An Application of the "Recursive Flexible Window" Methodology to Test for Financial Bubbles in a Major Stock Market

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Identifying and dating financial bubbles in real time is in the forefront of current empirical research. Their accuracy provides real time useful "warning alerts" to central bankers and fiscal regulators. The complexity of their nonlinear structure and the inherent sudden break mechanisms makes the econometric testing challenging. The new recursive flexible window methodology provided by Phillips, Shi, and Yu (2015) gives consistent results and delivers significant power gains when multiple bubbles occur. It successfully identifies well-known historical episodes of exuberance and collapse. In this paper we look at the Indian stock market indices, the SENSEX, and the NIFTY 50, to see if there is any evidence of a bubble there. We use monthly data for each series, with the Sensex data spanning April 1979 to October 2018 and NIFTY 50 data spanning July 1990 to October 2018. The existence of bubbles in this index will give us some indication of where bubbles are more likely to occur, and therefore provide evidence of potential economic (financial) crises.

Keywords: bubbles, recursive, window, simulation

INTRODUCTION

History is replete with incidents of financial crises which ex-post become a wakeup call for policy makers and the people. Experts have repeatedly said that the present crisis was preceded by "asset market bubbles" and / or "excessive credit expansion." But the fact of the matter remains that we do not have good quantitative markers which can ex-ante indicate the beginning of a "bubble" in the asset markets which may lead to a catastrophe later. If we had quantitative observable "warning signs" it might be possible to avoid some economic debacles. Thus, after the most recent global financial crisis of 2007-2009, the main thrust in the Basel III accord was to emphasize on more close and determinate market surveillance, so that bankers and policy makers could be forewarned of a possible impending implosion.

Cooper (2008) has said "Economists have taught us that it is unwise and unnecessary to combat asset price bubbles and excessive credit creation. Even if we were unwise enough to prick an asset price bubble, we are told it is impossible to see the bubble while it is in its inflationary phase." Thus, we must accept that there is no practical way to identify the beginning of a crisis. The task at hand is to try to identify possible quantitative markers from the data, that something is "wrong" and that a speculative bubble is probably taking shape. It will worsen if measures to "quell" it are not taken. That is where the Phillips, Shi, and Yu

(PSY henceforth, 2014) research comes into effect. This paper offers a powerful and credible "quantitative metric" to detect exuberance in financial data right where it is originating. This procedure helps us pinpoint the start of the problem and can thus help us monitor the markets. Once detected, policies to counter the exuberance can be promulgated and implemented.

Since we know that history has proven that it has a bad habit of repeating itself, this early warning diagnostic tool will come in handy in helping formulate policies to avert the impending crisis. The best part of this test is that it can be implemented on current data in real time and therefore try to detect the "fault lines."

In the economics literature we have multiple tests to detect the crisis ex-post, and then explain it. Gurkaynak (2008) is a helpful review of this documentation. But there was no test to identify ex-ante the origination of a bubble. Phillips, Wu, and Yu (PWY henceforth, 2011) presented a recursive method to detect exuberance in asset prices during an inflationary phase. The advantage here is that the early detection (ex-ante acknowledgement) can help banks, regulators, and policy makers to address the problem in its nascent state. PWY is effective in the early detection of bubble markers, provided there was a single bubble in the data sample. They proved the effectiveness of the test using NASDAQ (PWY, 2011) and the US housing bubble, (Phillips and Yu (2011)).

But then came the question of "economic reality" which showed that there usually were multiple recurring financial crises over long periods. Ahmed (2009) gave us evidence of sixty different financial crises, in the 17th century alone. Thus, the next step in the evolution of these detection tests was to create the one that could identify multiple bubbles in the same sample period. A test to clearly identify periodic collapsing and recovering economic data was simply not there. This recursive identification is extremely complex compared to identifying a single bubble. The main problem is computationally managing the non-linear structure of multiple breaks / bubbles in the data. With the presence of multiple break points in the data, the discriminatory power of the detectors goes down dramatically and hence the upswings and downswings are not decipherable in the same data stream. Thus, the challenge is twofold:

1. Come up with a statistical metric which can detect multiple factual fractures in the non-linear data stream and
2. Be powerful and effective enough so as not to have a false negative detection tolerance (to avoid unnecessary policies) and a high positive detection tolerance (to ensure good and early effective policy application.)

