US APR vs EU APR, and the Substitute Tax on Loan Effects on These Formulas

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In Italy, especially these days, we have been dealing with the common "mortgage constant", technically built under the principles of the compounded interest methods. This worldwide well-known financial technique clearly shows that every single rate is constant and composed by the sum of the principal part + the accrued interest. The rates of this kind of amortization plan result in an inversely proportional scheme: higher interest reimbursed at the beginning of the amortization plan, increasing rate by rate.

If this is unquestionable everywhere, the Italian Courts and many Italian banks officially deny that this amortization plan technique incorporates compound interest and anatocism, although the mortgage constant is on the compounded interest structure, as per the anatocism effect in it.

In order to defend this thesis about the absence of anatocism in compounded interest on constant rate, basically, all the other Senior Expert Witnesses of the Italian Courts recall EU APR formula, constantly declaring that it is equal to IRR formula, without any differences, without any exceptions.

Their statement, confirmed by many Italian Courts, is that EU APR formula is exactly corresponding to the IRR formula, determined using the IRR Excel function (XIRR). My study states that this is very incorrect, that the IRR formula (and function) is equal to the US APR, which assume as an APY, and different from EU APR, better to define as EAR.

Keywords: US APR vs EU APR formulas, compound interest and anatocism

THE ANNUAL PERCENTAGE RATE OF CHARGE: DEFINITION AND HISTORIC BACKGROUND

As per the most common financial literacy, the Annual Percentage Rate of Charge (APRC), also known as "*Annual Percentage Rate*" (APR) measures the cost of the credit extended to consumers, and it is generally determined by multiplying the interest rate charged per period by the number of periods in a year.

The law sets the US APR formula on a loan that is, basically, the following:

$$\text{APR} = r * m$$

where the US APR is the interest rate, *r*, charged per period multiplied by the number of periods in a year, *m*. This is the US APR formula stated by Rich & Rose (1997, p. 115) that generally defines the US APR as an effective periodic rate annualized without incorporating compounding.

This formula originates from the Appendix J of the Regulation Z where the US APR is generally computed in a two-staged process:

a) In the first stage, the periodic rate is actuarially determined using the "general equation" described in the relative sections of the Regulation Z;
b) In the second stage, the US APR is calculated by multiplying the periodic rate obtained from the first stage by the number of the compounding periods in a year.

According to the general equation, these two stages could be expressed through the following two mathematical expression, as per Vicknair study published in JFED in 2004:

$$0 = \boldsymbol{A_0} - \frac{\boldsymbol{P_1}}{(\boldsymbol{1} + \boldsymbol{i})^{\boldsymbol{t_1}} \times (\boldsymbol{1} + \boldsymbol{f_1} \times \boldsymbol{i})} - \frac{\boldsymbol{P_2}}{(\boldsymbol{1} + \boldsymbol{i})^{\boldsymbol{t_2}} \times (\boldsymbol{1} + \boldsymbol{f_2} \times \boldsymbol{i})} - \cdots - \frac{\boldsymbol{P_n}}{(\boldsymbol{1} + \boldsymbol{i})^{\boldsymbol{t_*} \times (\boldsymbol{1} + \boldsymbol{f_n} \times \boldsymbol{i})}}$$

and

$$\text{APR} = r * m$$

These equations represent any single advance transaction, assuming that the advance occurs at the beginning of the loan and with a minor change in terms and order. Where

A_0 = amount of the advance, net of prepaid finance charges
P_j = amount of the *jth* payment
i = periodic rate expressed as a decimal equivalent
f_j = fraction of a compounding period in the time interval from the advance to the *jth* payment
t_j = number of full compounding periods in the time interval from the advance to the *jth* payment
n = number of payments
m = number of compounding periods in a year

when f_j equals zero $(1 + f_j * i)$ equals one, the equation

$$0 = \boldsymbol{A_0} - \frac{\boldsymbol{P_1}}{(\boldsymbol{1} + \boldsymbol{i})^{\boldsymbol{t_1}} \times (\boldsymbol{1} + \boldsymbol{f_1} \times \boldsymbol{i})} - \frac{\boldsymbol{P_2}}{(\boldsymbol{1} + \boldsymbol{i})^{\boldsymbol{t_2}} \times (\boldsymbol{1} + \boldsymbol{f_2} \times \boldsymbol{i})} - \cdots - \frac{\boldsymbol{P_n}}{(\boldsymbol{1} + \boldsymbol{i})^{\boldsymbol{t_*} \times (\boldsymbol{1} + \boldsymbol{f_n} \times \boldsymbol{i})}}$$

is equivalent to the following other equation:

$$0 = \boldsymbol{A_0} - \frac{\boldsymbol{P_1}}{(\boldsymbol{1} + \boldsymbol{r})^{\boldsymbol{t_1}}} - \frac{\boldsymbol{P_2}}{(\boldsymbol{1} + \boldsymbol{r})^{\boldsymbol{r_2}}} - \cdots \frac{\boldsymbol{P_n}}{(\boldsymbol{1} + \boldsymbol{r})^{\boldsymbol{t_n}}}$$

This last equation equals *i* to *r* but only in the case the odd payments are not incorporated in the above-mentioned first stage equation, otherwise the equivalence $i = r$ becomes false. In fact, if *r* represents the compound rate for a whole period, then the rate corresponding to the fractional period *f* is correctly specified by the geometric mean $(1+r)^f$ and not by $f * r$, which assumes simple interest. Hence, the PV factor corresponding to $t + f$ periods is $1/(1+r)^f$ and not $1/(1+ f * r)$. Likewise, the PV factor corresponding to $t + f$ periods is $1/(1+r)^t * (1+r)^f$ or $1/(1+r)^{t+f}$ and not $1/(1+r)^t * (1 + r * r)$. Thus, the correct equation including the odd payment periods becomes the following:

$$0 = \boldsymbol{A_0} - \frac{\boldsymbol{P_1}}{(\boldsymbol{1} + \boldsymbol{r})^{\boldsymbol{t_1} + \boldsymbol{f_1}}} - \frac{\boldsymbol{P_2}}{(\boldsymbol{1} + \boldsymbol{r})^{\boldsymbol{t_2} + \boldsymbol{f_2}}} - \cdots \frac{\boldsymbol{P_n}}{(\boldsymbol{1} + \boldsymbol{r})^{\boldsymbol{t_n} + \boldsymbol{f_n}}}$$

It clearly appears that this last formula evidences in its denominator of each payment term the periodic rate corresponding to an odd payment period as a simple rather than compound rate.

However, the US APR represents the stated rate that show the cost of borrowing over an interest period as a function of the loan's face value. Therefore, the US APR misstates the true interest rate that, instead, corresponds to the cost of borrowing over an interest period as a function of the borrower's usable funds, where for "*usable funds*" it means the funds the borrower actually gets to use. For this reason, we use the different acronym of EAR (sometimes also written like "*EAPR – Effective Annual Percentage Rate*"), standing for "*Effective Annual Rate*", that measure, the true interest rate when compounding occurs more frequently than once a year. In other financial textbooks, the difference between the US APR and the EAR is explained through the difference between a nominal interest rate and its effective, sometimes called like "*real*" interest rate namely the returns in which interest is quoted in terms of the current power of purchasing, in general, of a currency. Thus, we can define the effective interest rate even like the returns with interest compounded once over the period of the quotation, while, on the contrary, the nominal interest rate assume better the valuation of a "quoted" rated, instead.

APR and EAR (both US and/or EU) differs each other, especially, because of the CRO and CLO policies of the lenders that define the level of the tolerance of the risk on the money lending, their cost of funding and the expected margin of profit coming from the lending of credit to consumers. These policies reflect – in the context of a loan – to a reduction of the usable funds below the face amount of the granted credit, due to, *id est*, closing costs, upfront cost, cost of funding transferred to the borrowers, discount interest or, even, from compensating balance requirements activity. Therefore, the interest rate concepts become the main aspect to be weighted on APR structure (both US and/or EU), because APR and, consequently, EAR, it represents the expression of many previous financial and economic aspects, especially related to the consumer credit market, and to behavioural finance field.

Beside this aspect, we should stress again that the APR (both US and/or EU) and the APY differs from each other even because of their different adjustment for the net cash flows related to a loan or investment rather than the stated principal and amortised payments. The net cash flows may differ from these stated amounts due to fees, points, compensating balances and so on. These discrepancies between the APR and the APY come to evidence even from the Truth Landing Act (TILA) and from the Truth in Saving Act.

Stated that CRO and CLO Areas determine ahead the terms of the loan contracts and provide the level of the foreseen interest rates, and the cost of funding deducted from the funds supplied to the borrower, a second level of observation is the term of reimbursement of the loan. In fact, normally, the lender asks interests from the borrower on the face value of the loan, and not on the effective sum lent, and this situation comes to determine an effective interest rate different, and consequently higher, than the nominal interest rate declared in the contract signed by the borrower and by the lender.

As just said before, the terms of reimbursement influence the effective interest rate, but it is not the only element effecting this evidence, because even the methodology of reimbursement plan of the loan effect the interest rate applied to the credit. Specifically, if the contract provide a reimbursement of the loan through interim annual payments of rates, such as through a number of rate in fraction of a year base, compounding occurs more than once a year, and so the effective interest rate arises.

