Regime-Switching Dynamic Nelson-Siegel Modeling to Corporate Bond Yield Spreads with Time-Varying Transition Probabilities

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The purpose of this study is to develop a regime-switching extension of the dynamic Nelson-Siegel term structure model and apply it to Japanese corporate bond spread data on an individual firm basis for the period April 1997 through December 2011. The results indicate that the estimated regime probability is closely linked to business and market sentiment. The regime-switching model is extended by adopting a time-varying transition probability matrix driven by leading macroeconomic indicators. The overall fit is improved by incorporating a time-varying transition probability matrix. Our results imply the importance of incorporating regime shifts into modeling the term structure of credit spreads.

INTRODUCTION

Recent studies on the term structure of credit spreads explain credit spread dynamism using an affine term structure model or a Nelson-Siegel model (Nelson-Siegel (1987)). These models are consistent with principal component analysis in which the first, second, and third factors explain over 90% of credit spread variation.

However, the question of whether the current term structure model explains 100% of the variation of the yield curve considering the non-normality or discontinuity of credit spreads remains unanswered.

In the research on the term structure of government bond yields, the models incorporating the jump process, or regime shifts, have considered the non-normality or discontinuity of government bond yield. The development of a term structure with regime shifts was earliest introduced by Hamilton (1989), Garcia and Perron (1996), and Gray (1996). The authors developed and estimated time series models to capture the dynamism of short-term interest rates. Landen (2000) solved the closed-form solution of bond price with two regime shifts using the Gaussian model, while Bansal and Zou (2002) studied a two-factor Cox-Ingersoll-Ross model with mean- reverting parameter shifts and shifts in both volatility and the market price of risk. Both studies insist that the regime-switching model has superior goodness of fit to the three-factor affine term structure model, and regime is closely linked with the business cycle and monetary policy.

However, as Litterman and Schenkman (1991) note, over 90% of the yield curve variation is explained by three principle components (level, slope, and curvature) of the term structure. One or two-factor models require modification even if they admit regime shifts.

To overcome this difficulty, Dai and Singleton (2000) developed a multi-factor affine term structure model with regime shifts. Dai et al. (2007) developed a discrete time three-factor regime-switching term structure model, which is composed of three factors and addresses the market price of risk with regime shifts. The authors also show the term structure of the historical volatility for each regime, which is useful for practitioners because the term structure of volatility is applicable to risk management and investment strategy.

Existing studies have developed Nelson-Siegel models with regime shifts for government bonds. Bernadell et al (2005) and Nyholm (2007) estimate the model for which the slope factor shifts into three regimes and investigate the relation between regime and the business cycle. Zhu and Rahman (2009) present and estimate a regime-switching macro-finance model of term structure with latent and macroeconomic factors, simultaneously examining the joint dynamics of the yield and macro factors. The authors note that two regimes do not fully explain the business cycle. Xiang and Zhu (2013) confirm that the empirical results support the two-regime Nelson–Siegel term structure model.

Moreover, Levant and Ma (2016) find that the factor loading parameter and latent factors conditional volatilities of the latent factors show significant switch. These authors use the Kalman filter, which can efficiently extract the level, slope, and curvature factors, and the Kim algorithm (Kim (1994)), which allows them to extract the states.

Since the Nelson-Siegel model produces three factors that capture the information in the entire term structure, the dynamic relationship can be analyzed among other asset classes. For example, Krishnan,et al. (2010) and Hua (2013) analyze the corporate credit spread and Shaw et al. (2014) extend the standard dynamic Nelson-Siegel model to a credit default swap. However, limited studies have been conducted on the regime-switching term structure model for credit spreads with the exception of Dionne et al. (2011)[1]. They assume that the possibility of change in regime might affect corporate bond spreads because it might influence the macroeconomic factors and risk- free interest rates. The authors examine the ability of observed macroeconomic factors and possibility that changes in regime explain a proportion of yield spreads caused by default risk, in the context of a reduced form model.

To summarize the previous research, the regime-switching term structure model represents a new field of research that began in early 2000.The model types include Nelson-Siegel and affine term structure models. In addition, the models extend regime shifts in the term structures of mean reversion, volatility, the market price of risk, etc. However, limited research addresses credit spreads.

The purpose of this study is to develop a regime-switching extension of the dynamic Nelson-Siegel model and apply it to Japanese corporate bond spread data on an individual firm basis. The regime-switching model is extended by adopting a time-varying transition probability matrix that is driven by the variables of the structural model proposed by Merton (1974) as well as leading macroeconomic indicators. The goodness of fit is investigated for the dynamic Nelson-Siegel model (baseline model) with and without regime-shifts, and with constant and time-varying transition probabilities.

The remainder of this paper is structured as follows. In Section 2, the data are described. Section 3 presents the dynamic Nelson-Siegel model, Kalman filter, and Kim algorithm. Section 4 provides the estimation results. Section 5 concludes with the implications of the results for future credit spread modeling.

DATA

Government bond yields

We used end-of-month price quotes for Japanese government bonds from April 1997 through December 2011.Price quotes for one and 20-year maturities were obtained from the Japan Bond Trading Company Limited. Because each month has different available maturities, we linearly interpolated nearby maturities to pool them into fixed maturities of three, six, 9, 12, 24, 36, 48, 60, 72, 84, 96, 108, and 120 months. The data on Government bond yield curve data were constructed using the Fisher, Nychka, and Zervos (1995) approach.

