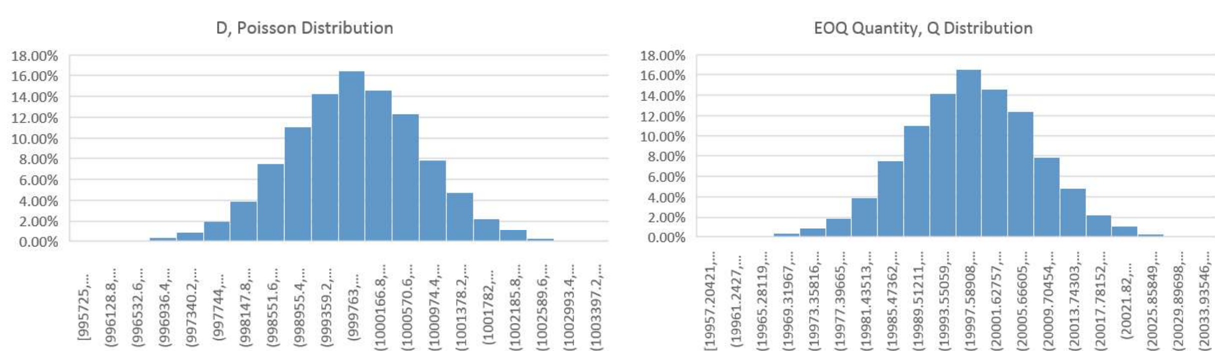
Risk Analysis and Stochastic Models

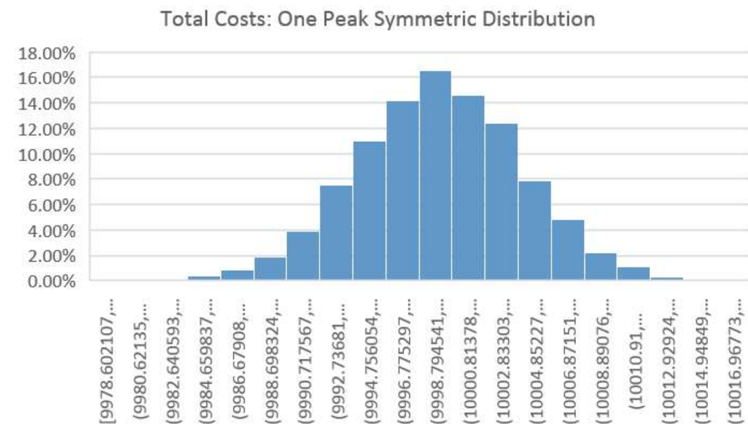
Med-Device also wants to be prepared for potential risks. Risk analysis becomes an important piece in inventory management and budgeting. Med-Device is not satisfied by sensitive analysis or scenario analysis; it prefers tools that are more advanced. With collected data and predictions of input variables, Med-Device uses stochastic models with Monte Carlo simulations to gain insights into cost control and inventory management. Med-Device has found that some input variables follow certain distributions as Table 3 summarizes.

Monte Carlo Simulation

Med-Device uses 10,000 iterations to simulate results of output. Graphs can present a range of outcome and probabilities that can be applied to risk analysis. Although in the planning and budgeting, expected values of input variables are used. Monte Carlo simulations can let Med-Device consider possible ranges of expected results. Figure 7 shows the histogram of simulations for M1 products.

FIGURE 7
M1 MONTE CARLO SIMULATION





Assume that D of M1 products has a Poisson distribution and that P and C are constant. Output variables Q and F—total costs—can be simulated using D. Simulation results show that both the EOQ number and minimum total costs are one-peak symmetric distributions. Figure 7 shows all graphs for input and output variables with distributions. It is also interesting to compare the deterministic results and stochastic results.