The PSY (2014 and 2015) papers present a recursive econometric technique to test financial bubbles in the data and separate them when multiple bubbles are present. Here the authors extend the PWY (2011) methodology, which is based on a sequence of forward recursive right tailed ADF unit root tests, using the Sup ADF (designated SADF) measure. This process allows for a dating strategy to identify the origination and termination dates of a specific bubble. This is achieved by using "backward regression techniques." The PSY (2014 and 2015) papers present an extension of the SADF tests, in form of a generalized SADF called the GSADF method. It includes a recursive backward regression technique, to time identify the origin and collapse of bubbles. It is a right tailed ADF test but has a flexible window width to separate one bubble from the next, to the next sequentially, since their lengths are bound to be different. In structure and logic, it is analogous to the left-sided recursive unit root test of Leybourne, Kim and Taylor (2007), this being a right-sided double recursive unit root test.

In this paper we implement this test on the Indian stock market indices, the SENSEX, and the NIFTY 50. We use monthly data for each series, with the Sensex data spanning April 1979 to October 2018 and NIFTY 50 data spanning July 1990 to October 2018. We find evidence of the existence of multiple bubble in this index, which correspond closely to reality. We believe that this information will be of interest to researchers in this field. Section 2 describes the reduced form model, the new rolling window recursive test and its limit theory. Section 3 elaborates the data stamping strategies, to separate single, double, and multiple bubbles in the same sample period. Section 4 is simulation results of the size, power and performance of the dating strategy tests. In section 5, we apply the PWY test, the sequential PWY test and the CUSUM test to the Indian SENSEX and NIFTY 50 data. Section 6 contains some concluding remarks.

ROLLING WINDOW TEST FOR BUBBLES

PSY (2015) develops the limit theories and consistency properties in case of single and multiple bubbles. PSY (2015, b) is a supplement describing the robustness checks of this testing procedure.
It originates with the standard asset pricing model:

$$P_t = \sum_{i=0}^{\infty} (\frac{1}{1+r_f}) i \, E_t(D_{t+i} + U_{t+i}) + B_t \tag{1}$$

where P_t = after dividend price of an asset
D_t = payoff (dividend) from the asset
r_f = risk free interest rate
U_t = unobservable fundamentals
B_t = bubble component
Here $P_t^f = P_t - B_t$ (market fundamentals) and B_t satisfies the sub martingale property

$$E_t = (B_{t+1}) = (1 + r_f) B_t \tag{2}$$

This equation sets up the alternative scenarios for the presence / absence of bubbles in the data. For example: If there are no bubbles, the B_t =0, then the degree of non-stationarity [I(0) or I(1)] of asset prices is controlled by asset payoffs or dividends (D_t) and the unobservable economic / market fundamentals. The advantage of the reduced form model is that it pretty much encompasses all standard formulations as intrinsic bubbles (Froot and Obstfeld, 1991), herd behavior (Abreu and Brunnermeier, 2003), time varying discounting (Phillips and Yu, 2011)[1].

A possible outcome would be like this: If D_t is an I(1) process, the U_t must be either I(0) or I(1) and asset prices can at the most be a I(1) process. But based on eq. (2), if there are bubbles, then asset prices will be explosive. Thus, when the fundamentals are I(1) and D_t is first difference stationary, we can infer bubbles if asset prices show evidence of explosive behavior. Eq (1) is one way to include a bubble variable in the standard asset pricing model, but the jury is still out on this.[2,3]

According to Phillips and Magdalinos (2007), explosive behavior in asset prices is a primary indicator of market exuberance, which can be identified in empirical tests using the "recursive testing procedure" like the right-side unit root test of PWY. This recursive procedure starts with a martingale null (with drift to capture long historical trends in asset data.) The model specification is:

$$y_t = dT^{-n} + \theta_{yt-1} + \epsilon_t \tag{3}$$

where $€_t$ is iid (0, $Ϭ^2$), $\Theta = 1$, and d is a constant, T is the sample size, and the parameter η controls the magnitude of the intercept and the drift, as $T \rightarrow \infty$. Solving eq. 3, gives us the deterministic trend, dt/T^n. Here there are three possibilities:

1. If n>0, the drift will be small compared to the linear trend.
2. If n>1/2, the drift is small relative to the martingale
3. If n=1/2, the output behaves like a Brownian motion, which is evident in many financial time series data.