If the periodic interest charge, r, is known, the EAR is found by using the following equation:

$$\text{EAR} = (1 + r)^{m} - 1$$

where m is the number of periods per year.

If we would compare the Annual Percentage Rate (of Charge) with its equivalent Annual Rate (EAR), the EAR will be always higher than the APR (both US and/or EU) due to interest compounding, whenever the compounding frequency is higher than once a year. In this case the formula will be

$$\text{EAR} = \left(1 + \frac{\text{APR}}{N}\right)^{N} - 1 (N = \text{ comp. freq. })$$

For the maximum precision, "*K*" or "*m*" replace the "*N*" of the above-cited formula in some financial books, *id est* Ross et al (2000), Melicher and Norton (2000) and Brealy et al (2001). EAR links US APR through mathematical conversion formulas as follows:

$$r_{\text{EAR}} = \left(1 + \frac{r_{\text{APR}}}{k}\right)^k - 1$$

and

Effective Rate per Compounding Interval = r_{apr}/k

where

APR = Annual Percentage Rate
EAR = Effective Annual Rate
k = number of compound interval per period

In case of continuously compounded EAR, we would have a compounding frequency to infinity, which means that we would have the highest possible EAR for a given US APR. In this case the formula would be the following:

$$\text{EAR}_\infty = \lim_{n\to\infty}\left(1 + \frac{\text{APR}}{N}\right)^N - 1 = e^{\text{APR}} - 1$$

If the US APR is known instead, you should divide the US APR by *m* and you should use the resulting number for *r* in the following equation:

$$r = \frac{\text{APR}}{m}$$

However, these formulas are not fully consistent, since decision-makers should be taught to think in terms of effective annual rates of return, and there is anyway an impairment when called to identify which of several loans has the lowest "true" interest cost when at least one of the loans being evaluated has an odd payment period. The logic consecution is that there are some flaws among these formulas, and, for this reason, it is necessary to proceed to a recap of them, for better comparison, as per following numbering:

$$0 = \boldsymbol{A_0} - \frac{\boldsymbol{P_1}}{(\boldsymbol{1+i})^{\boldsymbol{t_1}}\times(\boldsymbol{1+f_1\times i})} - \frac{\boldsymbol{P_2}}{(\boldsymbol{1+i})^{\boldsymbol{r_2}}\times(\boldsymbol{1+f_2\times i})} - \cdots - \frac{\boldsymbol{P_n}}{(\boldsymbol{1+i})^{\boldsymbol{t_0}\times(\boldsymbol{1+f_n\times i})}} \quad (1)$$

APR = i × m (2)

APR = r × m (3)

$$0 = A_0 - \frac{P_1}{(1+r)^{t_1+f_1}} - \frac{P_2}{(1+r)^{t_2+f_2}} - \cdots \frac{P_n}{(1+r)^{t_n+f_n}} \quad (4)$$

$$AER = \left(1 + \frac{APR}{m}\right)^m - 1 \quad (5)$$

The *equation #1* has its flaw easily in evidence through an empiric example: let assume that today we invest $ 100.00 for four and one-half periods at a periodic compound rate of 10.00%.

As per time-vale money formula, we will have:

$$\$ 100.00 * (1 + .01)^{4.5} = \$ 153.56$$

The *equation #5* correctly verifies what we already know, that is that the periodic rate of return on the investment is 10.00%, or

$$\$ 100.00 = \$ 153.56 / (1 + .01)^{4.5}$$

On the contrary, *equation #1* would give as periodic rate of return on the investment the different percentage of 9.973%, or

$$\$ 100.00 = \$ 153.56 / (1 + 0.09973)^4 * (1 + 0.5 * 0.9973)$$

To illustrate the effect on the periodic rate, US APR, and its annual effective rate (AER) of using *equation #1* rather than *equation #5*, we offer the reader of the David B. Vicknair example made in his study, as follows:

- ✓ Three possible consumer loans to be compared;
- ✓ Single net advance on October 1, 2001;
- ✓ $ 10,000 capital;
- ✓ 15 monthly instalments;
- ✓ 360-day year (30-day month) to compute the odd-days interest.

Loan “*U*” (Uniform payment periods)

- 11.45% nominal rate compounded monthly;
- 15 payments of $ 718.68 beginning on November 1, 2001 and ending on January 1, 2003;
- $f = 0$

Loan “*L*” (Long first payment period)

- 12.00% nominal rate compounded annually;
- 90 Odd days interest of $ 300.00 [10,000 * 12%/360 * 90]
- One payment of $ 11,536.00 on January 1, 2003;
- $f = 90/360$

Loan “*S*” (Short first payment period)

- 11.68% nominal rate compounded annually;
- 90 Odd days interest of $ 292.00 [10,000 * 11.68%/360 * 90]
- Three payments of $ 3,627.12 on January 1, 2002 – on July 1, 2002 and on January 1, 2003;
- $f = 90/180$

Therefore, Vicknair produced in hi study the following *Table 1* (p. 65 – JFED Vol. 30 Spring 2004 Ed.) that provides the comparison of the interest rates of these three hypothetical loans:

TABLE 1
COMPARATIVE INTEREST RATES FOR LOAN U, L, AND S

		Periodic Rate		Annual Percentage Rate		Annual Effective Rate	
	Nominal Rate	Regulation Z[1]	Actuarially Correct[2]	Regulation Z[3]	Actuarially Correct[4]	Regulation Z[5]	Actuarially Correct[6]
Loan U	11.450%	0.954%	0.954%	11.448%	11.448%	12.068%	12.068%
Loan L	12.000%	12.000%	12.110%	12.000%	12.110%	12.000%	12.110%

Loan S	11.680%	5.840%	5.869%	11.680%	11.738%	12.021%	12.083%

[1]Equation (1); [2]Equation (5); [3]Equation (2); [4]Equation (3); [5]Equation (6); [6]Equation (6)

We can observe from this *Table 1* that the periodic rate, Regulation Z APR and the corresponding Regulation Z-based AER on Loan "*U*" are identical to the analogous actuarially correct rates, because of the uniform payment periods.

Instead, the periodic rate on Loan "*L*" determined by *equation #1* understates the periodic rate determined by *equation #5* by approximately 11.0 basis points (12.000% versus 12.110%), while the corresponding understatement on Loan "*S*" is approximately 2.9 basis points (5.840% versus 5.869%). This discrepancy is due to compounding, because Loan "*L*" compounds annually and its periodic and annualized rates converge. However, since Loan "*S*" compounds semi-annually, the understatement in the periodic rate is magnified when it is annualized. Hence, Regulation Z APR understates the actuarially correct US APR on Loan "*S*" by approximately 5.8 basis points (11.680% versus 11.738%), while the actuarially correct AER is understated by approximately 6.1 basis points (12.021% versus 12.083%).

Table 1 gives further information about US APR and AER:

- Loan "*L*" has a lower Regulation Z-based APR than Loan "*U*" (12.000% versus 12.068%), but has a higher actuarially correct AER (12.110% versus 12.068%);
- Loan "*S*" has a lower Regulation Z-based AER than Loan "*U*" (12.021% versus 12.068%), but has a higher actuarially correct AER (12.083% versus 12.068%);
- Both Loan "*U*" and Loan "*S*" have a higher Regulation Z-based AER than Loan "*L*" (12.068% and 12.021% versus 12.000%), but their actuarially correct AERs are lower (12.068% and 12.083% versus 12.110%).

These evidences show that an uninformed decision-maker would always identify the lowest cost loan by merely comparing nominal rates or Regulation Z APRs.

To help understanding the reader about the evident flaw intrinsic incorporated in these formulas, we copy, again, the 2004 study of Vicknair through some figures and relative his comments on them, as follows:

FIGURE 1
UNDERSTATEMENT IN AER FOR LOAN "L" WITH CHARGE IN COMPOUNDING PERIODS

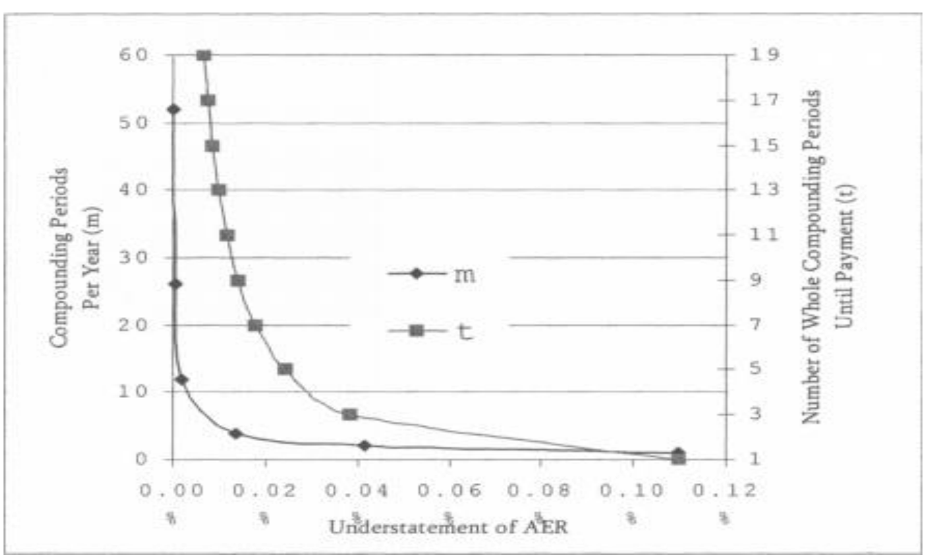

Figure 1 is a compound graph showing how the understatement in the AER for Loan "*L*" varies with changes in either the number of compounding periods in a year (*m*), or the number of whole compounding periods prior the payment (*t*), while holding other independent variables constant. Reading from the left axis, the left-hand data series shows that as *m* is increased the understatement approaches zero percent. The most significant reduction occurs when a change from annual (m =1) to semi-annual (m =2) compounding is made. Changes beyond monthly compounding have little effect on the understatement. Similarly, reading from the right axis, the right-hand data series shows that as *t* is increased the understatement also approaches zero percent. However, the reduction is more gradual than with a change in *m*.