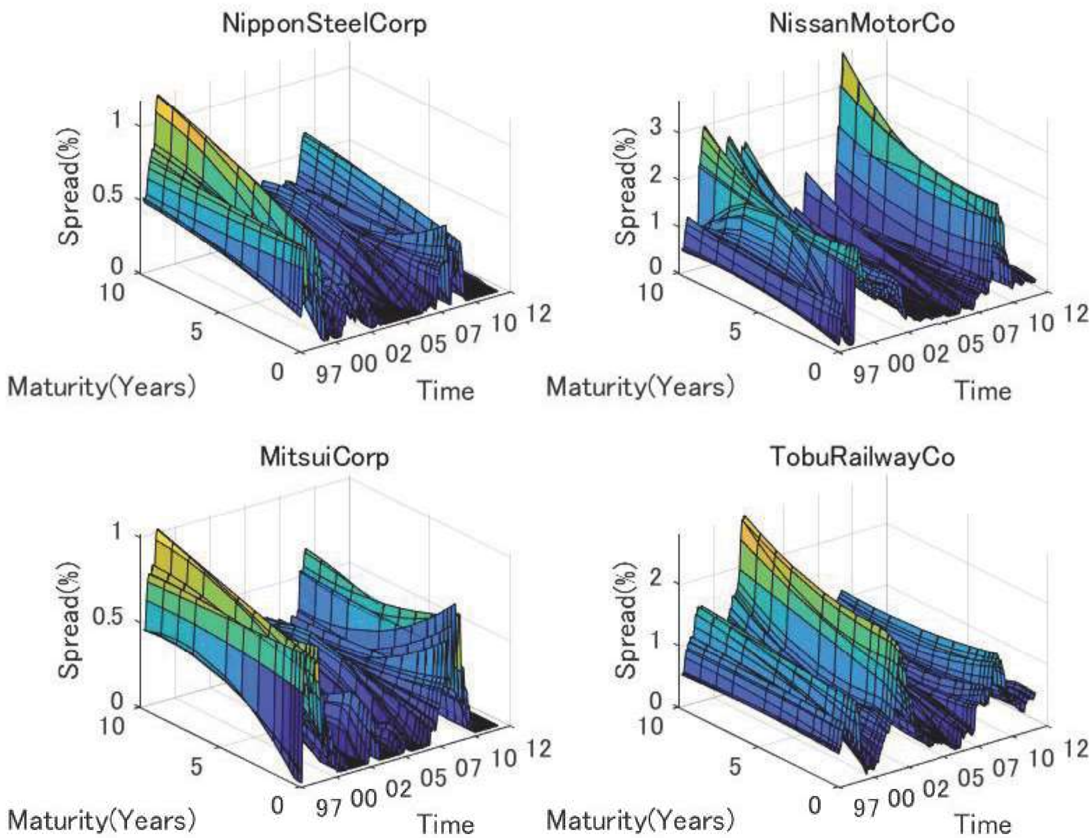
Corporate bond yields

We used over-the-counter bond transactions from the Japan Securities Dealers Association. Our sample was composed of industrial-, banking-, and service-sector firms in the Japan Securities Dealers Association at any time during the period April 1997 to December 2011. We required price data with short to long-term time to maturity of corporate bonds for many companies in each time period because the study focuses on the term structure of credit spreads. The criteria for selecting corporate bonds were constructed in the following manner.

1. Observation period: we selected firms with time series data over six years from April 1997 to December 2011. The data period started in April 1997 because the Japan Securities Dealers Association has published data since that date.
2. Time to maturity: we required corporate bonds of different maturities that have data for at least seven years for each month, in order to estimate the level, slope, and curvature factors of credit spreads.
3. Number of prices: we required a minimum of five bond prices with different maturities.

Based on the above criteria, our final sample was composed of 56 firms. This study estimated the corporate bond spread using the B-spline model of Steeley (1991)[2]. The corporate bond spread is created in such a way that corporate bond yield is subtracted from the same maturity of government bond yield. Figure 1 shows the time series of term structure of credit spreads of representative 4 firms among selected 56 names. The sample covers the period from September 1997 to December 2011. All credit spread data are monthly.

FIGURE 1
CREDIT SPREDS ACROSS COMPANIES AND TIME



Note: Figure 1 shows the time series of the term structure of credit spreads of four representative 4 firms (among the selected 56 firms)

Models and Estimation

To estimate the model, we introduce a unified state-space modeling approach that allowed us to simultaneously fit the credit curve at each point in time and estimate the underlying dynamics of the factors. This section explains the state-space representation of the term structure model and estimation methodology.

Dynamic Nelson-Siegel model

This subsection presents the dynamic Nelson-Siegel approach for which the term structure parameter is time-varying as the base model proposed by Diebold and Li (2006). First, we explain the dynamic Nelson-Siegel model and, then, develop the Nelson-Siegel model with regime shifts. The term structure factor of the Nelson-Siegel model is considered the unobservable state variable in state-space modeling. $\boldsymbol{\beta}_{j,t} = \left(\beta_{j1,t}, \beta_{j2,t}, \beta_{j3,t}\right)'$ for $j\left(j = 1, \ldots, 56\right)$.