The researcher needs to be careful and exercise caution because the emphasis here is on the alternative hypothesis, because departures from market fundamentals are the markers of interest. But as with all types of model specifications, we know that they are sensitive to intercepts, trends and trend breaks etc., as described in PSY (2014). Eq. 3 is tested for exuberance using the rolling window ADF approach or the recursive approach of the authors. The basic logic is that if the rolling window regression starts from the r_1^{th} fraction and ends with the r_2^{th} fraction (from sample size T), then $r_2 = r_1 + r_w$, where r_w is the size of the window. This model is:

$$\Delta y_t = \alpha_{r1,r2} + \beta_{r1,r2}\, y_{t-1} + \Sigma_{i=1}^{k} \gamma_{r1,r2}^{i}\, \Delta_{yt-1} + \epsilon_t \tag{4}$$

where k is the lag length, and ϵ_t is iid, with (0, $\sigma^2_{r1,r2}$). The basic form is reformulated to include the presence of "multiple bubbles" to separate the market switching time periods from explosion to contraction, and again explosion sequentially. They use the Sup ADF test called SADF. It is a recursive / repeated estimation procedure with window size r_w., where r_w goes from r_0 (smallest sample window fraction) to r_1 (largest sample window fraction), and sample end point $r_2 = r_w$, going from 0 to 1. The SADF statistic i:[4]

$$\text{SADF}\ (r_0)\ n= \sup_{r2\in[r0,1]} \text{ADF}^{r2}_{0}$$

Rolling window GSADF test: The ADF regression is run on eq. 4, recursively, but continuously on sub-samples of the data based on window width chosen according to $r_0, r_1, r_2 \ldots\ldots r_w$. The subsamples chosen here are more extensive than the SADF test. The difference here is that we allow the window width to change within the feasible range where $r_w = r_2 - r_1$. The GSADF statistic is:

$$\text{GSADF}\ (r_0) = \sup r_2 \in[r_0, 1]\ \{\text{ADF}^{r2}_{r1}\} \qquad r_1\in[0, r_2\text{-}r_0] \tag{5}$$

The GSADF statistic as given in eq. (5).[5] Here we see that the limit distribution of the GSADF holds (is identical), but with the intercept and the assumption of a random walk structure, we have no drift or small drift. The GSADF's asymptotic distribution depends on the "smallest window width size r_0." Care needs to be exercised on choosing the width of r_0. It depends on the number of observations in the sample.

Case (1): If T is small, r_0 must be made large enough to ensure the inclusion of an adequate number of observations.

Case (2): If T is large, r_0 should be set small, to be able to include different "explosive" burst in the data. The authors run simulations and derive the critical values (CV's). The conclusions are:

1. As r_0 decreases, CVs of the test statistic increases
2. For r_0 given, the CVs are constant in finite samples.
3. GSADF statistic CVs are larger than the SADF statistic, which is larger than the ADF statistic, and its concentration also increases, increasing confidence in the test outcomes. The backward SADF statistic is the sup value of the ADF sequence run over this interval.

$$\text{BSADF}\ r_2\ (r_0) = \sup_{r1\in[0,\ r2\text{-}r0]} \{\text{ADF}\ ^{r2}_{r1}\}$$

We see here that the ADF test is a special case of the backward sup ADF (BSADF) when r_1=0.

The empirical steps are:

1. Determine $\text{ADF}r_2$ and the sup ADF within the feasible range of r_2 (from r_0 to $r_{1.}$) The origination of the bubble is dated. This procedure imposes the condition that the bubble marker is the existence of a critical value greater than L_T = Log (T). This separates the short and temporary market blips (which happen all the time in real life) from actual exuberance. Dating is done using the formula:[6,7,8]

$$r_e^{\wedge} = inf_{r2\in[r0,1]}\{r_2 : ADF_{r2} > cv_{r2}^{\beta T}\} \tag{6}$$

and

$$r_f \quad = inf_{r2\in\left[r_e \quad + \frac{\log(T)}{T,1}\right]} \left\{r_2 : ADF_{r2} < cv_{r2\}}^{\beta T}\right\} \tag{7}$$

where $cv^{\beta T}_{r2}$ is the $100(1\text{-}\boldsymbol{\beta}_T)$ % critical value of the ADF statistic based on $[T_{r2}]$ observations. Here $\boldsymbol{\beta}_T \rightarrow 0$, as $T\rightarrow\infty$.