FIGURE 2
UNDERSTATEMENT IN THE AER FOR LOAN "U", LOAN "L" AND LOAN "S" WITH CHANGES IN F

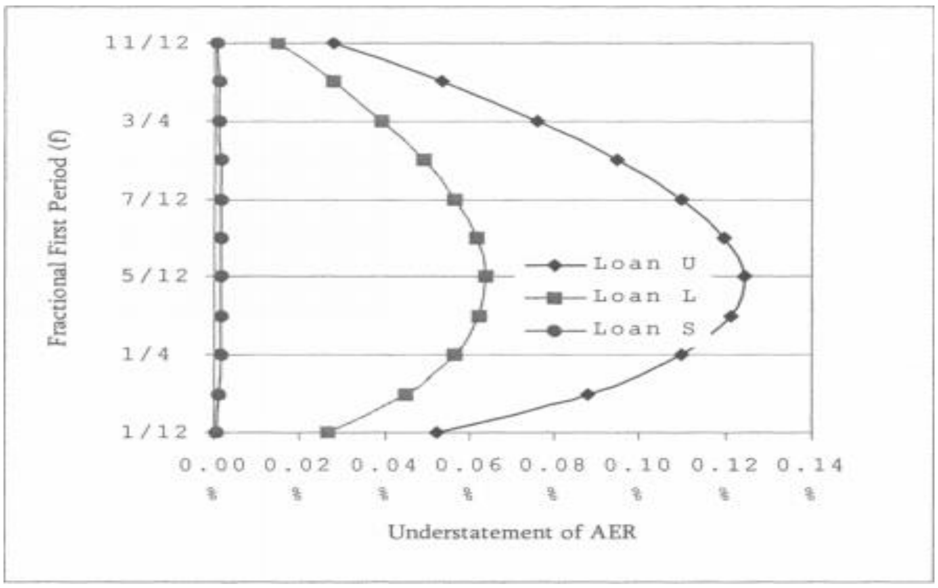

Figure 2 shows how the understatement in the AER for Loan "*U*", Loan "*L*" and Loan "*S*" varies with the changes in the value of *f*, while holding other independent variables constant. The understatement reaches its maximum point when the value of *f* is approximately 5/12, or approximately 13 days for Loan "*U*" 150 days for Loan "*L*", and 75 days for Loan "*S*". What is most striking about Figure 2 is the negligible effect that a change in the value of *f* has on the understatement for Loan "*U*", which is compounded monthly.

FIGURE 3
UNDERSTATEMENT IN AER FOR WITH CHANGES IN NOMINAL RATE

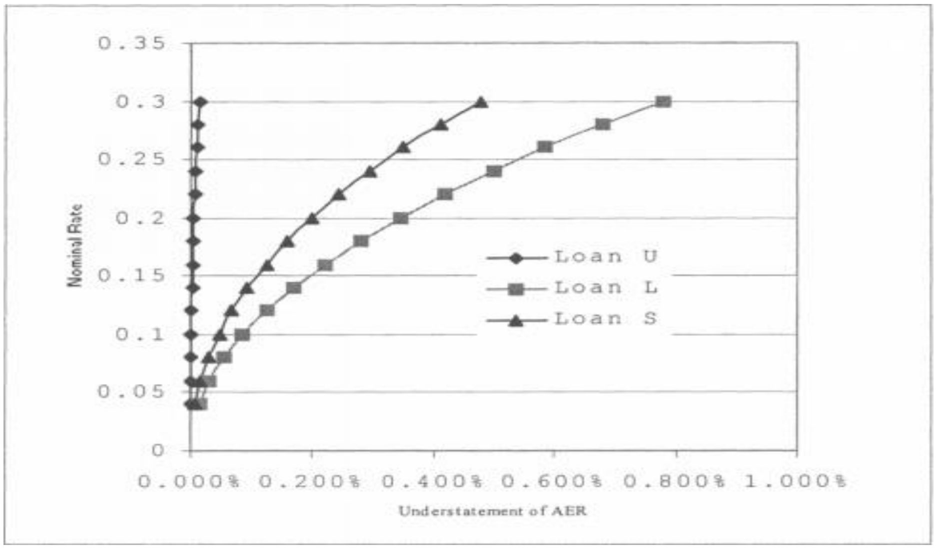

Figure 3 shows how the understatement in the AER varies with changes in the nominal annual rate, while holding other independent variables constant (including $f = 5/12$). As in Figure 2, what is most striking about Figure 3 is how the understatement for Loan "*U*" is unaffected by changes in the nominal rate.

Taken together, Figure 1, 2 and 3 suggest that the understatement in the AER is probably not material in transactions with low nominal rates, with monthly or more frequent payments, or with terms exceeding several years. Although interest rates vary over time, by type of transaction, and between lenders, most single advance consumer loans typically involve monthly payments and terms of two years or longer. However, in the case of short-term transactions, and, in particular, single payment loans with a term exceeding one year, but less than two years, such as Loan "*L*" in the example, the potential of significant understatement and impaired decision-making remains a concern. Therefore, when payment periods are uniform equation (see *Equation #1*) is actuarially correct and a Regulation Z APR can be used in the *Equation #6* to derive the corresponding actuarially correct AER.

This is why, sometimes, we recall the EAR, instead, as built as an annual effective yield (APY), as it considers the effects of periodic compounding coming because interest is assessed on previous periods, regardless the fact if the payment resulted to be unpaid, or not. Thus, since interest compounds, the APR formula will understate the true or effective interest cost, even because the lender considers each periodical payment due like a closing cost for the borrower and a *stand-alone* cash flow of revenue, on which it earns a separate profit or loss.

In contrast, from the borrower's perspective, the periodical payment of the rates becomes like a closing costs that reduces the usable funds below the face amount equal to the payment due, and, so, to raise the periodic effective cost of borrowing above the nominal interest rate.

Although it is possible to infer the periodic nominal rate from the loan documents and payment pattern, we must impute the periodic effective rate. The periodic effective rate is the rate that equates the periodical payments with the usable funds of the face value of the credit.

Mathematically, we can express the periodic effective rate by the internal rate of return mechanism, but it is not a completely correct operation, as Prof. Yuri K. Shestopaloff mathematically and empirically demonstrated in his book ("*Science of Inexact Mathematics – Investment Performance Measurement – Mortgages and Annuities Computing Algorithms Attribution Risk Valuation*", AKVY Press). For this reason, we must stay with what maths set about, that is the compounding situations and the periodic annualization occurring for every periodic interim payments requested by the lender to the borrower.

Therefore, starting by the APR and EAR (or AER, sometimes, from the acronym "*Annual Effective Rate*") formulas before evidenced, we can recapitulate them in these mathematical expressions:

$$APR = r \times m$$
$$AER = (1 + r)^m - 1$$
$$AER = \left(1 + \frac{APR}{m}\right)^m - 1$$
$$APR = \left[(1 + AER)^{\frac{1}{m}} - 1\right] \times m$$
$$r = APR/m$$
$$r = (1 + AER)^{\frac{1}{m}} - 1$$

where it becomes clear that the periodic effective rate is the common building block of the APR and the EAR, and where the difference between the APR and EAR resides in how the annualization occurs on the effective period rate. In fact, the APR annualizes the periodic effective rate without recognizing the effect of intra-period [*rectus*: interim] compounding, whereas, the AER annualizes the periodic effective rate by incorporating intra-period [*rectus*: interim] compounding. Mathematically speaking, we can determine that the APR is the result of applying a linear annualization process to the periodic effective rate, while the corresponding AER is the result of applying a geometric [*rectus*: "*progressive*"] annualization process to the same rate.

Unfortunately, due to the poor quality of many experts, especially Italians, it is very common to see that there is a general and vast unawareness of the true meaning of the interest-rate quotes. This evidence appears like a clear professional incompetence that brings to incorrect calculations and definitions.

As a partial defence in favour of the Italian experts, especially, we must note that, often, interest rate quotations are ambiguous because the terms may be stated in a misleading manner or are not stated explicitly. Exacerbating this problem is the fact that, often, the banks providing the rate details do not provide the right information regarding such matters. Knowledge of the conventions used for different types of rate quotes is essential when attempting to do time-value calculations, since effective rates are needed in actual time value calculations. Therefore, to be complete and unambiguous, any interest rate quotation or banking contract must indicate at least the following three terms:

a) The quoted amount (*id est*, the nominal interest rate);
b) The quotation period (*id est*, per year);
c) The number of times the rate is compounded during the quotation period, also called "the compounding frequency of the quotation" (*id est*, compounded quarterly or four times per year).

