An Input Output Analysis Model for the Spanish Economy Based on Working Hours

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In labor economics, the input-output models have had little development. In the second half of the 20th century, various input-output analyses focused on the use of time in the German economy were initiated and resulted in the Input output tables of time (TIOT). However, despite the fact that this methodology has been included in the manual published by Eurostat on the elaboration of input-output frameworks, there is little research and analysis with this type of tables.

In this study we obtained the TIOT for the Spanish economy in 2016 and on the basis of the analysis of Passinetti and Saffra on interindustry relationships,, it presents an analysis of input-output on the basis of the TIOT which gives rise to an open model of Leontief, and a model of shadow prices for the hours worked, which can be used for the analysis of the impacts on the economy that lead to changes in the distribution of hours worked in consequence of the extension of new technologies.

Keywords: time input output table, price input-output model, shadow-pricing inputs

INTRODUCTION

Since the initial impetus of Wassily Leontief and Richard Stone, numerous developments, models and applications have been based on either the Input Output Table (IOT) or the Social Accounting Matrix (SAM). In fact, Stone (1985) pointed out that the construction of input-output tables had been systematized in relation to the development of national accounting. For his part, in relation to the statistical development of the tables, he cited numerous works on the stability, adjustment and projection of technical coefficients, on prices, on capital coefficient matrices and on regional tables. With regard to the development of Leontief's open input-output model, he referred to the processes of endogenization of components of final demand (in particular of household consumption), the generalization of production functions (using changes in coefficients, functions with intermediate and primary factors, or cost functions according to the proposal for generalization by Diewert, 1971), and work on dynamic aspects of the model, both theoretical and applied in the context of simulation, control and optimization. And on the extensions of the input-output model, he highlighted the issue of environmental pollution (with coefficients of pollutant emissions and decontamination industries), income distribution (in the broader context of social accounting matrices), property and financial flows, and international trade (in the multinational/multisectoral models, among which Leontief's model of the world economy stands out. Miller and Blair (2009) are a good reference for understanding the historical evolution of Leontief's model and the many methodological extensions it has given rise to.

In the field of labor economics, however, input-output models have had a much more limited development. It was in the second half of the 20th century that various input-output analyses focusing on the use of time in the German economy were initiated, leading to the Time Input Output Table (TIOT) (Stänglin. R.;1973). However, despite the fact that this methodology was included in the manual published by Eurostat on the elaboration of input-output frameworks (Eurostat, 2008), there is hardly any research and analysis with this type of table.

In this article, the TIOT for the Spanish economy in 2016 are calculated. Based on Passinetti and Saffra's analysis of inter-industry relations, an input-output analysis model is proposed on the basis of the TIOT, which results in shadow prices for the hours worked, which in the fourth section are calculated for the Spanish economy.

The Leontief Model

Pasinetti L. (1983) explains the input-output model with the help of an example taken from Saffra (1960). It is an economic system limited to the production of three goods: grain (g), iron (f) and turkeys (t). The systematization of the operations of this system in physical terms gives rise to the following table:

TABLE 1
FLOWS OF GOODS IN PHYSICAL TERMS

	g	f	t	
g	240	90	120	450
f	12	6	3	21
t	18	12	30	60
450	21	60		

The matrix reflects in the first row the quantities of grain that grain producers sell to other grain or iron and turkey producers, in the second row the quantities of iron that iron producers sell to grain and turkey producers, and so on. By columns, we would have the purchases that grain producers make from grain, iron, and turkey producers, and so on.

The above table does not specify the use to which the goods will be put. Logically, one part of the goods will be used as a means of production and another part as a consumer good. In the example, it is assumed that the system employs 60 workers, 18 in the grain industry, 12 in the iron industry and 18 in the turkey industry. Since each worker consumes, on average, three quintals of grain and half a gross of turkey, the part of the production that is used for consumer goods is determined. Starting from the previous table we build another one in which such differentiation is made:

TABLE 2
FLOWS OF GOODS AND LABOR

	g	f	t	final sector	
g	186	54	30	180	450
f	12	6	3		21
t	9	6	15	30	60
Final sector	18	12	30		60
450	21	60			

In the table the productions are expressed in different units, grain and iron in quintals, and turkey in grosses. For this reason, the productions corresponding to each column cannot be added up. Given that trade between sectors requires a relationship between goods or prices, which in the example is established on the basis of 10 quintals of grain for one quintal of iron, for two grosses of turkey and 1,81818 man-years of work. Taking as a reference, 1, for the price of a quintal of iron; the price of a quintal of grain will be

0.1; the price of a quintal of turkey will be 0.5, and the annual wage per worker will be 0.555. Using these prices, the table is prepared in terms of values, where the sums per row coincide with the sum of columns.

TABLE 3
FLOWS OF GOODS AND SERVICES IN TERMS OF UNIT MEASUREMENT

	g	f	t	final sector (consumer)	Grand totals
g	18.6	5.4	3	18	45
f	12	6	3		21
t	4.5	3	7.5	15	30
Final sector (Value added)	9.9	6.6	16.5		(33)
45	21	30	(33)	96	

This table is called the inter-industry transaction matrix or **input-output table**. The value added is the surplus of the system, net product of the economic system or national income, which in this system is destined only to satisfy the consumption needs of the workers.