DATA STAMPING STRATEGIES

The idea is to identify bubbles in real time data and then look for the "markers" identifying those bubbles / episodes of market exuberance. The problem is that the standard ADF test can identify extreme observations, as r = [T_r], but cannot separate between a bubble phase observation from one which is part of a natural growth trajectory. Market growth is not an indication of bubbles. Thus, ADF tests may result in finding "pseudo bubble detection." Making this distinction is the major contribution of this test. The authors run backward sup ADF or backward SADF tests, to improve the chances of deciphering a bubble from a growth trajectory. The recursive test means running SADF backwards on the sample, increasing the sample sequence using a fixed sample r_2, but varying the initial point from 0 to (r_2-r_0). This gives the SADF statistic:

$$\{ADF^{r2}_{r1}\} \in |0, r_2 -0|$$

Bubbles are inferred from the backward SADF statistic or the BSADF $r_2(r_0)$. The origin of the bubbles, the date and timing, is the first observation whose BSADF statistic exceeds the critical value of the BSADF. The bubble ending date is the first observation whose BSADF is below the BSADF critical value. The intermediary period is the duration of the bubble. The origination / termination dates are calculated thus:

$$\hat{r_e} = inf_{r2\in[r0,1]}\{r_2: BSADF_{r2}(r_0) > scv^{\beta T}_{r2} \tag{8}$$

$$\hat{r_f} = r_{2\in}[inf_{\hat{r_e}+\left[\frac{\partial \log(T)}{T,1}\right]}\{r_2: BSADF_{r2}(r_0) > scv^{\beta T}_{r2} \tag{9}$$

where $scv^{\beta T}_{r2}$ is the 100(1-$\boldsymbol{\beta}_T$)% critical value of the sup ADF statistic, based on [T_{r2}] observations. $\boldsymbol{\beta}_T$ goes to zero, as the sample size approaches infinity. The distinction between the SADF and the GSADF (backward sup ADF) tests, both run over $r_2 \in [r_0,1]$ is given by the statistic as:

$$SADF\ (r_0) = sup_{r2\in[r0,1]}\ \{ADF_{r2}\}$$

and

$$GSADF\ (r_0) = sup_{r2\in[r0,1]}\ \{BSADF_{r2}(r_0)\}$$

SIMULATIONS

Simulations were performed to examine the credibility of the PWY, sequential PWY, CUSUM and the GSADF test, in terms of size and power, but most importantly their capability to identify multiple bubble episodes. The basic data generating process is given by:

$$y_t = dT^{-n} + \theta_{yt-1} + \epsilon_t \tag{10}$$

with δ =n =1. They examine two different models, namely Evans (1991) collapsing bubble and the PWY model. Simulations using the same data set, same number of observations / replications show that the size distortion of SADF > GSADF. The next question is the effect of the lag selection length. Both SADF and the GSADF have size distortion weakness. But its magnitude is small when we use a fixed lag length in recursive tests. But GSADF has smaller distortion than SADF and thus has a leg up on the latter in lowering the probability of "false detection." The authors recommend the fixed lag length use with the GSADF test for multiple bubbles. They find that the SADF test has an inherent weakness evidenced repeatedly. It could not identify bubbles when the full sample was used but was able to do so when the sample was truncated. But the recursive application of the GSADF test was able to identify multiple bubbles, without having to arbitrarily truncate / segment / re-select sample starting points. This is a major advantage of GSADF over the SADF procedure. Moreover, the results show that the bubble identification power of the GSADF test increases as the sample size increases.

EMPIRICAL APPLICATION

We use monthly data for the BSE Sensex for the period April 1979 to October 2018, for a total of 475 observations and for the Nifty 50 for the period July 1990 to September 2018 for a total of 339 observations. The BSE data set was obtained from the website of the BSE (https://www.bseindia.com/market_data.html). The Nifty50 data set was obtained from the website of NSE Indices limited (http://www.niftyindices.com/reports/historical-data), a subsidiary of National Stock Exchange of India Limited. The data used is the respective stock price index for the relevant month. We then conduct the SADF and the GSADF tests on the stock price index according to the basic model in eq.(1). The results are given in tables 1 and 2. Also given in each table are the critical values of the two tests obtained from a simulation exercise using 2000 replications of the data in each case.