Let make an empiric example to show the possible discrepancies that may result if an interest rate quote would be incorrectly interpreted, as follows:

- Loan: $ 1,000,000.00;
- Annual interest rate: 12.00%
- Full amortisation loan over 25 years;
- Instalments: annual, semi-annual, monthly.

Compounding periods per year	Effective Monthly Rate (1)	Monthly Loan Payment (2)	Total Payments over the Term of the Loan (3)	Total Interest Paid (4)	Effective Annual Interest Rate (5)
1 (annual compounding)	0.94887929%	$ 10,081.84	$ 3,024,552.03	$ 2,024,552.03	12.00000000%
2 (semi-annual compounding)	0.97587942%	$ 10,319.00	$ 3,095,698.66	$ 2,095,698.66	12.36000000%
3 (monthly compounding)	1.00000000%	$ 10,532.24	$ 3,159,672.43	$ 2,159,672.43	12.68250301%
Difference between amount calculated with monthly vs. annual compounding		4.50%	4.50%	6.70%	5.70%

(1) Calculations for the effective monthly rates are the follows:
$(1.12^{1/12} – 1) * 100\%$ → for the 12.00% per year compounded annually;
$[(1 + 0.12 / 2)^{1/6} – 1] * 100\%$ → for the 12.00% per year compounded semi-annually;
$0.12 / 12 * 100\%$ → for the 12.00% per year compounded monthly.
(2) The monthly loan payments are determined as follows:
$\$ 1,000.00 * r / [1 – 1 / (1 + r)^{300}]$ where r is the effective monthly rate as determined as above;
(3) The total payments over the term of the loan are equal to the monthly loan payment multiplied by 300 payments;
(4) The total interest paid is equal to the total payments determined above minus the initial principal amount of $ 1,000,000.00;
(5) The effective annual interest rate is calculated as $[(1 + 0.12 / m)^m – 1] * 100\%$ where m is the frequency of compounding per year.

The amortisation plan for the 12 months compounding per year for 300 instalments is the following:

Instalment #	Payment	Interest	Discount factor	Interest actualization	Principal	Balance

						1.000.000,00
1	10.532,24	10.000,00	1,0000	10.000,00	532,24	999.467,76
2	10.532,24	9.994,68	1,0000	9.994,68	537,56	998.930,19
3	10.532,24	9.989,30	1,0000	9.989,30	542,94	998.387,26
4	10.532,24	9.983,87	1,0000	9.983,87	548,37	997.838,89
5	10.532,24	9.978,39	1,0000	9.978,39	553,85	997.285,03
6	10.532,24	9.972,85	1,0000	9.972,85	559,39	996.725,64
7	10.532,24	9.967,26	1,0000	9.967,26	564,98	996.160,66
8	10.532,24	9.961,61	1,0000	9.961,61	570,63	995.590,02
9	10.532,24	9.955,90	1,0000	9.955,90	576,34	995.013,68
10	10.532,24	9.950,14	1,0000	9.950,14	582,10	994.431,58
11	10.532,24	9.944,32	1,0000	9.944,32	587,93	993.843,65
12	10.532,24	9.938,44	1,0000	9.938,44	593,80	993.249,85
13	10.532,24	9.932,50	1,0000	9.932,50	599,74	992.650,10
14	10.532,24	9.926,50	1,0000	9.926,50	605,74	992.044,36
15	10.532,24	9.920,44	1,0000	9.920,44	611,80	991.432,57
16	10.532,24	9.914,33	1,0000	9.914,33	617,92	990.814,65
17	10.532,24	9.908,15	1,0000	9.908,15	624,09	990.190,55
18	10.532,24	9.901,91	1,0000	9.901,91	630,34	989.560,22
19	10.532,24	9.895,60	1,0000	9.895,60	636,64	988.923,58
20	10.532,24	9.889,24	1,0000	9.889,24	643,01	988.280,57
21	10.532,24	9.882,81	1,0000	9.882,81	649,44	987.631,14
22	10.532,24	9.876,31	1,0000	9.876,31	655,93	986.975,21
23	10.532,24	9.869,75	1,0000	9.869,75	662,49	986.312,72
24	10.532,24	9.863,13	1,0000	9.863,13	669,11	985.643,60
25	10.532,24	9.856,44	1,0000	9.856,44	675,81	984.967,80
26	10.532,24	9.849,68	1,0000	9.849,68	682,56	984.285,24
27	10.532,24	9.842,85	1,0000	9.842,85	689,39	983.595,85
28	10.532,24	9.835,96	1,0000	9.835,96	696,28	982.899,56
29	10.532,24	9.829,00	1,0000	9.829,00	703,25	982.196,32
30	10.532,24	9.821,96	1,0000	9.821,96	710,28	981.486,04
31	10.532,24	9.814,86	1,0000	9.814,86	717,38	980.768,66

Instalment #	Payment	Interest	Discount factor	Interest actualization	Principal	Balance
32	10.532,24	9.807,69	1,0000	9.807,69	724,55	980.044,10
33	10.532,24	9.800,44	1,0000	9.800,44	731,80	979.312,30
34	10.532,24	9.793,12	1,0000	9.793,12	739,12	978.573,19
35	10.532,24	9.785,73	1,0000	9.785,73	746,51	977.826,68
36	10.532,24	9.778,27	1,0000	9.778,27	753,97	977.072,70
37	10.532,24	9.770,73	1,0000	9.770,73	761,51	976.311,19
38	10.532,24	9.763,11	1,0000	9.763,11	769,13	975.542,06
39	10.532,24	9.755,42	1,0000	9.755,42	776,82	974.765,24
40	10.532,24	9.747,65	1,0000	9.747,65	784,59	973.980,65
41	10.532,24	9.739,81	1,0000	9.739,81	792,43	973.188,21
42	10.532,24	9.731,88	1,0000	9.731,88	800,36	972.387,85
43	10.532,24	9.723,88	1,0000	9.723,88	808,36	971.579,49
44	10.532,24	9.715,79	1,0000	9.715,79	816,45	970.763,04
45	10.532,24	9.707,63	1,0000	9.707,63	824,61	969.938,43
46	10.532,24	9.699,38	1,0000	9.699,38	832,86	969.105,58
47	10.532,24	9.691,06	1,0000	9.691,06	841,19	968.264,39
48	10.532,24	9.682,64	1,0000	9.682,64	849,60	967.414,79
49	10.532,24	9.674,15	1,0000	9.674,15	858,09	966.556,70
50	10.532,24	9.665,57	1,0000	9.665,57	866,67	965.690,02
51	10.532,24	9.656,90	1,0000	9.656,90	875,34	964.814,68
52	10.532,24	9.648,15	1,0000	9.648,15	884,09	963.930,59
53	10.532,24	9.639,31	1,0000	9.639,31	892,94	963.037,65
54	10.532,24	9.630,38	1,0000	9.630,38	901,86	962.135,79
55	10.532,24	9.621,36	1,0000	9.621,36	910,88	961.224,90
56	10.532,24	9.612,25	1,0000	9.612,25	919,99	960.304,91
57	10.532,24	9.603,05	1,0000	9.603,05	929,19	959.375,72
58	10.532,24	9.593,76	1,0000	9.593,76	938,48	958.437,24
59	10.532,24	9.584,37	1,0000	9.584,37	947,87	957.489,37
60	10.532,24	9.574,89	1,0000	9.574,89	957,35	956.532,02
61	10.532,24	9.565,32	1,0000	9.565,32	966,92	955.565,10
62	10.532,24	9.555,65	1,0000	9.555,65	976,59	954.588,51
63	10.532,24	9.545,89	1,0000	9.545,89	986,36	953.602,15

Instalment #	Payment	Interest	Discount factor	Interest actualization	Principal	Balance
64	10.532,24	9.536,02	1,0000	9.536,02	996,22	952.605,93
65	10.532,24	9.526,06	1,0000	9.526,06	1.006,18	951.599,75
66	10.532,24	9.516,00	1,0000	9.516,00	1.016,24	950.583,50
67	10.532,24	9.505,84	1,0000	9.505,84	1.026,41	949.557,10
68	10.532,24	9.495,57	1,0000	9.495,57	1.036,67	948.520,43
69	10.532,24	9.485,20	1,0000	9.485,20	1.047,04	947.473,39
70	10.532,24	9.474,73	1,0000	9.474,73	1.057,51	946.415,88
71	10.532,24	9.464,16	1,0000	9.464,16	1.068,08	945.347,80
72	10.532,24	9.453,48	1,0000	9.453,48	1.078,76	944.269,04
73	10.532,24	9.442,69	1,0000	9.442,69	1.089,55	943.179,49
74	10.532,24	9.431,79	1,0000	9.431,79	1.100,45	942.079,04
75	10.532,24	9.420,79	1,0000	9.420,79	1.111,45	940.967,59
76	10.532,24	9.409,68	1,0000	9.409,68	1.122,57	939.845,02
77	10.532,24	9.398,45	1,0000	9.398,45	1.133,79	938.711,23
78	10.532,24	9.387,11	1,0000	9.387,11	1.145,13	937.566,10
79	10.532,24	9.375,66	1,0000	9.375,66	1.156,58	936.409,52
80	10.532,24	9.364,10	1,0000	9.364,10	1.168,15	935.241,38
81	10.532,24	9.352,41	1,0000	9.352,41	1.179,83	934.061,55
82	10.532,24	9.340,62	1,0000	9.340,62	1.191,63	932.869,92
83	10.532,24	9.328,70	1,0000	9.328,70	1.203,54	931.666,38
84	10.532,24	9.316,66	1,0000	9.316,66	1.215,58	930.450,80
85	10.532,24	9.304,51	1,0000	9.304,51	1.227,73	929.223,07
86	10.532,24	9.292,23	1,0000	9.292,23	1.240,01	927.983,06
87	10.532,24	9.279,83	1,0000	9.279,83	1.252,41	926.730,65
88	10.532,24	9.267,31	1,0000	9.267,31	1.264,93	925.465,71
89	10.532,24	9.254,66	1,0000	9.254,66	1.277,58	924.188,13
90	10.532,24	9.241,88	1,0000	9.241,88	1.290,36	922.897,77
91	10.532,24	9.228,98	1,0000	9.228,98	1.303,26	921.594,50
92	10.532,24	9.215,95	1,0000	9.215,95	1.316,30	920.278,21
93	10.532,24	9.202,78	1,0000	9.202,78	1.329,46	918.948,75
94	10.532,24	9.189,49	1,0000	9.189,49	1.342,75	917.606,00
95	10.532,24	9.176,06	1,0000	9.176,06	1.356,18	916.249,81