An input-output table as it is currently done is a more complex operation than the one described in the "Lessons on Production Theory", two types of tables are produced, the origin and the destination ones. In those of origin, the main and secondary productions are presented by branch of activity, and in that of destination, the matrix of inter-industrial flows is presented (Table nº 4). However, the table of destination differentiates the following sub-tables or matrices (Quadrants):

- Intermediate consumption differentiating between domestic (Quadrant I) and imported (Quadrant III)
- Value Added and its Components (Quadrant V)
- Domestic production
- Final demand differentiated by uses and by origin of employment between domestic (Quadrant II) and imported (Quadrant IV)

TABLE 4
DIAGRAM OF AN INPUT=OUTPUT TABLE

	Homogeneous branches	Components of final demand	Total
Domestic products	Quadrant I	Quadrant II	Domestic production
Imported products	Quadrant III	Quadrant IV	Imports
GVA and its components	Quadrant V		Value added
Total	Domestic production	Final demand	

But an input-output table is not an economic model per se, it is an analytical representation as complete as possible of the flows of goods and services that occur between the actors in an economic system. We speak of a **Leontief model** when we start from the fact that the economic system represented in the table is stationary, that is, it reproduces itself in the same way year after year, and this implies that the labour force employed by each sector is fixed, as is the technical knowledge or technology that determines that certain quantities of other goods are needed for the production of a given good. It also requires that consumption decisions do not vary from year to year, that goods are purchased in the same markets of origin, and that employers settle for the same rate of profit.

Leontief's model is therefore an analytical reworking of an input-output table (IOT), specifically a symmetrical input-output table that takes the following independent matrices from the representation of table no. 4: the matrix of intermediate domestic consumption (r), the matrix of final demand and the matrix of primary inputs. The intermediate consumption matrix accounts for the terms of trade between the different production branches. The final demand matrix records the part of the production of goods and services that is destined for final users (consumption demand, investment demand and external demand for goods produced in the national economy). And finally, the matrix of primary inputs where the payments made by companies and administrations for using the factors originating in production (income from work and business surpluses) are recorded. The matrix of primary inputs provides the Value Added of each branch, which is obtained by deducting the total of intermediate consumption from the value of the production. Each element x_{ij} of the matrix of intermediate consumption collects the consumption of products of branch i that branch j makes. If this consumption originates from companies resident in the territorial reference area of the input-output table, in other words, it is of an internal nature, it is referenced with the superscript r, and those imported from non-resident units are referenced with the superscript m. The output of a branch (X_j) is obtained as the sum of the elements that appear in each column: intermediate consumption of resident units, imports and value added (V). By rows, the destinations of domestic production (X_i) and imports (M_i) are shown. These destinations are intermediate demand (purchases made by other sectors) and final demand (D_i).

Given the accounting equilibrium of an IOT, where the production value by columns has to be equalized with the distributed or used production in each row, the formal structure of the IOT is represented through the following system of linear equations:

$$x_{11}^r + x_{12}^r + \ldots + x_{1n}^r + D_1^r = X_1$$

$$x_{21}^r + x_{22}^r + \ldots + x_{2n}^r + D_2^r = X_2$$

$$x_{n1}^r + x_{n2}^r + \ldots + x_{nn}^r + D_n^r = X_n$$

We define the technical coefficient a_ij as the ratio between the quantity consumed of an input and the production value of a branch: $\frac{x_{ij}}{X_j}$

And we obtain a new system of equations:

$$a_{11}^r X_1 + a_{12}^r X_2 + \ldots + a_{1n}^r X_n + D_1^r = X_1$$

$$a_{21}^r X_1 + a_{22}^r X_2 + \ldots + a_{2n}^r X_n + D_2^r = X_2$$

$$\ldots$$

$$a_{n1}^r X_1 + a_{n2}^r X_2 + \ldots + a_{nn}^r X_n + D_n^r = X_n$$

This new system of equations, in matrix notation, is expressed by:

$$A^r X + D^r = X$$

By operating conveniently it becomes:

$$D^r = (I - A^r)X$$

where, I is the Identity matrix and

$$X = (I - A^r)^{-1} D^r$$

The matrix $(I - A^r)^{-1}$ is known as the Leontief inverse matrix, whose elements A_{ij}^r are a measure of the production effort required of branch i by branch j to supply one unit of final demand of branch j. Each element of the Leontief inverse matrix thus represents the cumulative effects (direct and indirect) underlying the production structure that the IOT represents.

The Leontief model that we have developed is the one used for the Tables that today are elaborated by the statistical services, but we cannot forget that in its initial formulation the Leontief model considered that the intersectorial transactions were the product of physical quantities q_{ij} and their prices p_i, so that each $x_{ij} = q_{ij}p_i$, in the same way that the monetary value of what is produced in sector j would be $X_j = Q_j p_j$.

Taking into account this new notation, the system of equations described above would now be formulated (domestic and imported origins are dispensed with in order to simplify the notations)

$$q_{11}p_1 + q_{12}p_1 + \cdots + q_{1n}p_1 + d_1 p_1 = Q_1 p_1$$

$$q_{21}p_2 + q_{22}p_2 + \cdots + q_{2n}p_2 + d_2 p_2 = Q_2 p_2$$

$$q_{n1}p_n + q_{n2}p_n + \ldots + q_{nn}p_n + d_n p_2 = Q_n p_2$$

if we define the technical coefficient a_{ij} as the ratio between the physical quantity consumed of an input and the physical output of a branch: $\frac{q_{ij}}{Q_j}$ and we obtain a new system of equations:

$$a_{11}Q_1 + a_{12}Q_2 + \cdots + a_{1n}Q_n + d_1 = Q_1$$

$$a_{21}Q_1 + a_{22}Q_2 + \cdots + a_{2n}Q_n + d_2 = Q_2$$

$$a_{n1}Q_1 + a_{n2}Q_2 + \cdots + a_{nn}Q_n + d_n = Q_n$$

which in matrix notation, is expressed by:

$$AQ + D = Q$$

and the solution:

$$Q = (I - A)^{-1} D$$

The only difference with the previous model is that the goods are valued in physical units or, alternatively, in unit value indices.