Equation 1 provides the state equation.

$$\boldsymbol{\beta}_t - \boldsymbol{\mu} = \mathbf{A}\left(\boldsymbol{\beta}_{t-1} - \boldsymbol{\mu}\right) + \boldsymbol{\eta}_t, \tag{1}$$

$$\boldsymbol{\eta}_t \sim N\left(\mathbf{0}, \boldsymbol{Q}\right).$$

Equation 2 provides the observation equation.

$$\mathbf{s}_t = \mathbf{F}\boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t \tag{2}$$

$$\boldsymbol{\varepsilon}_t \sim N\left(\mathbf{0}, \boldsymbol{H}\right).$$

where

$\boldsymbol{T}$: length of the time series;

$\boldsymbol{\beta}_t = \left(\beta_{1,t}, \beta_{2,t}, \beta_{3,t}\right)' \in \mathbf{R}^3$: the term structure factor at time t;

$\boldsymbol{\mu} = \left(\mu_{\beta_1}, \mu_{\beta_2}, \mu_{\beta_3}\right)' \in \boldsymbol{R}^3$: mean reversion parameter of the term structure factor $\boldsymbol{\beta}_t$;

$\boldsymbol{A}$: matrix of the autoregressive process coefficient;

$\boldsymbol{\eta} \in \boldsymbol{R}^3$: disturbance term;

$\boldsymbol{Q}$: conditional covariance matrix of the term structure factor;

$\varepsilon \in \boldsymbol{R}^m$: error term of the observation equation;

$\mathbf{s}_t = \left(s^{(n_1)}{}_t, \ldots, s^{(n_m)}{}_t\right) \in \mathbf{R}^m$: credit spreads at time t for $n_i\left(i = 1, \ldots, m\right)$ time to maturity;

$\boldsymbol{F} \in \boldsymbol{R}^{m \times 3}$: coefficient matrix of the term structure factor. The element-by-element notation is as follows:

$$\boldsymbol{F} = \begin{pmatrix} 1 & \dfrac{1 - e^{-\lambda n_1}}{\lambda n_1} & \dfrac{1 - e^{-\lambda n_1}}{\lambda n_1} - e^{-\lambda n_1} \\ 1 & \dfrac{1 - e^{-\lambda n_2}}{\lambda n_2} & \dfrac{1 - e^{-\lambda n_2}}{\lambda n_2} - e^{-\lambda n_2} \\ \vdots & \vdots & \vdots \\ 1 & \dfrac{1 - e^{-\lambda n_m}}{\lambda n_m} & \dfrac{1 - e^{-\lambda n_m}}{\lambda n_m} - e^{-\lambda n_m} \end{pmatrix} \tag{3}$$

$\lambda \in \boldsymbol{R}$: parameters that determine the term structure factor, and

$\boldsymbol{H} \in \boldsymbol{R}^{m \times m}$: covariance matrix of observation error ε .The element-by-element notation is as follows:

$$\boldsymbol{H} = \begin{pmatrix} \sigma_\varepsilon^2(n_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_\varepsilon^2(n_m) \end{pmatrix} \tag{4}$$

Dynamic Nelson-Siegel model with regime shifts

The development of Markov switching extensions to time series modeling has provided a useful way of characterizing business cycle dynamics. The first approach is to restrict the transition probability to be constant over time. The problem is that the probability of switching one regime to another cannot depend on the behavior of the underlying economic fundamentals. Filardo (1994) and Diebold et al (1994) propose a class of Markov switching models in which the transition probability can vary with the fundamentals.

With regard to the Nelson-Siegel term structure model, Bernadell et al. (2005) and Nyholm (2007) propose a model in which the mean reversion of the "slope" factor depends on the regimes, and applies it to US treasury notes. The model in this study is supposed to shift the mean reversion of "level," "slope," and "curvature" in two regimes to enhance the goodness of fit. The model also incorporates the time varying transition probability in addition to the fixed transition probability.

Then, based on DNS, the model mean reversion vector μ shifts between the two regimes are constructed[3], Based on the above setting, the state-space representation is described as follows.

Equation 5 presents the modified state equation.

$$\boldsymbol{\beta}_t - \boldsymbol{\mu}^{S_t} = \mathbf{A}\left(\beta_t - \boldsymbol{\mu}^{S_t}\right) + \boldsymbol{\eta}_t, \tag{5}$$

$$\boldsymbol{\eta}_t \sim N(\mathbf{0}, \boldsymbol{Q}).$$

where

$S_t \in \{0,1\}$: variables representing regime at time t;

$\boldsymbol{\mu}^k = \left(\mu_{\beta 1}^k, \mu_{\beta_2}^k, \mu_{\beta_3}^k\right)' \in \boldsymbol{R}^3 (k = 0,1)$: mean reversion vector with regard to k;

The transition probability is constant over time:

$$p_{ik} = \Pr\left[S_t = k \middle| S_{t-1} = i\right]. \tag{6}$$

Fixed transition probability matrix is as follows:

$$\boldsymbol{P}^Z = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix} \in \boldsymbol{R}^{2\times 2}. \tag{7}$$

Time varying transition probability driven by macro variable Z_{t-1} at time $t-1$.

$$p_{ik,t} = \Pr\left[S_t = k \middle| S_{t-1} = i, Z_{t-1}\right]. \tag{8}$$

The transition matrix notation is described as follows.

$$\mathbf{P}^{Z_t} = \begin{pmatrix} p(Z_{t-1}) & 1-q(Z_{t-1}) \\ 1-p(Z_{t-1}) & q(Z_{t-1}) \end{pmatrix}. \tag{9}$$