Instalment #	Payment	Interest	Discount factor	Interest actualization	Principal	Balance
96	10.532,24	9.162,50	1,0000	9.162,50	1.369,74	914.880,07
97	10.532,24	9.148,80	1,0000	9.148,80	1.383,44	913.496,63
98	10.532,24	9.134,97	1,0000	9.134,97	1.397,28	912.099,35
99	10.532,24	9.120,99	1,0000	9.120,99	1.411,25	910.688,11
100	10.532,24	9.106,88	1,0000	9.106,88	1.425,36	909.262,75
101	10.532,24	9.092,63	1,0000	9.092,63	1.439,61	907.823,13
102	10.532,24	9.078,23	1,0000	9.078,23	1.454,01	906.369,12
103	10.532,24	9.063,69	1,0000	9.063,69	1.468,55	904.900,57
104	10.532,24	9.049,01	1,0000	9.049,01	1.483,24	903.417,34
105	10.532,24	9.034,17	1,0000	9.034,17	1.498,07	901.919,27
106	10.532,24	9.019,19	1,0000	9.019,19	1.513,05	900.406,22
107	10.532,24	9.004,06	1,0000	9.004,06	1.528,18	898.878,04
108	10.532,24	8.988,78	1,0000	8.988,78	1.543,46	897.334,58
109	10.532,24	8.973,35	1,0000	8.973,35	1.558,90	895.775,68
110	10.532,24	8.957,76	1,0000	8.957,76	1.574,48	894.201,20
111	10.532,24	8.942,01	1,0000	8.942,01	1.590,23	892.610,97
112	10.532,24	8.926,11	1,0000	8.926,11	1.606,13	891.004,84
113	10.532,24	8.910,05	1,0000	8.910,05	1.622,19	889.382,64
114	10.532,24	8.893,83	1,0000	8.893,83	1.638,41	887.744,23
115	10.532,24	8.877,44	1,0000	8.877,44	1.654,80	886.089,43
116	10.532,24	8.860,89	1,0000	8.860,89	1.671,35	884.418,08
117	10.532,24	8.844,18	1,0000	8.844,18	1.688,06	882.730,02
118	10.532,24	8.827,30	1,0000	8.827,30	1.704,94	881.025,08
119	10.532,24	8.810,25	1,0000	8.810,25	1.721,99	879.303,09
120	10.532,24	8.793,03	1,0000	8.793,03	1.739,21	877.563,88
121	10.532,24	8.775,64	1,0000	8.775,64	1.756,60	875.807,28
122	10.532,24	8.758,07	1,0000	8.758,07	1.774,17	874.033,11
123	10.532,24	8.740,33	1,0000	8.740,33	1.791,91	872.241,20
124	10.532,24	8.722,41	1,0000	8.722,41	1.809,83	870.431,37
125	10.532,24	8.704,31	1,0000	8.704,31	1.827,93	868.603,44
126	10.532,24	8.686,03	1,0000	8.686,03	1.846,21	866.757,24

Instalment #	Payment	Interest	Discount factor	Interest actualization	Principal	Balance
127	10.532,24	8.667,57	1,0000	8.667,57	1.864,67	864.892,57
128	10.532,24	8.648,93	1,0000	8.648,93	1.883,32	863.009,25
129	10.532,24	8.630,09	1,0000	8.630,09	1.902,15	861.107,10
130	10.532,24	8.611,07	1,0000	8.611,07	1.921,17	859.185,93
131	10.532,24	8.591,86	1,0000	8.591,86	1.940,38	857.245,55
132	10.532,24	8.572,46	1,0000	8.572,46	1.959,79	855.285,76
133	10.532,24	8.552,86	1,0000	8.552,86	1.979,38	853.306,38
134	10.532,24	8.533,06	1,0000	8.533,06	1.999,18	851.307,20
135	10.532,24	8.513,07	1,0000	8.513,07	2.019,17	849.288,03
136	10.532,24	8.492,88	1,0000	8.492,88	2.039,36	847.248,67
137	10.532,24	8.472,49	1,0000	8.472,49	2.059,75	845.188,92
138	10.532,24	8.451,89	1,0000	8.451,89	2.080,35	843.108,56
139	10.532,24	8.431,09	1,0000	8.431,09	2.101,16	841.007,41
140	10.532,24	8.410,07	1,0000	8.410,07	2.122,17	838.885,24
141	10.532,24	8.388,85	1,0000	8.388,85	2.143,39	836.741,85
142	10.532,24	8.367,42	1,0000	8.367,42	2.164,82	834.577,03
143	10.532,24	8.345,77	1,0000	8.345,77	2.186,47	832.390,56
144	10.532,24	8.323,91	1,0000	8.323,91	2.208,34	830.182,22
145	10.532,24	8.301,82	1,0000	8.301,82	2.230,42	827.951,80
146	10.532,24	8.279,52	1,0000	8.279,52	2.252,72	825.699,08
147	10.532,24	8.256,99	1,0000	8.256,99	2.275,25	823.423,83
148	10.532,24	8.234,24	1,0000	8.234,24	2.298,00	821.125,83
149	10.532,24	8.211,26	1,0000	8.211,26	2.320,98	818.804,84
150	10.532,24	8.188,05	1,0000	8.188,05	2.344,19	816.460,65
151	10.532,24	8.164,61	1,0000	8.164,61	2.367,63	814.093,01
152	10.532,24	8.140,93	1,0000	8.140,93	2.391,31	811.701,70
153	10.532,24	8.117,02	1,0000	8.117,02	2.415,22	809.286,48
154	10.532,24	8.092,86	1,0000	8.092,86	2.439,38	806.847,10
155	10.532,24	8.068,47	1,0000	8.068,47	2.463,77	804.383,33
156	10.532,24	8.043,83	1,0000	8.043,83	2.488,41	801.894,92
157	10.532,24	8.018,95	1,0000	8.018,95	2.513,29	799.381,63

Instalment #	Payment	Interest	Discount factor	Interest actualization	Principal	Balance
158	10.532,24	7.993,82	1,0000	7.993,82	2.538,43	796.843,21
159	10.532,24	7.968,43	1,0000	7.968,43	2.563,81	794.279,40
160	10.532,24	7.942,79	1,0000	7.942,79	2.589,45	791.689,95
161	10.532,24	7.916,90	1,0000	7.916,90	2.615,34	789.074,61
162	10.532,24	7.890,75	1,0000	7.890,75	2.641,50	786.433,11
163	10.532,24	7.864,33	1,0000	7.864,33	2.667,91	783.765,20
164	10.532,24	7.837,65	1,0000	7.837,65	2.694,59	781.070,61
165	10.532,24	7.810,71	1,0000	7.810,71	2.721,54	778.349,08
166	10.532,24	7.783,49	1,0000	7.783,49	2.748,75	775.600,33
167	10.532,24	7.756,00	1,0000	7.756,00	2.776,24	772.824,09
168	10.532,24	7.728,24	1,0000	7.728,24	2.804,00	770.020,09
169	10.532,24	7.700,20	1,0000	7.700,20	2.832,04	767.188,05
170	10.532,24	7.671,88	1,0000	7.671,88	2.860,36	764.327,69
171	10.532,24	7.643,28	1,0000	7.643,28	2.888,96	761.438,72
172	10.532,24	7.614,39	1,0000	7.614,39	2.917,85	758.520,87
173	10.532,24	7.585,21	1,0000	7.585,21	2.947,03	755.573,83
174	10.532,24	7.555,74	1,0000	7.555,74	2.976,50	752.597,33
175	10.532,24	7.525,97	1,0000	7.525,97	3.006,27	749.591,06
176	10.532,24	7.495,91	1,0000	7.495,91	3.036,33	746.554,73
177	10.532,24	7.465,55	1,0000	7.465,55	3.066,69	743.488,04
178	10.532,24	7.434,88	1,0000	7.434,88	3.097,36	740.390,68
179	10.532,24	7.403,91	1,0000	7.403,91	3.128,33	737.262,34
180	10.532,24	7.372,62	1,0000	7.372,62	3.159,62	734.102,73
181	10.532,24	7.341,03	1,0000	7.341,03	3.191,21	730.911,51
182	10.532,24	7.309,12	1,0000	7.309,12	3.223,13	727.688,38
183	10.532,24	7.276,88	1,0000	7.276,88	3.255,36	724.433,03
184	10.532,24	7.244,33	1,0000	7.244,33	3.287,91	721.145,12
185	10.532,24	7.211,45	1,0000	7.211,45	3.320,79	717.824,33