The Leontief model includes another system of equations, which refers to prices, in the IOT each column is a function of production, which also gives rise to a system of equations:

$$q_{11}p_1 + q_{21}p_2 + \cdots + q_{n1}p_n + VAB_1 = Q_1 p_1$$

$$q_{12}p_1 + q_{22}p_2 + \cdots + q_{n2}p_n + VAB_2 = Q_2 p_2$$

$$q_{1n}p_1 + q_{2n}p_2 + \cdots + q_{nn}p_n + VAB_n = Q_n p_n$$

which can also be formulated in terms of technical coefficients of physical units:

$$a_{11}p_1 + a_{21}p_2 + \ldots + a_{n1}p_n + \frac{VAB_1}{Q_1} = p_1$$

$$a_{12}p_1 + a_{22}p_2 + \ldots + a_{n2}p_n + \frac{VAB_2}{Q_2} = p_2$$

$$a_{1n}p_1 + a_{2n}p_2 + \ldots + a_{nn}p_n + \frac{VAB_n}{Q_n} = p_n$$

Considering now l' the vector whose i-th element indicates for each sector the proportion that the GVA represents of the total production Q_i:

That is, if: $v = (v_1, v_2, \ldots, v_n)$, $v_i = \frac{VAB_i}{Q_i}$

Therefore, in matrix notation, that would be:

$$A'P + v' = P$$

To which the solution is:

$$P = (I - A')^{-1}v'$$

This solution is known as the **Leontief pricing model**.

Input-Output Tables in Working Hours (TIOT)

The hours of work that have been carried out in each sector can be distributed between the intermediate and final uses of the production. That is to say, taking as a reference an Input-Output table valued in monetary terms:

$$x_{11}^r + x_{12}^r + \cdots + x_{1n}^r + D_1^r = X_1$$

$$x_{21}^r + x_{22}^r + \cdots + x_{2n}^r + D_2^r = X_2$$

$$x_{n1}^r + x_{n2}^r + \ldots + x_{nn}^r + D_n^r = X_n$$

considering $H_1, H_2, \ldots, H_n$ the vector of hours worked in each sector, the distribution by use of the hours worked would be:

$$H_1 \frac{x_{11}^r}{X_1} + H_1 \frac{x_{12}^r}{X_1} + \cdots + H_1 \frac{x_{1n}^r}{X_1} + H_1 \frac{D_1^r}{X_1} = H_1$$

$$H_2 \frac{x_{21}^r}{X_2} + H_2 \frac{x_{22}^r}{X_2} + \cdots + H_2 \frac{x_{2n}^r}{X_1} + H_2 \frac{D_2^r}{X_2} = H_2$$

$$H_n \frac{x_{n1}^r}{X_n} + H_n \frac{x_{n2}^r}{X_n} + \ldots + H_n \frac{x_{nn}^r}{X_n} + H_n \frac{D_n^r}{X_n} = H_n$$

defining $h_i j = H_i \frac{x_{ij}^r}{X_j}$ as the hours destined to produce the goods i demanded by sector j, and $D_j^h = H_j \frac{d_j^r}{X_j}$ the hours destined to satisfy the final demand of good j, we have an input-output table expressed in working hours, whose notation would be:

$$h_{11} + h_{12} + \cdots + h_{1n} + D_1^h = H_1$$

$$h_{21} + h_{22} + \cdots + h_{2n} + D_2^h = H_2$$

$$h_{n1} + h_{n2} + \ldots + h_{nn} + D_n^h = H_n$$

Taking to the quotient $P_j^h = \frac{X_j}{H_j}$ the value of merchandise j of each hour (or thousands of hours) worked in the sector. The multiplication of each h_ij by each P_j^h will give as a result the interindustrial consumption $x_i^r j$, just as it will occur with the product of the worked hours for the final demand of j D_j^h and the value P_j^h, from which the final demand of j (D_j^r) will result.

Transforming the constructed table into hours of work to monetary units (millions of euros) involves a commodity price for each hour of work performed in each sector, or shadow price of the hour worked, since it includes all costs (including the remuneration of the employers) that the production system performs to put one hour of final production on the market.

Starting from the table of worked hours, we elaborate the **open Leontief model**, defining the technical coefficient a_{ij}^h as the relation between the consumed quantity of an input and the production value of a branch: $\frac{h_{ij}}{H_j}$ and we obtain a new system of equations:

$$a_{11}^h H_1 + a_{12}^h H_2 + \ldots + a_{1n}^h H_n + D_1^h = H_1$$

$$a_{21}^h H_1 + a_{22}^h H_2 + \ldots + a_{2n}^h H_n + D_2^h = H_2$$

$$\ldots$$

$$a_{n1}^h H_1 + a_{n2}^h H_2 + \ldots + a_{nn}^h H_n + D_n^h = H_n$$

This new system of equations, in matrix notation, is expressed by:

$$A^h H + D^h = H$$

By operating conveniently, it becomes:

$$D^h = (I - A^h)H$$

where, I is the Identity matrix and

$$H = (I - A^h)^{-1} D^h$$

The elements A_{ij}^h of the matrix $(I - A^h)^{-1}$ constitute a measure of the effort in working hours required from branch i by branch j to produce one hour of final demand.