$p(Z_{t-1})$ and $q(Z_{t-1})$ are given as the following logit function[4].

$$p(Z_{t-1}) = \Pr\left[S_t = 0 \middle| S_{t-1} = 0, Z_{t-1}\right] = \frac{\exp(p_0 + p_1 Z_{t-1})}{1 + \exp((p_0 + p_1 Z_{t-1}))} \tag{10}$$

$$q(Z_{t-1}) = \Pr\left[\mathrm{S_t} = 1 \middle| \mathrm{S_{t-1}} = 1, Z_{t-1}\right] = \frac{\exp(q_0 + q_1 Z_{t-1})}{1 + \exp((q_0 + q_1 Z_{t-1}))} \tag{11}$$

For the remainder of this paper, the dynamic Nelson-Siegel model with regime shifts for which the transition probability is $\boldsymbol{P}^Z$ is abbreviated to DNSRS(fp). The dynamic Nelson-Siegel model with regime shifts for which the transition probability $\mathbf{P}_t^Z$ is dependent on time t and abbreviated as DNSRS(tvp).

Parameter estimation by state-space representation and Kim Filter

The latent factors for the DNSRS(fp) and DNSRS(tvp) models are the term structure factors and the unobserved state S_t. A method suggested.by Kim and Nelson (1999) was used to construct the likelihood function and to carry out the relevant estimation of the model parameters.This methodology relies on an iterative step where the Hamilton filter, the procedure used to estimate the regime switching part of the model, is used to ensure that the dimension of the Kalmam filter collapses to a tractable dimension. Further details can be found in Kim and Nelson (1999).

The following are the variables of the state-space model and dimension at each time step.

ψ_t: denotes the vector of information set (credit spreads) available as of time t. ψ_t and S_t are mutually independent;

$\beta_{t|t-1} = E[\beta_t | \psi_{t-1}] \in \mathbf{R}^3$: expectation(estimate) of term structure factor β as of t based on information set ψ_{t-1};

$\beta_{t|t} = E[\beta_t | \psi_t] \in \mathbf{R}^3$: expectation(estimate) of the term structure factor β conditional on information set ψ_t up to t;

$\mathbf{P}_{t|t-1} = E[(\beta_t - \beta_{t|t-1})(\beta_t - \beta_{t|t-1})' | \psi_{t-1}] \in \mathbf{R}^{3\times3}$: covariance matrix of the term structure factor β conditional on information set ψ_{t-1} up to t;

$\mathbf{P}_{t|t} = E[(\beta_t - \beta_{t|t})(\beta_t - \beta_{t|t})' | \psi_t] \in \mathbf{R}^{3\times3}$: covariance matrix of the term structure factor β conditional on information set ψ_{t-1} up to t;

$\boldsymbol{f}_{t|t-1} \in \boldsymbol{R}^{m\times m}$: conditional variance of the prediction error $e_{t|t-1}^{(i,k)}$ conditional on information set ψ_t;

$\beta_{t|t-1}^i = E[\beta_t | \psi_{t-1}, S_{t-1} = i] \in \mathbf{R}^3$: expectation (estimate) of the term structure factor β conditional on information up to t based on information set ψ_t and $S_{t-1} = i$;

$\beta_{t|t-1}^k = E[\beta_t | \psi_{t-1}, S_t = k] \in \mathbf{R}^3$: expectation (estimate) of the term structure factor β conditional on information up to t based on information set ψ_{t-1} and $S_{t-1} = k$;

$\beta_{t|t-1}^{(i,k)} = E[\beta_t | \psi_{t-1}, S_t = k, S_{t-1} = i] \in \mathbf{R}^3$: expectation (estimate) of the term structure factor β conditional on information up to t based on information set ψ_{t-1} and $S_{t-1} = i$; and

$\varepsilon_{t|t-1}^{(i,k)} = s_t - \mathbf{F}\beta_{t|t-1}^{(i,k)} \in \mathbf{R}^m$:prediction error β conditional on information up to t based on information set ψ_{t-1} and $S_t = k$;

The following describe the algorithm of the parameter estimation of the state-space model.
We initialized the Nelson-Siegel term structure factors and their variances as in the non-switching case. To initialize the unobserved state probability $\Pr(S_1 = i | \psi_1)$ we calculate the steady-state or unconditional probability $\pi_k(k = 0{,}1)$ and set the initial probability;

$$\Pr\left[S_1 = 0 | \psi_1\right] = \frac{1-q}{2-p-q} \tag{12}$$

$$\Pr\left[S_1 = 1 | \psi_1\right] = \frac{1-q}{2-p-q} \tag{13}$$

for DNSRS(fp) model,

$$\Pr\left[S_1 = 0 | \psi_1\right] = \frac{1-q(Z_0)}{2-p(Z_0)-q(Z_0)} \tag{14}$$

$$\Pr\left[S_1 = 1 | \psi_1\right] = \frac{1-q(Z_0)}{2-p(Z_0)-q(Z_0)} \tag{15}$$

for DNSRS(tvp) model where p, q and Z are defined in the above transition probability matrix.
Given the realizations of the term structure factors at t and $t-1$, the Kalman filter was expressed in the following steps.