Instalment #	Payment	Interest	Discount factor	Interest actualization	Principal	Balance
186	10.532,24	7.178,24	1,0000	7.178,24	3.354,00	714.470,33
187	10.532,24	7.144,70	1,0000	7.144,70	3.387,54	711.082,79
188	10.532,24	7.110,83	1,0000	7.110,83	3.421,41	707.661,38
189	10.532,24	7.076,61	1,0000	7.076,61	3.455,63	704.205,75
190	10.532,24	7.042,06	1,0000	7.042,06	3.490,18	700.715,56
191	10.532,24	7.007,16	1,0000	7.007,16	3.525,09	697.190,48
192	10.532,24	6.971,90	1,0000	6.971,90	3.560,34	693.630,14
193	10.532,24	6.936,30	1,0000	6.936,30	3.595,94	690.034,20
194	10.532,24	6.900,34	1,0000	6.900,34	3.631,90	686.402,30
195	10.532,24	6.864,02	1,0000	6.864,02	3.668,22	682.734,08
196	10.532,24	6.827,34	1,0000	6.827,34	3.704,90	679.029,18
197	10.532,24	6.790,29	1,0000	6.790,29	3.741,95	675.287,23
198	10.532,24	6.752,87	1,0000	6.752,87	3.779,37	671.507,86
199	10.532,24	6.715,08	1,0000	6.715,08	3.817,16	667.690,70
200	10.532,24	6.676,91	1,0000	6.676,91	3.855,33	663.835,37
201	10.532,24	6.638,35	1,0000	6.638,35	3.893,89	659.941,48
202	10.532,24	6.599,41	1,0000	6.599,41	3.932,83	656.008,65
203	10.532,24	6.560,09	1,0000	6.560,09	3.972,15	652.036,50
204	10.532,24	6.520,36	1,0000	6.520,36	4.011,88	648.024,62
205	10.532,24	6.480,25	1,0000	6.480,25	4.052,00	643.972,63
206	10.532,24	6.439,73	1,0000	6.439,73	4.092,52	639.880,11
207	10.532,24	6.398,80	1,0000	6.398,80	4.133,44	635.746,67
208	10.532,24	6.357,47	1,0000	6.357,47	4.174,77	631.571,90
209	10.532,24	6.315,72	1,0000	6.315,72	4.216,52	627.355,37
210	10.532,24	6.273,55	1,0000	6.273,55	4.258,69	623.096,69
211	10.532,24	6.230,97	1,0000	6.230,97	4.301,27	618.795,41
212	10.532,24	6.187,95	1,0000	6.187,95	4.344,29	614.451,12
213	10.532,24	6.144,51	1,0000	6.144,51	4.387,73	610.063,39
214	10.532,24	6.100,63	1,0000	6.100,63	4.431,61	605.631,79
215	10.532,24	6.056,32	1,0000	6.056,32	4.475,92	601.155,86
216	10.532,24	6.011,56	1,0000	6.011,56	4.520,68	596.635,18

Instalment #	Payment	Interest	Discount factor	Interest actualization	Principal	Balance
217	10.532,24	5.966,35	1,0000	5.966,35	4.565,89	592.069,29
218	10.532,24	5.920,69	1,0000	5.920,69	4.611,55	587.457,74
219	10.532,24	5.874,58	1,0000	5.874,58	4.657,66	582.800,08
220	10.532,24	5.828,00	1,0000	5.828,00	4.704,24	578.095,84
221	10.532,24	5.780,96	1,0000	5.780,96	4.751,28	573.344,55
222	10.532,24	5.733,45	1,0000	5.733,45	4.798,80	568.545,76
223	10.532,24	5.685,46	1,0000	5.685,46	4.846,78	563.698,98
224	10.532,24	5.636,99	1,0000	5.636,99	4.895,25	558.803,72
225	10.532,24	5.588,04	1,0000	5.588,04	4.944,20	553.859,52
226	10.532,24	5.538,60	1,0000	5.538,60	4.993,65	548.865,87
227	10.532,24	5.488,66	1,0000	5.488,66	5.043,58	543.822,29
228	10.532,24	5.438,22	1,0000	5.438,22	5.094,02	538.728,27
229	10.532,24	5.387,28	1,0000	5.387,28	5.144,96	533.583,31
230	10.532,24	5.335,83	1,0000	5.335,83	5.196,41	528.386,90
231	10.532,24	5.283,87	1,0000	5.283,87	5.248,37	523.138,53
232	10.532,24	5.231,39	1,0000	5.231,39	5.300,86	517.837,68
233	10.532,24	5.178,38	1,0000	5.178,38	5.353,86	512.483,81
234	10.532,24	5.124,84	1,0000	5.124,84	5.407,40	507.076,41
235	10.532,24	5.070,76	1,0000	5.070,76	5.461,48	501.614,93
236	10.532,24	5.016,15	1,0000	5.016,15	5.516,09	496.098,84
237	10.532,24	4.960,99	1,0000	4.960,99	5.571,25	490.527,59
238	10.532,24	4.905,28	1,0000	4.905,28	5.626,97	484.900,62
239	10.532,24	4.849,01	1,0000	4.849,01	5.683,24	479.217,39
240	10.532,24	4.792,17	1,0000	4.792,17	5.740,07	473.477,32
241	10.532,24	4.734,77	1,0000	4.734,77	5.797,47	467.679,85
242	10.532,24	4.676,80	1,0000	4.676,80	5.855,44	461.824,41
243	10.532,24	4.618,24	1,0000	4.618,24	5.914,00	455.910,41
244	10.532,24	4.559,10	1,0000	4.559,10	5.973,14	449.937,27
245	10.532,24	4.499,37	1,0000	4.499,37	6.032,87	443.904,40
246	10.532,24	4.439,04	1,0000	4.439,04	6.093,20	437.811,21

Instalment #	Payment	Interest	Discount factor	Interest actualization	Principal	Balance
247	10.532,24	4.378,11	1,0000	4.378,11	6.154,13	431.657,08
248	10.532,24	4.316,57	1,0000	4.316,57	6.215,67	425.441,41
249	10.532,24	4.254,41	1,0000	4.254,41	6.277,83	419.163,58
250	10.532,24	4.191,64	1,0000	4.191,64	6.340,61	412.822,97
251	10.532,24	4.128,23	1,0000	4.128,23	6.404,01	406.418,96
252	10.532,24	4.064,19	1,0000	4.064,19	6.468,05	399.950,91
253	10.532,24	3.999,51	1,0000	3.999,51	6.532,73	393.418,18
254	10.532,24	3.934,18	1,0000	3.934,18	6.598,06	386.820,12
255	10.532,24	3.868,20	1,0000	3.868,20	6.664,04	380.156,08
256	10.532,24	3.801,56	1,0000	3.801,56	6.730,68	373.425,40
257	10.532,24	3.734,25	1,0000	3.734,25	6.797,99	366.627,41
258	10.532,24	3.666,27	1,0000	3.666,27	6.865,97	359.761,44
259	10.532,24	3.597,61	1,0000	3.597,61	6.934,63	352.826,81
260	10.532,24	3.528,27	1,0000	3.528,27	7.003,97	345.822,84
261	10.532,24	3.458,23	1,0000	3.458,23	7.074,01	338.748,83
262	10.532,24	3.387,49	1,0000	3.387,49	7.144,75	331.604,08
263	10.532,24	3.316,04	1,0000	3.316,04	7.216,20	324.387,87
264	10.532,24	3.243,88	1,0000	3.243,88	7.288,36	317.099,51
265	10.532,24	3.171,00	1,0000	3.171,00	7.361,25	309.738,27
266	10.532,24	3.097,38	1,0000	3.097,38	7.434,86	302.303,41
267	10.532,24	3.023,03	1,0000	3.023,03	7.509,21	294.794,20
268	10.532,24	2.947,94	1,0000	2.947,94	7.584,30	287.209,90
269	10.532,24	2.872,10	1,0000	2.872,10	7.660,14	279.549,76
270	10.532,24	2.795,50	1,0000	2.795,50	7.736,74	271.813,01
271	10.532,24	2.718,13	1,0000	2.718,13	7.814,11	263.998,90
272	10.532,24	2.639,99	1,0000	2.639,99	7.892,25	256.106,65
273	10.532,24	2.561,07	1,0000	2.561,07	7.971,17	248.135,47
274	10.532,24	2.481,35	1,0000	2.481,35	8.050,89	240.084,59
275	10.532,24	2.400,85	1,0000	2.400,85	8.131,40	231.953,19
276	10.532,24	2.319,53	1,0000	2.319,53	8.212,71	223.740,48
277	10.532,24	2.237,40	1,0000	2.237,40	8.294,84	215.445,65