TABLE 1
BSE SENSEX

	Test Statistic	Finite Sample Critical Values		
Number of observations = 474		90%	95%	99%
SADF	8.6552	1.2037	1.4447	2.0588
GSADF	8.6552	2.0352	2.2836	2.8509

TABLE 2
NIFTY 50

	Test Statistic	Finite Sample Critical Values		
Number of observations = 339		90%	95%	99%
SADF	4.5858	1.1390	1.4196	2.0308
GSADF	4.9365	1.9611	2.1967	2.7155

Both tests find evidence of bubbles or explosive sub-periods over the long-term data for both indices (test statistics in each case exceed the critical values for both test statistics considered). We then conduct a bubble monitoring exercise for each index using the backward ADF test and its critical value (using the PWY strategy), and the backward SADF statistic and its critical value (using the PSY strategy). This is done in figures 1–4 which are given in the appendix of the paper.

FIGURE 1
BSE SENSEX BACKWARD ADF (SADF) STATISTIC

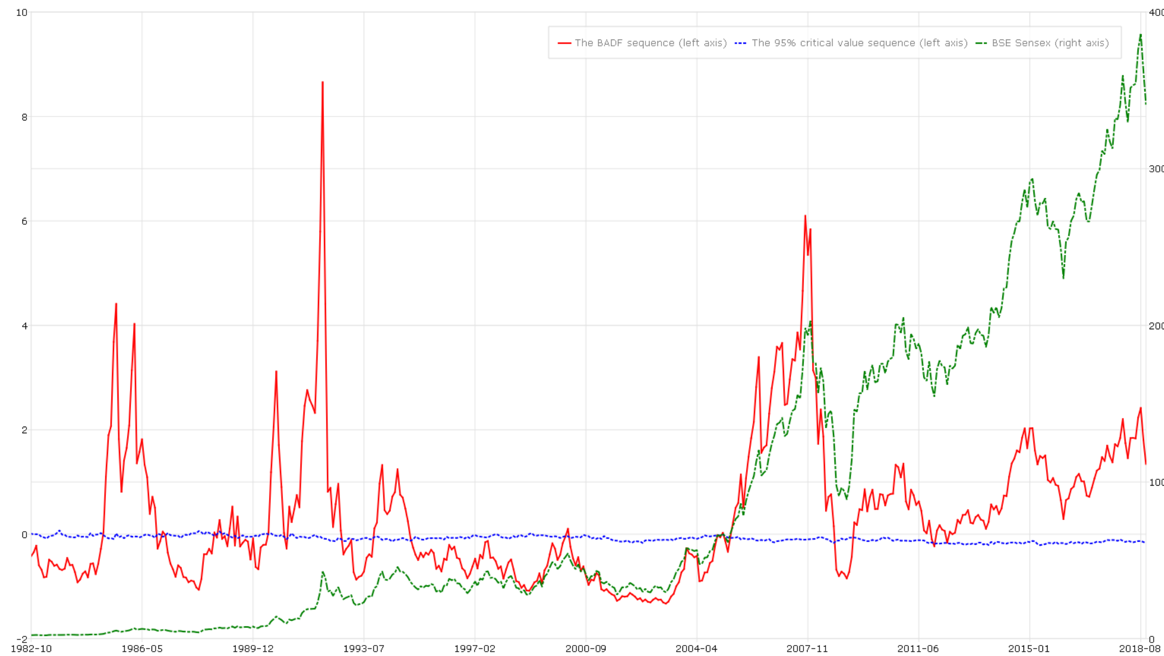

FIGURE 2
BSE SENSEX BACKWARD SADF (GSADF) STATISTIC

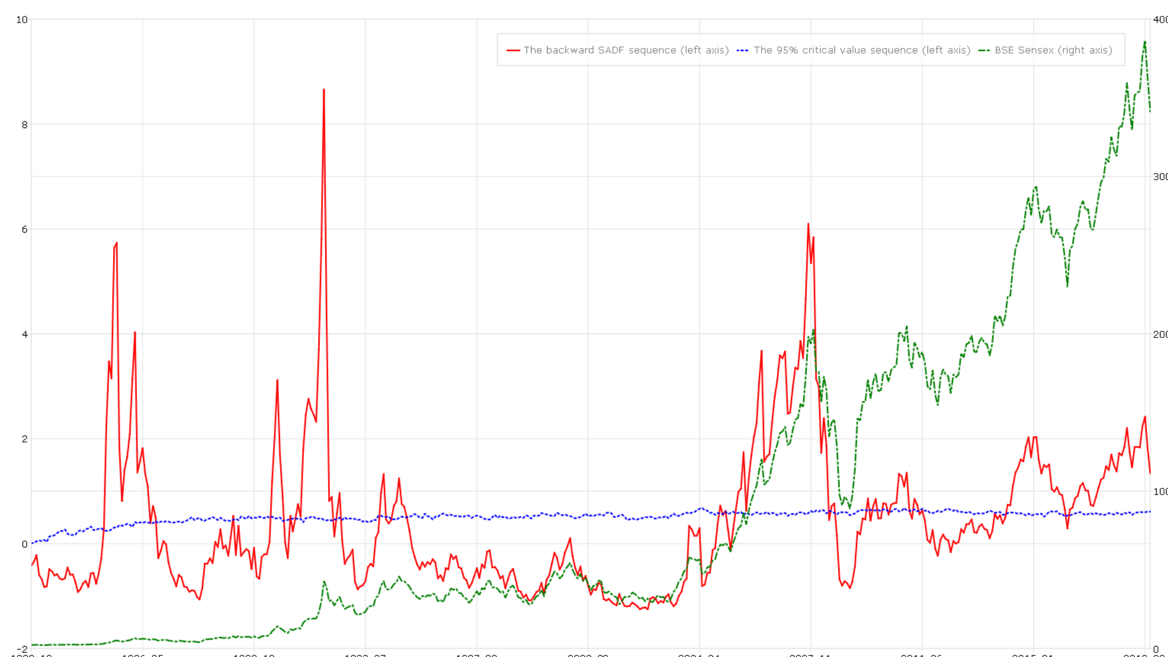

FIGURE 3
NIFTY 50 BACKWARD ADF (SADF) STATISTIC

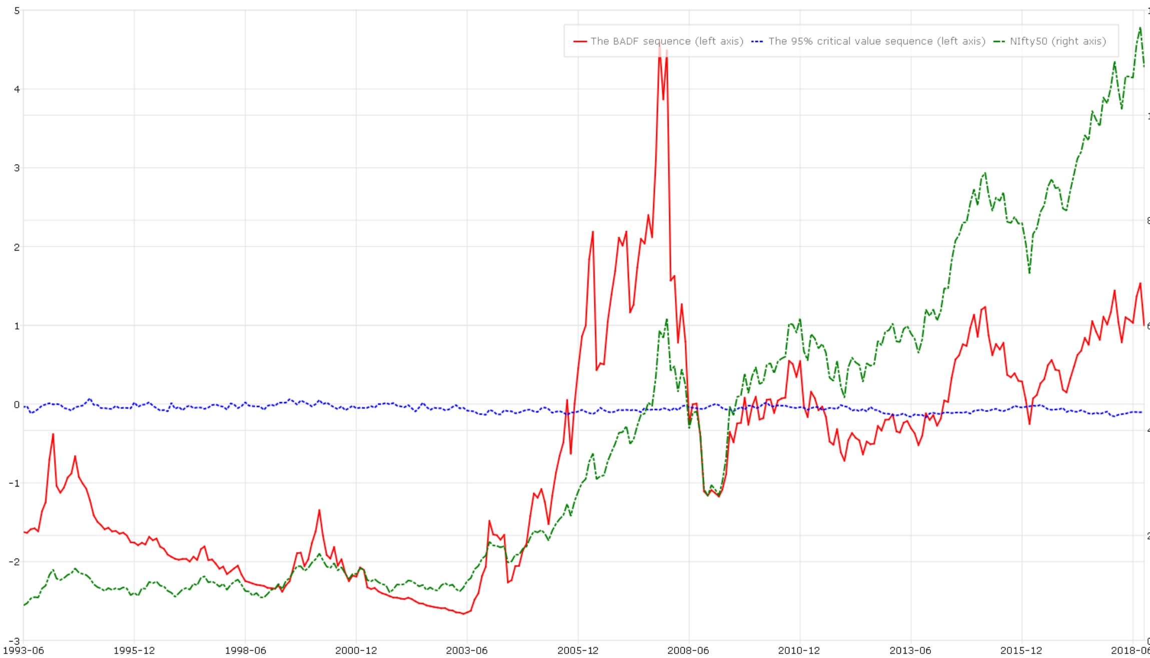

FIGURE 4
NIFTY 50 BACKWARD SADF (GASDF) STATISTIC

In each figure the solid line represents the relevant test statistic, and the dark broken line represents the critical value, and the light broken line represents the BSE Sensex index. Figures 1 and 3 present results from the use of the backward ADF test from the PWY paper, and figures 2 and 4 present results from the use of the backward SADF statistics from the PSY paper. In Figure 1 we look at the BSE Sensex and the existence of a bubble (test statistic greater than the critical value) is evident in the mid-1980s,and again in the late 1980s to early 1990s. The period from mid-2004 to the end of our sample period in October 2018 is clearly very volatile, and some of this volatility is prior to the financial crisis. Figure 2 shows a bubble again for the mid-1980s and late 1980s to early 1990s (just like in figure 1), and a volatile period after mid-2004. The ability of the BADF statistic to detect multiple bubbles is suspect, and therefore the results in Figure 2 (based on the PSY paper) are more reliable.