An Input-Output Model Based on TIOT

Let us now try the exercise of drawing up a table for a modern economic system. Our table must therefore include a production model in which the production of each sector is the result not only of the physical capital invested, but also of a workforce with different qualifications and a production technology determined by the combination of raw materials, intermediate goods and services required to place a final production on the market. Production takes place in a context of determining the price of labor on the basis of a wage agreement, in which employers and labor unions intervene, and a price of products based on a target or margin on working capital required by the production process. Consumption patterns, as occurs in a SAM, will end up being determined by the social class of the households, which in this simplification will be based on the qualification levels of the employees.

For didactic purposes, we will consider an economic system that, as in Passinetti (1985), we will limit to three goods and services the production of three goods: grain (g), iron (f) and services (s). The systematization of the operations of this system in physical terms gives rise to the following table of hours worked:

TABLE 5
TABLE OF HOURS WORKED (THOUSANDS)

	g	f	s	FD	Total
g	900	100	0	1000	2000
f	10	800	20	270	1000
s	0	10	40	450	500
2000	1000	500			

The matrix reflects in the first row the hours that the grain producers self-consume and those that the production incorporates in selling to iron and service producers, and so on.

Taking into account the qualifications required in each sector.

TABLE 6
TABLE OF STAFF QUALIFICATIONS

	g	f	s
unqualified	1	1	0
qualified	0	0.02	1
managers	0	0.01	0.04

The hours offered for each qualification will then be:

TABLE 7
HOURS WORKED BY SECTOR

	g	f	s
unqualified	2000	970.873786	0.00000
qualified	0	19.417476	480.76923
managers	0	9.708738	19.23077

If the full-time working period in a year is 1760 hours, the necessary workers will be:

TABLE 8
WORKERS BY SECTOR

	g	f	s
unqualified	1136.364	551.632833	0.00000
qualified	0.000	11.032657	273.16434
managers	0.000	5.516328	10.92657

The company is made up of families of unqualified, qualified and managerial workers, and each one of them will consume the goods produced but in different quantities. We will assume that the unqualified workers will consume more wheat than the other workers, that the managers consume less manufactures (iron), while they hire more service days (restaurants, doctors, sanitary, etc.) than the other workers:

TABLE 9
TABLE OF HOUSEHOLD CONSUMPTION

	unqualified	qualified	managers
g	2	1	0,5
f	2	2	1
s	0	2	4

The quantities consumed by households, expressed in units, would be:

	Quantities consumed by households
g	3668.411
f	3960.830
s	2322.162

The working hours required to produce one unit of each service and product consumed would therefore be:

	Working hours per unit of service and product consumed
g	0.2725976
f	0.0681675
s	0.1937849

In other words, each unit of grain consumed required 0.27 working hours, the manufacture of iron 0.07 working hours, and services 0.19 working hours.

We are going to incorporate a pricing model, for these we are going to establish some assumptions about how this society is organized and sets its prices.

Firstly, there is a wage negotiation that signs wage agreements (conventions) between workers and employers. There are therefore three collective agreements, one for each sector, which, taking as a reference 1 monetary unit of annual salary for grain workers, have reached the following agreements:

TABLE 10
TABLE OF WAGES IN COLLECTIVE CONVENTIONS

	unqualified	qualified	managers
g	1	0	0
f	1	1,2	1,5
s	0	1,2	1,5

With the wage table, the Workers' Remuneration (RA) is determined:

	RA
g	1136.3636
f	573.1465
s	344.1871

In terms of hours, each sector's pay will be:

	RA per Hour worked
g	0.5681818
f	0.5731465
s	0.6883741

To assess flows between sectors and consumption in the final sectors, hypotheses have to be made about how prices would be determined. This hypothesis, taking the ideas of Safra's model (1960), is based on the idea that companies obtain a surplus margin on the working capital that they count at the beginning of the productive process, this working capital, is what they advance to acquire the raw materials and services that are needed, and to pay the workers, considering $RA_j = w_j L_j$, the hypothesis is formulated as follows:

$$S_j = (x_{1j}P_1 + x_{2j}P_2 + \ldots + x_{nj}P_n + RA_j)\pi$$

where π is the margin they are trying to get for the expected cash flow. That can also be presented with the technical coefficients in hours $a_{i,j}^h$:

$$\frac{S_j}{H_j} = (a_{1j}^h P_1 + a_{2j}^h P_2 + \ldots + a_{nj}^h P_n + \frac{RA_j}{H_j})\pi$$

By adapting Leontief's price model to the behavior of producers in the economic system, the equation for sector "j" would become:

$$P_j = a_{1j}^h P_1 + a_{2j}^h P_2 + \ldots + a_{nj}^h P_n + (a_{1j}^h P_1 + a_{2j}^h P_2 + \ldots + a_{nj}^h P_n + \frac{RA_j}{H_j})\pi + \frac{RA_j}{H_j})$$

it can be simplified,

$$P_j = (a_{1j}^h P_1 + a_{2j}^h P_2 + \ldots + a_{nj}^h P_n)(1+\pi) + (\frac{RA_j}{H_j})(1+\pi)$$

or if desired:

$$\frac{P_j}{1+\pi} = a_{1j}^h P_1 + a_{2j}^h P_2 + \ldots + a_{nj}^h P_n + \frac{RA_j}{H_j}$$