Instalment #	Payment	Interest	Discount factor	Interest actualization	Principal	Balance
278	10.532,24	2.154,46	1,0000	2.154,46	8.377,78	207.067,86
279	10.532,24	2.070,68	1,0000	2.070,68	8.461,56	198.606,30
280	10.532,24	1.986,06	1,0000	1.986,06	8.546,18	190.060,12
281	10.532,24	1.900,60	1,0000	1.900,60	8.631,64	181.428,48
282	10.532,24	1.814,28	1,0000	1.814,28	8.717,96	172.710,52
283	10.532,24	1.727,11	1,0000	1.727,11	8.805,14	163.905,39
284	10.532,24	1.639,05	1,0000	1.639,05	8.893,19	155.012,20
285	10.532,24	1.550,12	1,0000	1.550,12	8.982,12	146.030,08
286	10.532,24	1.460,30	1,0000	1.460,30	9.071,94	136.958,14
287	10.532,24	1.369,58	1,0000	1.369,58	9.162,66	127.795,48
288	10.532,24	1.277,95	1,0000	1.277,95	9.254,29	118.541,19
289	10.532,24	1.185,41	1,0000	1.185,41	9.346,83	109.194,36
290	10.532,24	1.091,94	1,0000	1.091,94	9.440,30	99.754,07
291	10.532,24	997,54	1,0000	997,54	9.534,70	90.219,37
292	10.532,24	902,19	1,0000	902,19	9.630,05	80.589,32
293	10.532,24	805,89	1,0000	805,89	9.726,35	70.862,97
294	10.532,24	708,63	1,0000	708,63	9.823,61	61.039,36
295	10.532,24	610,39	1,0000	610,39	9.921,85	51.117,51
296	10.532,24	511,18	1,0000	511,18	10.021,07	41.096,44
297	10.532,24	410,96	1,0000	410,96	10.121,28	30.975,17
298	10.532,24	309,75	1,0000	309,75	10.222,49	20.752,68
299	10.532,24	207,53	1,0000	207,53	10.324,71	10.427,96
300	10.532,24	104,28	1,0000	104,28	10.427,96	-0,00

The first consideration about this simulation of amortisation plan and about the data above-evidenced is that with monthly compounding, both monthly and total payments are almost 4.50% higher than with annual compounding. If the calculations for effective annual rates is the well-known interest rate conversion formula, they are not useful to calculate the monthly loan payments or interest charges.

All these mathematical evidences should support the laws regulating the lending money activity, and, mostly, to stop any possible unethical behaviour of the lenders that, naturally, could be inclined to charge exorbitant interest rate on loans, since lenders business would be making business through making a rate of return on the money they have to lend. So, APR and EAR formulas should come to be a general guidance for the legislator in order to legislate the conduct and the behaviour of the lenders, especially in two specific matters:

1) Usury;
2) Disclosure, limpidity and correctness.

As per common definition, "*usury*" is the act of lending money at an excessively high interest rate, and it is a concept already existing by the time of the ancient Christians, Jews, Islamic societies and ancient Greece, where usury was considered straight immoral. More in general, a better definition of usury could be the taking of excessive interest, without considering the concept of the rate. In fact, originally, usury was the taking of ***any*** interest for the use of a fungible, namely, something that was consumed in its use. Actually, the lender is not operating any labour, expense or risk on the capital lent, and this – in theory – should not allow the lender to get back any extra sum exceeding the capital itself lent to the borrower. In practice, the lender, instead, has the right to receive a counterclaim exceeding the capital he lent, because the privation of the availability of his capital needs an equal reward. Precisely, the justification for the lender to ask for more money than what lent is, substantially, because of:

a) The actual loss sustained by the lender on account of the loan;
b) The foregoing of profit that would certainly have accrued if the lender had not parted with the thing;
c) Real and unusual risk incurred by the lender;
d) The penalty agreed upon by the lender and borrower in case of default;
e) The legal interest in the case of money lent.

These elements are extrinsic titles which are normally admitted to be present in the loan of money and which justify the taking of interest by the lender. In fact, the loss sustained, the profit foregone, the risk incurred, have all tangible value and justice decrees that compensation be made for them. Whoever goes beyond the right given to him by these extrinsic titles is guilty of usury. Thus, usury is not so much a disease as a symptom of a serious disorder in the economic life of a people: for this reason, governments should not pursue usurer, but rather they should fight what it could beget usury.

Restrictions on interest rates, bans on usury, are about as old as giving "interest-loans".

As how, correctly, Prof. Seckelman recalled in his study, many of the words and notions go back about 2000 years to Roman law; others roughly 1000 years to the banking system developed in what is today northern Italy, and again others to legal constructions used at the end of the Middle Ages and later to accommodate the need for loans with restrictions on interest.

The Latin words for interest were *fenus* and *usura*. With bans on interest *fenus* got lost, but *usura* is still to be found as *usury* and *usure*, the English and French words for (demanding) interest higher than allowed or deemed exorbitant. Under bans on interest, any interest was usury. In times of such a ban Roman law recognised alternative titles connected with lending money, claims for a reward. One is *interesse* of the lender in repayment of a loan in time. In other general words, we can define "usury" as contracting for or receiving something in excess of the amount allowed by law for the loan or forbearance of money, with primarily moral issue, as usury becomes to be a violation of the virtue of justice, more than just a violation of a law.

Even the title of the lender to demand a penalty to the borrower in case the repayment was delayed is part of the concept of "usury". One can easily imagine that such a fine could be proportional to the capital due and to the delay period and thus increased "linearly" with time. This means that the money lent stayed constant, that it was not a growth quantity; the penalty could be something different and separate. When interest-bearing loans were generally permitted, this title changed into what today is a legal claim on a return from money lent, instead.

The English word for it is *interest* and similar terms are used in Romanic languages. This change in the nature of the claim turns the money lent into a quantity that grows with interest "exponentially" with time. Interest not paid becomes part of the capital and bears interest. Such growth of debts is documented *e.g.* for England in the 13th century.

The words used today in many of the languages of the Western World for "*interest*" in its modern meaning of "*capital interest*", a return on capital lent, stem from words used for circumventions of bans on such interest, and thus point to sources of income with a similar effect, but legally of a different nature, resulting in different mathematical rules for and economic returns from them.

This to proof that history has shown many different behaviour by Governors about this phenomenon, under ethics, social, civil, financial and economic effects and point of views.

This dynamic management of the usury valuation changed through the centuries from country to country, from culture to culture, till nowadays where the laws developed following, as much as they could, the growth of the ethics of people. Besides being a violation of law, usury can also be a violation of social justice and it is always a violation of charity principles.

Although usury is a worldwide phenomenon, and, basically, with equally behaviour in all lenders that try to make excessive profit on money lend, usury laws are quite different from country to country, especially because of the concept of usury each country has and because of the different laws, from country to country, existing about the consumer credit market regulation. For instance, U.S.A. – apparently - focused its attention to the mechanism related to the investment of money [*rectus*: saving], and this point of view had surely influenced the U.S. Legislator on determine an indicator of cost of lending money considering the effective revenue of the lending. While, in the European Countries, it seems that their attention about usury focused on the opposite point of view of the problem that is the borrower side, for whom the fulcrum is the charge of its debit, and not the revenue of the lenders credit.

Instead, it appears that both US. Legislator and EU Legislator commonly agreed to defend the consumer credit borrower's rights introducing some rules preventing potential frauds operated by lenders, "playing" on the mechanism before illustrated about the differences intervening between nominal and effective interest rates.

To fight this potential problem in money lending, we had different legislative steps by during World War II in the U.S., when, mainly, each State of the Union expressed and compared credit costs, through various methods. By the early 1960s, the attention over the disclosure methods took place, especially after Washington believed that curbs on consumer credit could help to stabilize inflation. The problem related to disclosure on credit consumer enhanced with the actions of the U.S. Senator from Illinois, Paul H. Douglas that was very convinced about the importance of a correct consumer credit information that would benefits the entire financial market.

In 1965, Mors said, "*the result of the varying disclosure practises is that consumers do not obtain easily comparable information from alternative suppliers of credit*": it was the beginning of a constant growth of interest in consumer knowledge, perception, or awareness of interest rate since the Consumer Credit Labeling Bill introduced in the U.S.A. in the 1960 Congress.

Until that time, in the United States the most common method to calculate the dollar amount of credit charges was through the finance cost in terms of dollars and cents per hundred dollars of initial balance per year (we call this method "*add-on rate*"). Even in many U.S. States, the usury laws stated the legal ceilings for credit charges using this specific method that was widely used because it is very simple and intuitive for everybody: a pencil and the back of an envelope were more than adequate for add-on calculations. The add-on rate formula (the total finance charge) was added onto the loan principal to derive the total amount that would be paid over the life span of the loan and it could easily and rapidly estimate the finance charge and the monthly payment.

This method was very effective:

Step 1) $ 10,000.00 x 5% = $ 500.00 per year for 3 years = $ 1,500.00 finance charge

Step2) $ 10,000.00 + $ 1,500.00 = $ 11,500.00

Step 3) $ 11,500.00 divided by 36, a bit under $320.00 per month for the payments.

Using a pencil and envelope, actual monthly payments = $ 319.45

Thus, with such mechanism, everyone was able to get the picture of the monthly instalment of the payments due, per unit of time, very quickly. The concepts around the effective interest rate and actualization of the finance charges were not perceived still at that time, and, moreover, nobody were feeling the needing to create a different methods to calculate the effective cost for the consumer credit lending. This situation got complicated during the 1960s because there was a widespread inability to estimate the dollar charges on instalment credit transactions and the gap between the effective annual rate and the add-on rate (dollar charge) results was getting more sensible than before.