$$\beta_{t|t-1}^{(i,k)} = (\boldsymbol{I}-\boldsymbol{A})\boldsymbol{\mu}^k + \boldsymbol{A}\beta_{t-1|t-1}^{i}, i,k = 0{,}1 \tag{16}$$

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{A}\boldsymbol{P}_{t-1|t-1}\boldsymbol{A}' + \boldsymbol{Q} \tag{17}$$

$$\varepsilon_{t|t-1}^{(i,k)} = s(t) - \mathbf{F}\beta_{t|t-1}^{(i,k)}, i,k = 0{,}1 \tag{18}$$

$$f_{t|t-1} = \boldsymbol{F}\boldsymbol{P}_{t|t-1}\boldsymbol{F}' + \boldsymbol{H} \tag{19}$$

The regime-dependent term structure factor and regime non-dependent mean squared error of the forecast were updated using the following formula.

$$\beta_{t|t}^{(i,k)} = \beta_{t|t-1}^{(i,k)} + \mathbf{P}_{t|t-1}\mathbf{F}'\mathbf{f}_{t|t-1}^{-1}\varepsilon_{t|t-1}^{(i,k)}, i,k = 0{,}1 \tag{20}$$

$$\mathbf{P}_{t|t} = \left(1 - \mathbf{P}_{t|t-1}\mathbf{F}'\mathbf{f}_{t|t-1}^{-1}\mathbf{F}\right)\mathbf{P}_{t|t-1} \tag{21}$$

The transition probability $\Pr\left[S_t = k, S_{t-1} = i | \psi_{t-1}\right] \in \mathbf{R}$ at time $t-1$ was calculated as follows.

$$\Pr\left[S_t = k, S_{t-1} = i | \psi_{t-1}\right] = p_{ik} \times \Pr\left[S_{t-1} = i | \psi_{t-1}\right], i,k = 0{,}1 \tag{22}$$

for the DNSRS(fp) model.

$$\Pr\left[S_t = k, S_{t-1} = i | \psi_{t-1}\right] = p_{ik,t} \times \Pr\left[S_{t-1} = i | \psi_{t-1}\right], i,k = 0{,}1 \tag{23}$$

for the DNSRS(tvp) model.

Consider the joint density $f\left(s_t, S_t = k, S_{t-1} = i \middle| \psi_{t-1}\right) \in \mathbf{R}$;

$$f\left(s_t, S_t = k, S_{t-1} = i \middle| \psi_{t-1}\right) = f\left(s_t \middle| S_t = k, S_{t-1} = i \middle| \psi_{t-1}\right) \Pr\left[S_t = k, S_{t-1} = i, \middle| \psi_{t-1}\right] \tag{24}$$

where the conditional density $f\left(s_t \middle| S_t = k, S_{t-1} = i, \middle| \psi_{t-1}\right)$ was obtained based on the prediction error decomposition.

$$f\left(s_t \middle| S_t = k, S_{t-1} = i, \middle| \psi_{t-1}\right) = (2\pi)^{-\frac{m}{2}} \mid \mathbf{f}_{t|t-1} \mid^{-\frac{1}{2}} \exp\left(-\frac{1}{2} \varepsilon_{t|t-1}^{(i,k)'} \left(\mathbf{f}_{t|t-1}\right)^{-1} \varepsilon_{t|t-1}^{(i,k)}\right) \tag{25}$$

Moreover, the marginal density $f\left(s(t) \middle| \psi_{t-1}\right) \in R$ is obtained using the following formula.

$$f\left(s_t \mid \psi_{t-1}\right) = \sum_{i=0}^{1} \sum_{k=0}^{1} f\left(s_t \mid S_t = k, S_{t-1} = i \middle| \psi_{t-1}\right) \times \Pr\left[S_t = k, S_{t-1} = i, \middle| \psi_{t-1}\right] \tag{26}$$

Once S_t was observed at the end of time t, the probability was updated to obtain the following.

$$\Pr\left[S_t = k, S_{t-1} = i \mid \psi_{\text{t-1}}\right] = \frac{f\left(s_t, S_t = k, S_{t-1} = i \mid \psi_{\text{t-1}}\right)}{f\left(s_t \middle| \psi_{\text{t-1}}\right)}, i, k = 0{,}1 \tag{27}$$

We obtain the “filtered" probabilities” Pr $\left[S_t = k \middle| \psi_{\text{t}}\right] \in \mathbf{R}$ using the sum-over-states approach.

$$\Pr\left[S_t = k \mid \psi_{\text{t}}\right] = \sum_{i=0}^{1} \Pr\left[S_t = k, S_{t-1} = i \mid \psi_{\text{t}}\right], i = 0{,}1 \tag{28}$$

In addition $\beta_{t|t}^{k}$ was calculated as follows.

$$\beta_{t|t}^{k} = \frac{\sum_{i=0}^{1} \Pr\left[S_t = k, S_{t-1} = i \mid \psi_t\right] \beta_{t|t}^{(i,k)}}{\Pr\left(S_t = k \mid \psi_t\right)}, \mathrm{i, k} = 0{,}1 \tag{29}$$

From the filter we obtained the density of S_t and log likelihood of parameter set $\boldsymbol{\theta}$.

$$l(\theta) = \sum_{t=1}^{T} \log\left(f\left(s_t \mid \psi_{t-1}\right)\right) \tag{30}$$

The parameter sets to be estimated with $\boldsymbol{\theta}$ were described as follows.