In matrix form, a new identity matrix (I_π) should be defined, such as:

$$I_\pi = \begin{bmatrix} \frac{1}{1+\pi} & 0 & \ldots & 0 \\ 0 & \frac{1}{1+\pi} & \ldots & 0 \\ . & . & \ldots & . \\ 0 & 0 & \ldots & \frac{1}{1+\pi} \end{bmatrix}$$

And the solution to the pricing model would be:

$$P = (I_\pi - A^{h'})^{-1} \frac{RA}{H}$$

In our example, we build the technical coefficient matrix A^h:

0.450	0.10	0.00
0.005	0.10	0.04
0.000	0.01	0.08

Assuming a margin of π=0.05, we build I_π to be:

0.952381	0.000000	0.000000
0.000000	0.952381	0.000000
0.000000	0.000000	0.952381

The matrix $((I_\pi - A'))$ would be

0.502381	-0.005000	0.000000
-0.100000	0.852381	-0.010000
0.000000	-0.040000	0.872381

Its inverse:

1.9928495	0.0116962	0.0001341
0.2339238	1.1751887	0.0134710
0.0107258	0.0538842	1.1469059

The prices would be calculated:

The solution of the price model results in the shadow price of the hour of grain work being 1.14 agricultural wage units, that of iron 0.81 agricultural wage units and each hour of services is remunerated at 0.82 agricultural wage units.

Table nº 11 finally shows the table of Flows of goods and services in monetary units (in this case in terms of the annual salary of unqualified workers) which is what the statistical services of that economic system would obtain when they prepare the destination table.

TABLE 11
FLOWS OF GOODS AND SERVICES IN TERMS OF UNIT OF MEASUREMENT

	g	f	t	final sector (consumption)	Grand totals
g	1025.187087	113.909676	0.000000	1139.0968	2278.1935
f	8.157397	81.573969	16.31479	220.2497	815.7397
s	0.000000	8.264781	33.05912	371.9151	413.2390
RA	1136.3636	573.1465	344.1871		(2053.697)
GOS	108.48541	38.84475	19.67805		(167.0082)
GVA	1244.8490	611.9913	363.8651		(2220.705)
Total	2278.1935	815.7397	413.2390		(3507.172)

As it is a closed economy, without an external sector, and with only one final use, the remunerations and the capital gains end up being destined to private consumption.

Determination of Shadow Prices of Hours Worked in the Spanish Economy

Based on the simplified input-output table for the Spanish economy in 2016 (Table nº 12). The last row of the table shows the hours worked by the sectors of the Spanish economy (in thousands) from the National Accounts of Spain.

We carried out the conversion of the Intermediate Consumption (Table nº 13) and the Final Demand of the Input Output Table for Spain in 2016 to working hours (Table nº 14).

Table nº 15 shows the technical coefficients for Spain's 2016 TIOT and table nº 16 shows the inverse of Leontief's TIOT.

With the vector of final demand (Table nº 14) and the inverse of Leontief (Table nº 16), we obtain the Leontief model in our table expressed in working hours:

	Working Hours
Agricultural	1489314
Industry	3328423
Energy	402711
Constr	2137690
Distr.hostel	10261542
ICT	803467
Finance	593147
Real.estate	341901
Prof.serv	3858281
Admin.Serv	6403412
Others	2512676

In order to reproduce the price model for the input-output table in Spanish hours in 2016, we calculated the rates of gross surplus on working capital in each sector:

	GOS.Rate
Agricultural	1.36
Industry	0.84
Energy	0.49
Constr	0.31
Distr.hostel	0.41
ICT	0.50
Finance	0.55
Real.estate	5.84
Prof.serv	0.35
Admin.Serv	0.14
Others	0.53

It should be borne in mind that in the table published by the INE for 2016, the total supply is obtained as the sum of final production and imports, in other words, it is not a domestic table, but rather a table of totals, where interindustrial consumption is the sum of both domestically produced and imported goods and services. The distribution of hours worked by sector, which is a domestic concept, therefore presents the limitation that the x_{ij} are totals and not domestic (x_{ij}^{r}) as would have been desirable. For the purposes of accounting equilibrium, the same occurs, since it is necessary to consider imports within the primary inputs. If the price model is to be reproduced, it ends up being necessary to consider the imports within the Value Added, and also within the surplus, since it is calculated from the difference between the Value Added and the Workers' Remuneration (*RA*). This is what has led to the definition of a new Value Added (*VA2*) from Supply (*PT*) and Inter-industrial Consumption (*IC*) and to obtaining an Operating Surplus (*GOS2*) The working capital used by the economy (*KC*), would be obtained by subtracting the Gross Operating Surplus from the Supply at basic prices, and as both magnitudes include imports, it would be the working capital that is anticipated to meet the payments made within the economy.

Table nº 17 shows the matrix I_π calculated with the surplus over working capital rate (π) for each sector. In Table nº 18 the matrix $(I_\pi - A')$, and in Table nº 19, its inverse $(I_\pi - A')^{-1}$.

The price model gives as a solution the shadow prices of the hours worked in the economy, calculated from the Supply at basic prices and the hours worked in each sector:

	Estimated hourly shadow price
Agricultural	0.0416884
Industry	0.2174689
Energy	0.2216567
Constr	0.0562520
Distr.hostel	0.0447717
ICT	0.1127851
Finance	0.1260495
Real.estate	0.4371066
Prof.serv	0.0526510
Admin.Serv	0.0357474
Others	0.0297362

CONCLUSIONS

Although one of the limitations of the current input-output analysis is the lack of matrices valued in physical consumption, this can be avoided by transforming the IOT into TIOT. If this type of exercise is carried out, the price model returns a product price of the hour worked in each sector, which can be considered a shadow price of the hour worked. The sectors with the highest shadow prices of the hour worked are, in this order: real estate, energy, industry, finance and ICT, the lowest shadow prices of the hour worked are public administrations, agriculture, the distribution sector (commerce and transport) and hotels and restaurants, professional services and construction.