A similar bubble monitoring exercise is carried out for the Nifty50 index in figures 3 and 4. Figure 3 indicates the existence of multiple bubbles. These bubbles occur from 2004 till about 2008 and then a small one around 2010 and after 2014. Figure 4 indicates almost the same pattern

Results from the backward ADF and the backward SADF statistic are quite similar for the BSE Sensex and the Nifty50. The backward SADF statistics (Figures 2 and 4) are considered more reliable for investigating multiple bubbles. Both indices have evidence for bubbles in the 2000s, up to about 2008 and again after 2014. We clearly do have evidence for a bubble in the 1990s and therefore we can conclude that there is evidence to support the widely referred to technology bubble in the late 1990s and early 2000s. There is limited evidence to indicate the existence of bubbles in 2007 -09 around the time of the financial crisis.

CONCLUSION

The new test, the GSADF procedure is a recursive test, able to detect multiple bubbles. It's a rolling window, right sided ADF unit root test, with a double sup-window selection criterion. The SADF test is good, but it cannot credibly detect multiple bubbles over the same sample data set. The GSADF test overcomes this weakness and has significant discriminatory power in detecting multiple bubbles. It makes it very relevant in studying the "time trajectory" of long historical data sets. We have evidence for the existence of bubbles in the1980s for both indices, thus providing evidence of a volatile stock market in the 1980s. There is evidence in favor of bubbles in the early to mid-2000s, before the time of the financial crises of 2008. Bubbles again reappear from about 2014 onwards. This is likely due to fall in the value of the rupee and other changes in the Indian economy and the economy of its neighbors and major trading partners.