The Consumer Credit Labeling Bill required a single percentage disclosure for the cost of credit, but in this bill was not defined accurately this percentage, and during the following years (1960-1964), the U.S. Legislator tried to front this issue, especially to find a comprehensive unit price for credit consumer. The main aim of the Congress was to full disclosure of the cost of credit so that consumers could make intelligent choices in the marketplace.

This is why, in 1968, in U.S.A., the Legislator issued a federal law called “Truth in Landing Act”, also known through the acronym “TILA” (15 U.S.C. §§ 1601-1667f, as amended) to promote the correct information between creditors and debtors in consumer credit contracts.

The U.S. Legislator, by this law, required creditors, who deal with consumers, to make certain written disclosure concerning finance charges and related aspects of credit transactions, and to comply with other mandates.

One of these mandates is the mandatory disclosure, through advertisement, of the preliminary economic conditions of the loan, and the APR applied to the offered loan to the consumers.

By this law, the U.S. Legislator meant to protect the safety of the consumer credit market, because the correct disclosure of the conditions let the borrowers to better understand if that loan was suitable [*rectus*: affordable] and, so, in theory, better preventing the probability of default of the loan.

Another major mandate on the TILA is to insure that the American consumers receive the whole truth about the price they asked to pay for credit.

Beside this important aspect, there is a second one, equally effective as the first one here exposed, about the correct regulation of the domestic consumer credit market. In fact, if the borrowers receive a correct information about the costs of the loan they need, they even would be able to compare better the offers of the loans on the market, and entice, on the counterpart, all the banks to offer competitive prices for their product offers to the consumers.

The entire consumer credit market would benefit of such hypothetical scenario, and these would be the first and immediate consequences, in theory.

- Less probability of default of the borrowers/loans;
- Less risks in lending money for the banks;
- Stronger liquidity and solvability of the credit system;
- More competitive prices (APRs);
- More reduction of the funding costs for the banks;
- Less funding costs transferred to the borrowers;
- Less difference among face value of the loans than their effective *traditio*.

The guideline expressed by the TILA had been implemented by the Consumer Financial Protection Bureau (CFPB) in Regulation Z of the Act, and, after, by the recent Dodd–Frank Wall Street Reform and Consumer Protection Act (commonly referred to as Dodd–Frank) on 2010. These two last acts stress the importance of the ethic and correctness of the counterparties involved in consumer credit market, to guarantee both the lenders and the borrowers.

The aim of TILA law is to let the lenders deserve to earn a fair rate of return to compensate them for their time and for the risk that the borrower will not repay the interest and/or principal on time or in full.

At the same time, this law is to let the borrowers to understand what it means subscribing a contract for a loan, and its consequences, in general, plus the exact cost they will have to front for all the period of the loan received by the creditors.

This general idea underlies the concepts of an ethic moneymaking, valid for both sides: creditors and debtors, equivalently. For this reason, the APR and the EAR find themselves applied on a large scale, especially, in the Depository Institutions and in the Markets, where savings accounts and certificates of deposits (CDs) are offered to individual savers.

To entice individuals to save with them, these financial institutions often state annual percentage rates but compound the interest more frequently than once a year, and this is another reason why the APR and the EAR result to be fundamentals, because the EAR reflects the true interest rate [*rectus*: yield] of the savings.

Of course, since most financial institutions depend on the spread between their cost of obtaining funds and their lending rates, they must balance the effective annual rates at which they borrow and lend.

The most common loans, typically, provide for monthly compounding so that the EAR is higher than the APR, and more frequently is the compounding period, greater is the spread between the APR and the

EAR values. The compounding of the interest is a relevant factor that influences the APR and the EAR, and, through them, the US APR and the following European Union APR of late '80es, that will be analysed hereafter in this paper.

A new recent debate at the Italian courts is about a sort of "mysterious" hidden costs intrinsic in the amortisation plan for the mortgage constant. Apparently, the Italian experts found during the last years that all the mortgage constants have an implicit hidden cost (in Italian "***occulto***") due to the kind of interest regime applied into it (see, e.g. *Court of Rome n. 2188/2021 of Jan. 21st 2021*). This newborn "theory" is not a new theory at all, and, instead, it demonstrates, again, the poorest quality of the Italian experts about these matters.

To answer to all these "experts" about this "*occulto*" cost in a mortgage constant, we can recall David A. Stangeland and Charles E. Mossman ("*An Effective Method for Teaching and Understanding Interest Rate Conversions*", Journal of Financial Education, Vol. 32, Spring 2006, pp. 103 – 107). In this study, the authors evidenced the correct technical quotation terms and right distinctions about the given (g) and desired (d) rates, précising the following terms of equation:

r_g = the quoted amount of the given interest rate;
L_g = the quotation period from the given interest rate;
m_g = the compounding frequency of the given interest rate;

and

r_d = the quoted amount we need to calculate for our interest rate;
L_d = the quotation period from the desired interest rate;
m_d = the compounding frequency of the desired interest rate.

The authors made even an empiric example to demonstrate what they defined the "***implied effective rate***" and that would be our "*occulto*" cost of this newborn "theory" in Italy (!!!). The example used is the following:

r_g = .20 = 20.00%;
L_g = 1 year;
m_g = 4

These information stands for a quoted amount of 20.00% (rate per year), with 1-year quotation period, compounded quarterly and it means that the interest is compounded every quarter of the one-year quotation period (0.25 years).

For what it concerns the desired rate, we know the following:

L_d = 0.5 years (or 6 months);
m_d = 6 (monthly compounding implies the desired rate is compounded 6 times in the half-year quotation period).

Therefore, once we have completed the conversion process, we will have solved for the desired quoted rate r_d, which is currently unknown. Starting by these evidences, the authors introduce the above-mentioned "*implied effective rate*" with this definition: given the quotation of any rate, we can divide the quoted amount, r_g, by the compounding frequency, m_g, to get the implied effective rate, using the following notations:

r_{ieg} = the amount of the implied interest rate determined from the given interest rate
L_{ieg} = the quotation period for the given rate's implied effective rate.

r_{ied} = the amount of the implied interest rate to be determined for the desired interest rate
L_{ied} = the quotation period for the desired rate's implied effective rate.

By definition for effective rates, the compounding frequency for both r_{ieg} and r_{ied} must always be exactly 1 (hence notation is not required for these compounding frequencies). Expressing the process to determine r_{ied} in formula form, we divide the quoted amount by the compounding frequency as follows:

$$r_{ieg} = \frac{r_g}{m_g}$$

The authors found the quotation period for an implied effective rate in a similar manner:

$$L_{ieg} = \frac{L_g}{m_g}$$

$$L_{ied} = \frac{L_d}{m_d}$$

The authors evidenced that taken together, the following equations:

$$r_{ieg} = \frac{r_g}{m_g} \text{ and } L_{ieg} = \frac{L_g}{m_g}$$

give us the implied effective rate. This implied effective rate has a quotation period equal to the compounding period of our original given quoted rate. The same authors, starting by these equations, arrived to convert this implied effective rate from the given quotation to the implied effective rate for the desired quotation as follows:

$$(1 + r_{ieg}) \wedge \frac{L_{ied}}{L_{ieg}} = (1 + r_{ied})$$

which implies that

$$r_{ied} = (1 + r_{ieg}) \wedge \frac{L_{ied}}{L_{ieg}} - 1$$

To convert the implied effective rate for the desired quotation, r_{ied}, to the desired quote amount, r_d, the authors used a variation of the equation $r_{ieg} = \frac{r_g}{m_g}$ and they have multiplied the desired rate's implied effective rate, r_{ied}, by the desired rate's compounding frequency, m_d, to get the desired quoted rate.

$$r_d = r_{ied} * m_d$$

Instead, many textbooks only indicate the following formula for converting between interest rates:

$$r_E = \left(1 + \frac{r_s}{m}\right)^m - 1$$

Unfortunately, while this formula is occasionally useful, it does not handle many of the required interest rate conversions necessary for input into time-value calculations. It is for this reason that the above-mentioned equations found by Stangeland and Mossman are more recommended instead of this last general equation, and the three-steps above mentioned will always give the desired rate quotation.

Most over, this is why the Italian experts find an "*occulto*" cost, that, instead, is due to the fragility and generality of this last equation commonly used to convert interest rate up to the compounding periods.

CONCLUSIONS

I assume that EU APR formula is on compounded interest as well, but "cleaned" of the daily compounding of the interest, as per US APR [rectus: APY], and fixed on a yearly basis, so that the compounding mechanism would start after one year, as per general EU law contrary to anatocism. Furthermore, in Italy, anatocism is a civil violation, and no one has interest in making it known despite the common "constant mortgage" contains a typical compounded interest amortization plan, as consequence of anatocism. Mainly, this situation comes from the EU APR formula, based upon the study of Prof.

Seckelmann (Final report on tender n° XXIV/96/U6/21 SECKELMANN, R., Methods of calculation, in the European Economic Area, of the annual percentage rate of charge, Final Report 31 October 1995, Contract n° AO 2600/94/00101). Anyway, up to now, all EU Countries apply Prof. Seckelmann EU APR formula, denying the natural compounding interest mechanism in it, though.