The Leontief model determines the working hours that different sectors would perform to meet an additional demand of one working hour to produce consumer goods in all sectors of the economy:

	Hourly effort increase of 1 hour of the final demand of each sector
Agricultural	1.537781
Industry	2.216204
Energy	1.437933
Constr	1.665644
Distr.hostel	4.066973
ICT	1.362496
Finance	1.577714
Real.estate	1.067771
Prof.serv	3.541281
Admin.Serv	1.275883
Others	1.489219

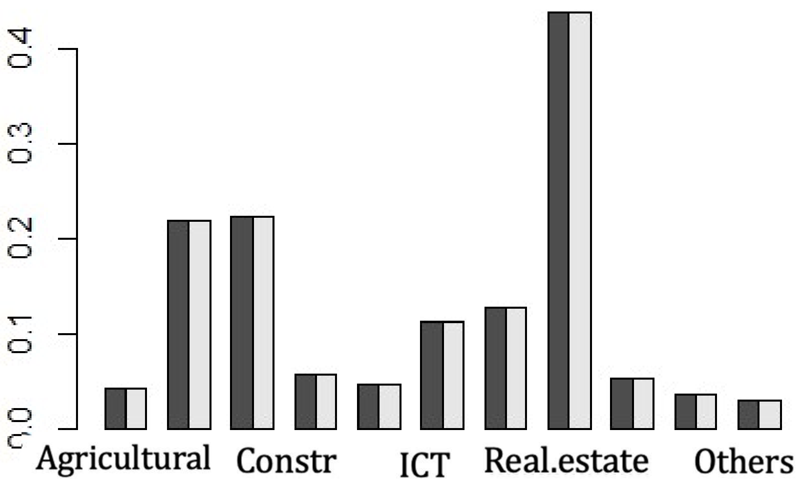
It is the distribution and hospitality sector that has to work the most hours to satisfy a demand for an additional hour of work in each sector, followed by professional services and industry.

After evaluating the production of each sector in terms of the hour worked or shadow hourly prices or product prices for the hour worked, we evaluate in single figures the flows between sectors and the consumption of the final sectors of the TIOT. In this regard, we can make assumptions about how these prices would be determined in the economic system. This hypothesis, taking the ideas of Safra's model (1960), is based on the idea that companies obtain as a surplus a margin on the working capital that they count on when starting the productive process, this working capital, is the one they advance to acquire the raw material and services that are needed, and to pay the workers.

If we determine a base scenario of hourly shadow prices or commodity prices for the hour worked, for illustrative purposes the example is given of how the hours worked would be determined assuming a rate for the Gross Surplus of 0.50 in all sectors, except agriculture (1.2), industry (0.8) and real estate (5). Table nº 20 shows matrix I_{π} corresponding to this scenario.

The price model, bearing in mind the limitations of this analysis, reproduces the shadow prices for hours worked in the Spanish economy in 2016.

FIGURE 1
SHADOW PRICES OF HOURS WORKED IN 2016 AND IN THE BASE SCENARIO



Based on the working hypothesis for the base scenario, the macroeconomic effects of a 10% reduction in hours worked in industry are simulated (Table nº 21).

Ultimately, modelling the Input Output Table on the basis of hours worked is an analytical tool to answer questions such as how to orient domestic demand for goods or exports to obtain a greater amount of hours worked, substitute imports for domestic production, or assess the impact on shadow prices of reductions in hours worked in a given sector, or the effects of a wage increase.

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APPENDIX

TABLE 12
SIMPLIFIED INPUT OUTPUT TABLE FOR THE SPANISH ECONOMY IN 2016

	Agricultural	Industry	Energy	Constr	Distr. hostel	ICT	Finance	Real.estate	Prof.serv	Admin.Serv	Others	FD	Demand
Agricultural	1723	29815	48	16	948	0	2	0	153	336	29	29018	62087
Industry	12109	191383	17013	22083	45674	5540	1147	277	11920	13442	2645	400597	723829
Energy	1379	17918	19291	1087	12707	1526	47	960	1987	3414	1246	27701	89264
Constr	370	2347	819	19869	4490	793	250	3748	1243	1322	329	84670	120249
Distr.hostel	4954	63230	7110	7994	66552	4623	1613	1127	14390	8833	2508	276493	459427
ICT	12	2936	1372	1369	5439	11795	1751	640	3197	3767	764	57577	90619
Finance	1167	4501	820	1391	6535	1278	14756	5360	2877	3610	845	31627	74766
Real.estate	4	2260	546	724	17378	2491	2530	848	3977	1953	1809	114928	149447
Prof.serv	478	19594	3817	10326	36284	10132	5998	3808	37337	10936	4604	59827	203142
Admin.Serv	42	1329	492	450	3779	1213	148	89	1281	5331	616	214136	228905
Others	62	1608	415	53	3144	1267	323	276	3532	1380	5493	57164	74718
RA	3971	56671	8095	26411	122235	19563	19794	4711	68139	146126	28008	0	0
VA	27989	120294	33011	54571	249005	37543	41138	130963	109605	174440	52892	0	0
Prod	50290	457214	84754	119932	451934	78200	69703	148096	191499	228765	73780	0	0
Impor	11797	266615	4510	317	7492	12419	5063	1352	11644	140	938	0	0
Supply	62087	723829	89264	120249	459427	90619	74766	149447	203142	228905	74718	0	0
Hours	1489314	3328423	402711	2137690	10261542	803467	593147	341901	3858281	6403412	2512676	0	0