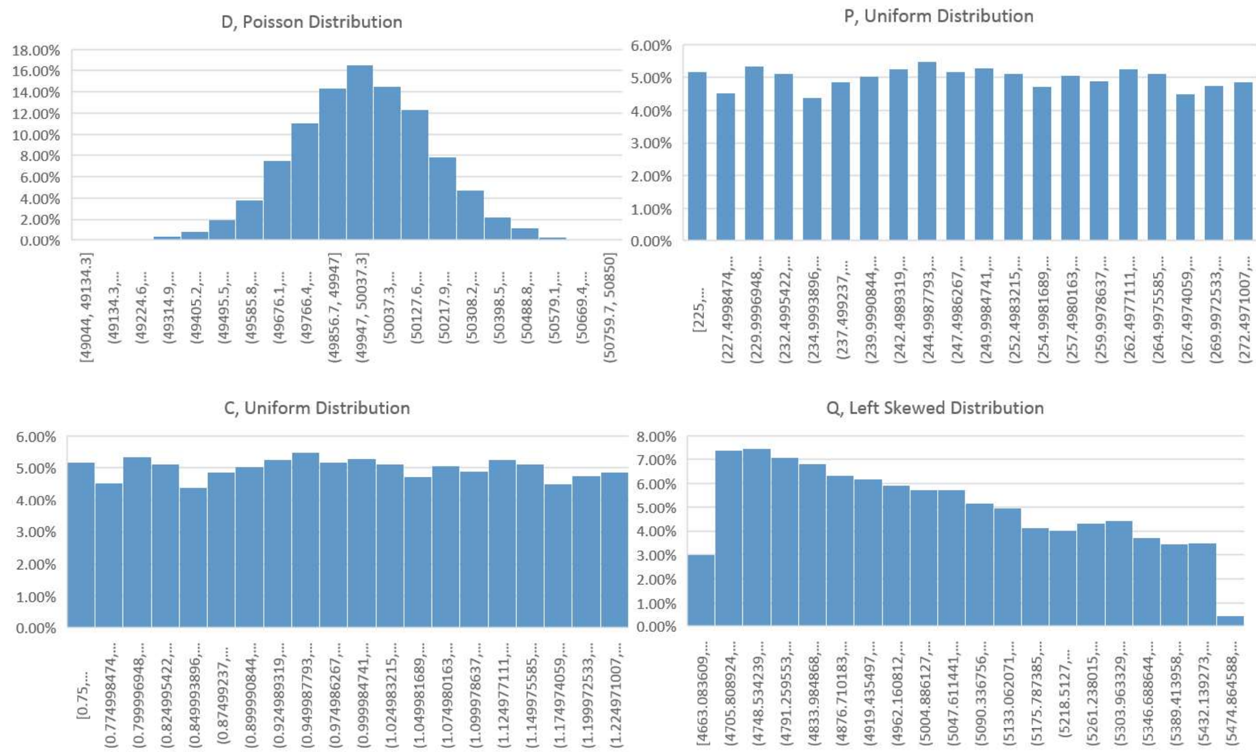
Table 4 compares these numerical results. Deterministic models using the expected value of inputs have very similar results of output EOQ and minimum total costs as stochastic models have. More interestingly, it is also possible to use simulation results to conclude 68.5% of possible outcomes falling within one standard deviation range and 95.1% of possible outcomes falling within two standard deviation range for both output EOQ and minimum total costs as shown in Table 5.

TABLE 4
SUMMARY OF M1 MONTE CARLO SIMULATIONS

	M1-Disposable Medical Gloves	
	Expected Value	Monte Carlo Simulation
Annual Demand	1,000,000 pairs	Poisson Distribution of 1,000,000
Ordering Cost/order	$100	$100
Carrying Cost/each	$.50	$.50
EOQ Quantity	20,000	20,000, symmetric distribution
Minimum Costs	$10,000	$10,000, symmetric distribution

Assume that M2 has a Poisson distribution for D, a uniform distribution for P, and a uniform distribution for C. Therefore, M2 has a more complex stochastic model for its EOQ number and minimum total costs. Simulation results indicate that the EOQ number is a left-skewed distribution and minimum total costs is more like a uniform distribution. Figure 8 shows all graphs for input and output variables with distributions. Table 6 compares numerical results of simulations for all input and output variables. The simulation results provide very useful information. 58.77% of possible outcomes fall within one standard deviation range, and 99.5% of possible outcomes fall within two standard deviation range for the output EOQ. 57.73% of possible outcomes fall within one standard deviation range, and 100% of possible outcomes fall within two standard deviation range for minimum total costs. Table 7 has a summary of the information.

FIGURE 8
M2 MONTE CARLO SIMULATION



Completed simulations have rich information for Med-Device in its business planning and budgeting. From risk analysis perspectives, Monte Carlo simulations consider input variables with distributions and provide Med-Device a more comprehensive analysis than the traditional scenario or sensitivity analysis does. In Med-Device inventory management and financial budgeting, Med-Device can use ranges generated by applying expected values and standard deviations. These ranges can allow Med-Device to consider unexpected changes in input variables such as demand changes, carrying cost changes, and ordering cost changes. Med-Device can, therefore, be better prepared in its operating budget, financial budget, and cash flow. However, scenario or sensitivity analysis fails to give such information.