$\boldsymbol{\theta}^{DNSRS(fp)} = \left(\boldsymbol{\mu}^k, \lambda, \boldsymbol{A}, \boldsymbol{Q}, p, q, \boldsymbol{H}\right)$

$\boldsymbol{\theta}^{DNSRS(tvp)} = \left(\boldsymbol{\mu}^k, \lambda, \boldsymbol{A}, \boldsymbol{Q}, p_0, q_0, p_1, q_1, \boldsymbol{H}\right)$

Then, we found parameter $\boldsymbol{\theta}$ set to maximize the log-likelihood function $l(\theta)$. We calculate term structure factor $\beta_{t|t}$ and the estimated credit spreads $\hat{s}_t$ using estimated $\beta_{t|t}^{k}$.

$$\beta_{t|t} = \sum_{k=0}^{1} \Pr\left[S_t = k \mid \psi_t\right] \beta_{t|t}^{k} \tag{31}$$

$$\hat{s}_t = \mathbf{F} \beta_{t|t} \tag{32}$$

Once we finished calculating the maximum of $l(\theta)$, all parameters were estimated.

Further we were able to calculate smoothed probability $\Pr[S_t = k \mid \psi_T] \in \mathbf{R}$ at time T using all the information in the sample. Under the given parameter set $\boldsymbol{\theta}$, we were able to calculate smoothed joint probabilities $\Pr[S_t = k, S_{t+1} = i \mid \psi_t]$ for $t = T-1, T-2\ldots,1$

$$\Pr\left[S_t = k, S_{t-1} = i \mid \psi_T\right] = \frac{\Pr[\mathrm{S}_{t+1} = i \mid \psi_{\mathrm{T}}] \times \Pr[\mathrm{S}_t = k \mid \psi_t] \times p_{ik}}{\Pr[\mathrm{S}_{t+1} = i \mid \psi_t]}, k, i = 0,1 \tag{33}$$

for the DNSRS(fp) model.

$$\Pr\left[S_t = k, S_{t-1} = i \mid \psi_T\right] = \frac{\Pr[\mathrm{S}_{t+1} = i \mid \psi_{\mathrm{T}}] \times \Pr[\mathrm{S}_t = k \mid \psi_t] \times p_{ik,t}}{\Pr[\mathrm{S}_{t+1} = i \mid \psi_t]}, k, i = 0,1 \tag{34}$$

for the DNSRS(tvp) model.

By summing over all of the states we were able to calculate smoothed probability $\Pr[S_t = k \mid \psi_T]$.

$$\Pr\left[S_t = k \mid \psi_{\mathrm{T}}\right] = \sum_{i=0}^{1} \Pr[\mathrm{S}_t = k, S_{t-1} = i \mid \psi_{\mathrm{T}}], k = 0,1 \tag{35}$$

Estimation results

This section describes the estimation results for DNSRS(fp) and DNSRS(tvp). First we discuss the DNSRS(fp) model by interpreting theregime probability of the model. Second, the estimation results of the parameter estimates of DNSRS(fp) are demonstrated. Next we discuss the DNSRS(tvp). We demonstrate how to select the driving factor of the time-varying transition probability,and then we examine the parameter estimates of the DNSRS(tvp) model. Finally, the goodness of fit of the models is evaluated using the AIC.

Regime identification: Interpretation of regime probability

This section examines the meaning of the estimated regime probability for the shape of the term structure of credit spreads. We make three assumptions concerning regimes to examine the relationship between the estimated regime probability and the term structure of credit spreads.

1. Assumption 1: the regime with a high value for the mean reversion of the level factor μ_{β_1} is regime 0, and that with a low value for the mean reversion of the level factor μ_{β_1} is regime 1.
2. Assumption 2: the regime with a high value for the mean reversion of the slope factor μ_{β_2} is regime 0, and that with a low value for the mean reversion of the slope factor μ_{β_2} is regime 1.
3. Assumption 3: the regime with a high value for the mean reversion of the curvature factor μ_{β_3} is regime 0, and that with a low value for the mean reversion of the curvature factor μ_{β_3} is regime 1.

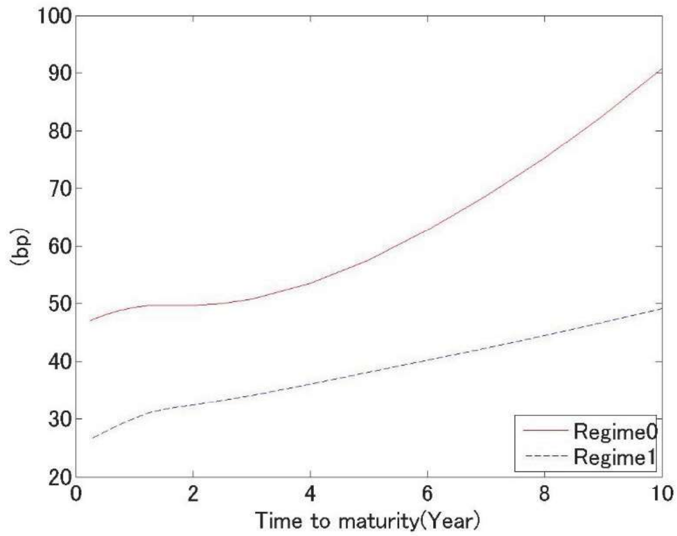
Under Assumption 1, we expect that in regime 0, the level of the term structure of credit spreads will be high while, in regime 1, that of the term structure of credit spreads will be low.