TABLE 13
TABLE OF INTER-INDUSTRIAL CONSUMPTION OF THE SPANISH ECONOMY IN 2016 IN HOURS

Agricultural	Industry	Energy	Constr	Distr.hostel	ICT	Finance	Real.estate	Prof.serv	Admin.Serv	Others
41326	715185	1149	372	22733	0	43	0	3675	8065	698
55683	880046	78232	101544	210024	25476	5273	1274	54812	61810	12164
6223	80839	87030	4905	57328	6884	212	4330	8963	15404	5619
6578	41721	14567	353219	79816	14097	4441	66622	22095	23510	5842
110655	1412269	158808	178550	1486483	103253	36027	25179	321402	197285	56020
108	26034	12167	12140	48223	104578	15524	5672	28349	33399	6771
9257	35705	6505	11035	51842	10141	117064	42525	22821	28636	6705
8	5170	1248	1655	39758	5699	5789	1940	9099	4469	4138
9088	372149	72492	196129	689142	192433	113929	72318	709147	207702	87449
1186	37180	13749	12580	105717	33927	4126	2490	35840	149130	17235
2095	54079	13969	1772	105726	42601	10855	9295	118764	46425	184724

TABLE 14
VECTOR OF THE FINAL DEMAND OF THE SPANISH ECONOMY IN 2016 IN HOURS

	Final Demand
Agricultural	696069
Industry	1842087
Energy	124974
Constr	1505183
Distr.hostel	6175612
ICT	510503
Finance	250911
Real.estate	262928
Prof.serv	1136301
Admin.Serv	5990252
Others	1922369

TABLE 15
TECHNICAL COEFFICIENTS OF THE SPANISH ECONOMY IN 2016 IN HOURS

	Agricultural	Industry	Energy	Constr	Distr.hostel	ICT	Finance	Real.estate	Prof.serv	Admin.Serv	Others
Agricultural	0.03	0.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Industry	0.04	0.26	0.19	0.05	0.02	0.03	0.01	0.00	0.01	0.01	0.00
Energy	0.00	0.02	0.22	0.00	0.01	0.01	0.00	0.01	0.00	0.00	0.00
Constr	0.00	0.01	0.04	0.17	0.01	0.02	0.01	0.19	0.01	0.00	0.00
Distr.hostel	0.07	0.42	0.39	0.08	0.14	0.13	0.06	0.07	0.08	0.03	0.02
ICT	0.00	0.01	0.03	0.01	0.00	0.13	0.03	0.02	0.01	0.01	0.00
Finance	0.01	0.01	0.02	0.01	0.01	0.01	0.20	0.12	0.01	0.00	0.00
Real.estate	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00
Prof.serv	0.01	0.11	0.18	0.09	0.07	0.24	0.19	0.21	0.18	0.03	0.03
Admin.Serv	0.00	0.01	0.03	0.01	0.01	0.04	0.01	0.01	0.01	0.02	0.01
Others	0.00	0.02	0.03	0.00	0.01	0.05	0.02	0.03	0.03	0.01	0.07

TABLE 16
LEONTIF'S INVERSE OF SPAIN'S TIOT 2016

	Agricultural	Industry	Energy	Constr	Distr.hostel	ICT	Finance	Real.estate	Prof.serv	Admin.Serv	Others
Agricultural	1.04	0.32	0.09	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.00
Industry	0.06	1.42	0.39	0.09	0.04	0.07	0.03	0.04	0.03	0.02	0.01
Energy	0.01	0.05	1.30	0.01	0.01	0.02	0.00	0.02	0.01	0.00	0.00
Constr	0.01	0.04	0.08	1.20	0.01	0.03	0.02	0.24	0.01	0.01	0.00
Distr.hostel	0.13	0.80	0.87	0.19	1.21	0.27	0.15	0.20	0.15	0.06	0.04
ICT	0.00	0.02	0.06	0.01	0.01	1.16	0.04	0.03	0.01	0.01	0.00
Finance	0.01	0.03	0.04	0.01	0.01	0.03	1.25	0.16	0.01	0.01	0.01
Real.estate	0.00	0.01	0.01	0.00	0.01	0.01	0.01	1.01	0.00	0.00	0.00
Prof.serv	0.03	0.30	0.46	0.17	0.12	0.40	0.33	0.37	1.25	0.05	0.06
Admin.Serv	0.00	0.03	0.07	0.01	0.02	0.06	0.02	0.02	0.02	1.03	0.01
Others	0.01	0.05	0.09	0.01	0.02	0.09	0.04	0.05	0.05	0.01	1.08

TABLE 17
MATRIX I_{π}ESCENARIO BASE

	Agricultural	Industry	Energy	Constr	Distr.hostel	ICT	Finance	Real.estate	Prof.serv	Admin.Serv	Others
Agricultural	0.42	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Industry	0.00	0.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Energy	0.00	0.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr	0.00	0.00	0.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Distr.hostel	0.00	0.00	0.00	0.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
ICT	0.00	0.00	0.00	0.00	0.00	0.66	0.00	0.00	0.00	0.00	0.00
Finance	0.00	0.00	0.00	0.00	0.00	0.00	0.65	0.00	0.00	0.00	0.00
Real.estate	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00	0.00	0.00
Prof.serv	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.74	0.00	0.00
Admin.Serv	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.88	0.00
Others	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.65