TABLE 5
PERCENTAGES OF INTERVALS FOR M1 SIMULATION RESULTS

	M1-Disposable Medical Gloves	
	EOQ Quantity	Minimum Costs
Expected Value	20,000	$10,000
Standard Deviation	10	$5
Within One Standard Deviation	68.51%	68.51%
	[19990, 20010]	[9995,10005]
Within Two Standard Deviations	95.15%	95.14%
	[19980,20020]	[9990,10010]

TABLE 6
SUMMARY OF M2 MONTE CARLO SIMULATIONS

	M2- Injection Needles	
	Expected Value	Monte Carlo Simulation
Annual Demand	50,000 Units	Poisson Distribution of 50,000
Ordering Cost/order	$250	$250
Carrying Cost/each	$1	$1
EOQ Quantity	5,000	5,028, Left Skewed Distribution
Minimum Costs	$5,000	$4,992, Uniform Distribution

TABLE 7
PERCENTAGES OF INTERVALS FOR M2 SIMULATION RESULTS

	M2- Injection Needles	
	EOQ Quantity	Minimum Costs
Expected Value	5,028	$4,992
Standard Deviation	223	$502
Within One Standard Deviation	58.77%	57.73%
	[4805, 5251]	[4490,5494]
Within Two Standard Deviations	99.5%	100%
	[4582,5474]	[3988,5996]

DISCUSSIONS AND CONCLUSIONS

This paper reviews one-product inventory management and multiple-product inventory management cases. Inventory Management a very important subject as materials or merchandise costs account for a large portion of total costs faced by manufacturing and merchandising companies. Building inventories high can be too risky for any company facing financial difficulties such as cash flow problems. In the meantime, maintaining low-level inventories may result in stock-out and back-order outcomes that may hurt businesses in the long run. An effective and efficient inventory management system always aims at minimizing total costs caused by inventory related costs and maintaining healthy level inventories.

The theoretical analysis in this paper demonstrates how to find optimal solutions to inventory related problems with given conditions. Due to complex models and conditions, deterministic models are popular and widely applied. Companies can incorporate these models and results in their inventory planning and financial budgeting. However, deterministic models fail to assess risks or unexpected outcomes effectively and efficiently, and that may create problems for businesses. Scenario and sensitive analysis can shed a light on possibly unexpected outcomes but cannot provide a comprehensive analysis or may provide a misleading result.

With the development of computer technology and data analytics software, it becomes possible to integrate input variables with various distributions observed or predicted into financial analysis. Moreover, Monte Carlo simulations apply these distributions and can better serve decision makers and financial analysts. Tools and methods shown in this paper will not only provide a useful analysis in the real world but also allow users to assess business risks. The numerical results in this paper have demonstrated the merits of stochastics models with Monte Carlo simulations in inventory management. A company like Med-Device that wants to have a better inventory management system for its products can benefit from stochastic simulations without limits.

In summary, this paper provides accounting researchers and practitioners with theoretical developments, examples of deterministic and stochastic models, numerical results, and analysis about optimal inventory management. The analysis and tools shown in the paper can be applied to real business problems. Future research will include studies of multiple products in multiple warehouses and activity-based costing systems. With more data becoming available, Big Data platforms and analysis tools are being integrated into applications of stochastics simulations, which can impact inventory management and analysis greatly.

CONTRIBUTIONS AND LIMITATIONS

This study may have important implications for the body of inventory management in cost accounting. First, this study extends the current knowledge of inventory management, especially concerning cost accounting issues. This study serves as an addition to building a cumulative tradition of research on inventory management in cost accounting. Second, this study attempts to use stochastic simulations to optimize inventory management. There are not many case studies or instructional materials in cost accounting employing stochastic simulations to inventory management. This study serves as a good addition. Finally, this study may provide a framework for the development of an instrument for inventory management optimization in cost accounting. Only a few studies have been published on the development of an instrument to evaluate the inventory management optimization. Thus, the measurement scales utilized in this study can serve as the starting point for further refining the instrument of inventory management optimization in cost accounting.

However, this study may also have some limitations. The possibility of a biased perception of stochastic simulations should be considered. As a means of performance improvement in inventory management, stochastic simulations have been utilized as a major tool or technique through various academic writings, especially in management science fields. As a result, stochastic simulations have been viewed as one of the major catalysts for optimizing inventory management without practical assessment of the actual impact of its implementation in businesses. Second, it is necessary for a researcher to make tradeoffs between the explanatory power and scope of research. Although this study attempts to reasonably infer variables, actual practicability in cost accounting curriculum may still be regarded as a potential threat to internal validity. Finally, this study is a strong mathematic theory based research work. The assumptions in the model still need to be widely tested in real business world. Because a theory based research study does not fully address practicability issues, it may not capture the complex interrelationships between variables that come into effect over time in real business applications.

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