In regime 0, the slope of the term structure for credit spreads is expected to be flat while, in regime 1, that of the term structure of credit spreads is expected to be steep. In regime 0, the curvature of the term structure of credit spreads is assumed convex, while, in regime 1, that of the term structure of credit spreads is assumed concave.

Under the above assumptions, an individual firm is categorized according to whether it belongs to regime 0 under smoothed probability $\Pr[S_t = 0 \mid \psi_T] > 0.5$ or to regime 1 under smoothed probability $\Pr[S_t = 0 \mid \psi_T] < 0.5$. Figures 2, 3, and 4 show the average term structure of credit spreads for level, slope, and curvature respectively, using the above methods.

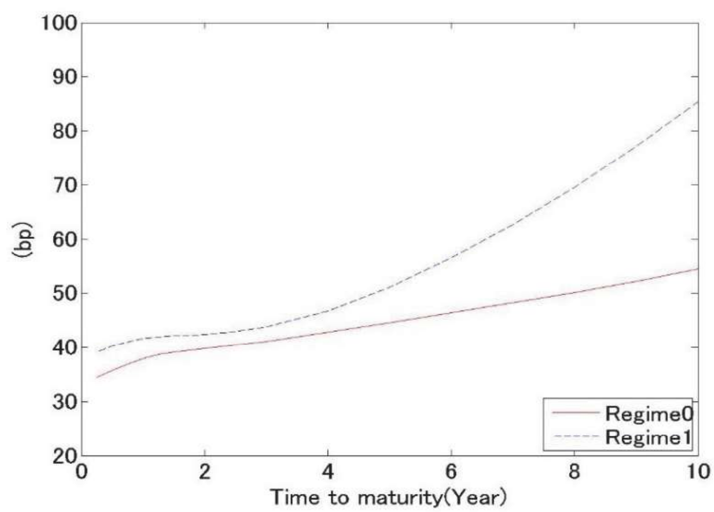
If the regime is classified by the level μ_{β_1} (Figure 2), this indicates high credit spreads for regime 0 and low credit spreads for regime 1, while, under regime 0, the curve becomes steep, particularly for the long end. If the regime is classified by the slope μ_{β_2} (Figure 3), this indicates flat credit spreads for regime 0 and steep credit spreads for regime 1. If the regime is classified by the curvature μ_{β_3} (Figure 4), this indicates a concave credit curve and an upward steep credit curve for regime 1. Given the above results, we conclude that the regime of the DNSRS(fp) model suggests "slope" shifts among the driving factors of credit spreads.

FIGURE 2
TERM STRUCTURE OF CREDIT SPREADS BY REGIME (IDENTIFY REGIME BY μ_{β_1})



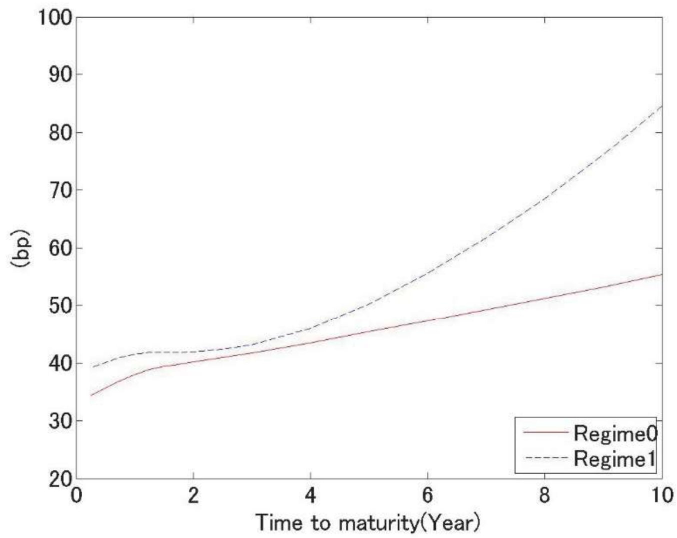
Note: Assuming a high value, μ_{β_1} is categorized as regime 0 and, assuming a low value, it is categorized as regime 1. The individual firms are categorized as regime 0 or regime 1, and an average of the sample is taken.

FIGURE 3
TERM STRUCTURE OF CREDIT SPREADS BY REGIME (IDENTIFY REGIME BY μ_{β_2})



Note: Assuming a high value, μ_{β_2} is categorized as regime 0 and, assuming a low value, is categorized as regime 1. The individual firm is categorized as regime 0 or regime 1, and an average of the sample is taken.

FIGURE4
TERM STRUCTURE OF CREDIT SPREADS BY REGIME (IDENTIFY REGIME BY μ_{β_3})



Note: Assuming a high value, μ_{β_3} is categorized as regime 0 and, assuming a low value, is categorized as regime 1. The individual firm is categorized as regime 0 or regime 1, and we take an average of the sample.

Examination of parameter estimates of DNSRS(fp) model

This section describes the estimation results of 56 firms. Table 1 indicates the estimation results of the mean reversion parameters of the DNSRS(fp) model,while,Table1 shows that regime 0 and 1 are classified based on the previous section and show the mean, the median, the first quartile, and the third quartile. Table 1 shows each sign of $\mu^0_{\beta_1}$, $\mu^1_{\beta_1}$, $\mu^0_{\beta_2}$, $\mu^1_{\beta_2}$, $\mu^0_{\beta_3}$, $\mu^1_{\beta_3}$ indicates the opposite direction.