TABLE 18
MATRIX $(I_\pi - A')$

	Agricultural	Industry	Energy	Constr	Distr.hostel	ICT	Finance	Real.estate	Prof.serv	Admin.Serv	Others
Agricultural	0.40	-0.04	0.00	0.00	-0.07	0.00	-0.01	0.00	-0.01	0.00	0.00
Industry	-0.21	0.28	-0.02	-0.01	-0.42	-0.01	-0.01	0.00	-0.11	-0.01	-0.02
Energy	0.00	-0.19	0.45	-0.04	-0.39	-0.03	-0.02	0.00	-0.18	-0.03	-0.03
Constr	0.00	-0.05	0.00	0.60	-0.08	-0.01	-0.01	0.00	-0.09	-0.01	0.00
Distr.hostel	0.00	-0.02	-0.01	-0.01	0.56	0.00	-0.01	0.00	-0.07	-0.01	-0.01
ICT	0.00	-0.03	-0.01	-0.02	-0.13	0.53	-0.01	-0.01	-0.24	-0.04	-0.05
Finance	0.00	-0.01	0.00	-0.01	-0.06	-0.03	0.45	-0.01	-0.19	-0.01	-0.02
Real.estate	0.00	0.00	-0.01	-0.19	-0.07	-0.02	-0.12	0.14	-0.21	-0.01	-0.03
Prof.serv	0.00	-0.01	0.00	-0.01	-0.08	-0.01	-0.01	0.00	0.55	-0.01	-0.03
Admin.Serv	0.00	-0.01	0.00	0.00	-0.03	-0.01	0.00	0.00	-0.03	0.85	-0.01
Others	0.00	0.00	0.00	0.00	-0.02	0.00	0.00	0.00	-0.03	-0.01	0.58

TABLE 19
MATRIX $(I_\pi - A')^{-1}$

	Agricultural	Industry	Energy	Constr	Distr.hostel	ICT	Finance	Real.estate	Prof.serv	Admin.Serv	Others
Agricultural	2.83	0.53	0.07	0.07	0.91	0.03	0.08	0.05	0.35	0.03	0.07
Industry	2.60	4.72	0.36	0.32	4.64	0.19	0.33	0.26	2.04	0.18	0.39
Energy	1.37	2.43	2.43	0.39	4.27	0.28	0.34	0.28	2.26	0.23	0.46
Constr	0.26	0.46	0.05	1.72	0.79	0.05	0.07	0.06	0.57	0.04	0.07
Distr.hostel	0.15	0.25	0.05	0.07	2.12	0.04	0.06	0.08	0.41	0.04	0.08
ICT	0.30	0.53	0.10	0.17	1.22	1.93	0.16	0.18	1.30	0.14	0.30
Finance	0.17	0.30	0.05	0.14	0.80	0.15	2.33	0.22	1.17	0.06	0.19
Real.estate	1.00	1.76	0.43	2.70	4.39	0.55	2.34	7.58	5.50	0.29	0.93
Prof.serv	0.12	0.20	0.03	0.05	0.52	0.04	0.06	0.06	2.00	0.04	0.13
Admin.Serv	0.05	0.09	0.02	0.02	0.18	0.02	0.02	0.02	0.14	1.18	0.03
Others	0.05	0.08	0.02	0.03	0.20	0.02	0.03	0.03	0.19	0.02	1.74

TABLE 20
MATRIX I_π SCENARIO SIMULADO

	Agricultural	Industry	Energy	Constr	Distr.hostel	ICT	Finance	Real.estate	Prof.serv	Admin.Serv	Others
Agricultural	0.45	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Industry	0.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Energy	0.00	0.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr	0.00	0.00	0.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Distr.hostel	0.00	0.00	0.00	0.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00
ICT	0.00	0.00	0.00	0.00	0.00	0.67	0.00	0.00	0.00	0.00	0.00
Finance	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.00	0.00	0.00	0.00
Real.estate	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00	0.00	0.00
Prof.serv	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.00	0.00
Admin.Serv	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.00
Others	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67

TABLE 21
SIMULATION OF A 10% REDUCTION IN THE HOURS WORKED BY THE INDUSTRY

	Projected.GVA	*Base.GVA*	*Variation per one*	*Base.Prices per hour*	*Projected.Prices per hour*
Agricultural	*39664.31*	*39785.8*	*-0.0030535*	*0.0416884*	*0.0415470*
Industry	*385810.34*	*386908.4*	*-0.0028380*	*0.2174689*	*0.2167458*
Energy	*36063.84*	*37520.9*	*-0.0388332*	*0.2216567*	*0.2408654*
Constr	*54835.90*	*54888.2*	*-0.0009528*	*0.0562520*	*0.0561487*
Distr.hostel	*256215.68*	*256497.5*	*-0.0010987*	*0.0447717*	*0.0446777*
ICT	*49909.08*	*49961.8*	*-0.0010552*	*0.1127851*	*0.1125895*
Finance	*46182.20*	*46201.5*	*-0.0004178*	*0.1260495*	*0.1259574*
Real.estate	*132062.71*	*132314.2*	*-0.0019007*	*0.4371066*	*0.4362451*
Prof.serv	*121183.87*	*121248.6*	*-0.0005338*	*0.0526510*	*0.0525868*
Admin.Serv	*174556.11*	*174580.9*	*-0.0001420*	*0.0357474*	*0.0357163*
Others	*53797.78*	*53829.8*	*-0.0005949*	*0.0297362*	*0.0296993*