ENDNOTES

1. See Shi (2011) for an overview of this literature.
2. Cochrane (2005) debates the rationale of including "bubble components" in an asset pricing model, while Cooper (2008) expresses bewilderment at the literatures attempt to rationalize the well accepted NASDAQ bubble, as an accurate reflection of the changing market times and environment.
3. Interestingly, the experts agree more on the presence of market exuberance leading to panics, either rationally or irrationally. It is based on changing economic fundamentals, arising from behavior alterations of market players, or due to changing discount rates over time etc.
4. Then there is the Markov-switching test of Hall, et.al (1999), to detect explosive behavior in the data sample, but it is open to suspicion since Shi (2013) found it to be susceptible to "false detection of explosiveness." Also, according to Funke et.al. (1994) and Van Norden and Vigfusson (1998), general filtering algorithms cannot differentiate between spurious explosiveness (the marker being high variance) as opposed to generic explosive behavior. The general approach of SADF is also used by Busetti and Taylor (2004) and Kim (2000) among others, to study "market bubbles" but the simulation study done by Homm and Breitung (2012) finds the PWY (SADF) test to be the most powerful metric in detecting multiple bubbles.
5. Eq. (5), Theorem 1, from PSY (2014)

6. The data process before the origination of the bubble is assumed to be a random walk for convenience, and it is the usual practice, but not necessary for the asymptotic properties to hold.
7. The authors have proven the consistency of ($\hat{r}_e$, $\hat{r}_f$) in PY (2009). Also see Phillips and Solo (1992) and Phillips and Shi (1994).
8. This sequential procedure (for proper and credible application) requires a long set of observations, the longer the better, to re-initialize the test process after a bubble.

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