This result shows the regime with a high value of μ_{β_2} and flat shape for the term structure of credit spreads. Many firms have a high level of credit spreads and convex curvature. On the other hand, for the regime with a low value of μ_{β_2} and steep shape for the term structure of credit spreads, there are also many firms with a low level of credit spreads and a concave curvature.

TABLE 1
PARAMETER ESTIMATES: THE DNSRS (fp) MODEL

	mean	median	the first quartile	the third quartile
$\mu_{\beta_1}^0$	0.054	0.047	0.029	0.084
$\mu_{\beta_1}^1$	−0.041	−0.037	−0.078	−0.003
$\mu_{\beta_2}^0$	0.045	0.039	0.010	0.096
$\mu_{\beta_2}^1$	−0.051	−0.045	−0.082	−0.023
$\mu_{\beta_3}^0$	0.059	0.063	0.017	0.100
$\mu_{\beta_3}^1$	−0.063	−0.065	−0.100	−0.038

Note1: Estimation results of the μ parameter of the DNSRS(fp) model.

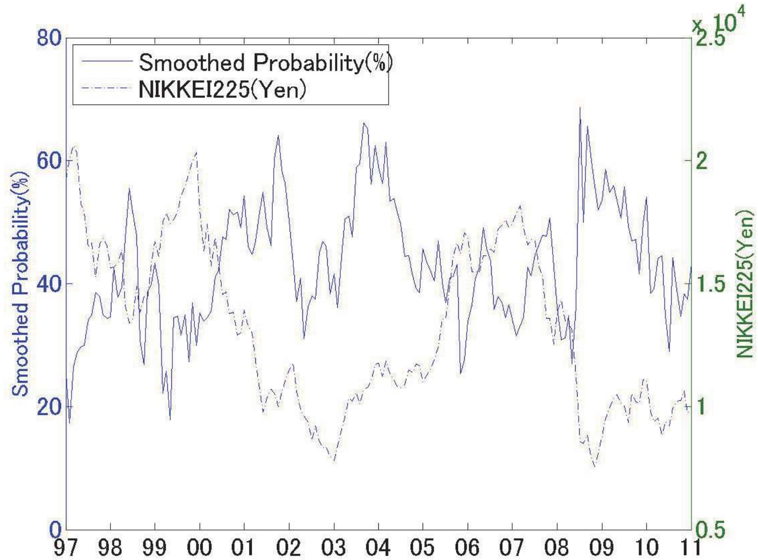
Note2: The regime with a high value of μ_{β_2} is considered to be regime 0. The regime with a low value is considered to be regime 1. The mean, the median, the first quartile, and the third quartile are calculated.

Regime Probability and Macroeconomic Environment

This section investigates the time series for the estimated regime probability. The left axis of Figure 5 shows the mean of 56 firms' times series of smoothed probability for regime 1 of the DNSRS(fp) model. The right axis of Figure 5 shows the NIKKEI225 as a proxy for economic sentiment.

Figure 5 shows that the smoothed probability and NIKKEI225 are correlated:the correlation coefficient is about -0.6. The smoothed probability increased during the financial crises in the first and second halves of the year 2000. Examining the corporate bond market in the first half of 2000, we encountered a series of events, such as the Mycal default, the bankruptcy of Enron Corporation, and synchronized terrorist attacks that caused deterioration in market sentiment. Firms reduced both their production and profits. In the financial crisis of the second half of 2000, the stock index dropped sharply following the Lehman Brothers bankruptcy. Reflecting the deterioration of business and market sentiment, corporate bond investors tended to sell off risky assets, and the spreads widened significantly. Such investor attitude toward risk aversion led to a sell off, particularly for the long end of the credit curve, and the term structure of credit spreads is now considered steeper.

FIGURE5
SMOOTHED PROBABILITY AND NIKKEI225



Note: The left axis shows the mean of 56 firms' times series of smoothed probabilities for regime 1 of the DNSRS(fp)model. The right axis shows NIKKEI225.

Selecting the driving factors of time varying transition probability

In this subsection, we explain how to select the macro economic variables used in DNSRS(tvp). The selection of such variables, which drive transition probability, determines the explanatory power of the model. The candidates for these variables are, economic conditions, credit fundamentals, and market sentiments all of which are considered driving factors of credit spread. These variables have been examined as related to the level of the credit spreads in previous studies,but it is not clear whether they affect the slope of the term structure of credit spreads. Han and Zhou (2015) use the structural model to demonstrate how the slope of a firm's term structure of credit spreads depends on both firm-specific and macroeconomic variables. Candidates are selected among returns of the NIKKEI stock index (Nikkei), NIKKEI volatility index (Nikkei Vol), two year Japanese government bond yield (Risk-free rate), Interest rate volatility (Rates Vol),Ted Spread, and firm equity volatility(Equity Vol)[5]. Next a logit regression model[6] is used for estimation. More concretely the smoothed probability is used to estimate logit regression as dependent variables, whereas the above macro variables are used as independent variables. Firstly, we explain the relationship between the candidate variables and regime shifts of the credit curve.

Nikkei

Following Collin-Dufresne et al. (2001), Stock index is used to proxy for the overall state of the economy. If this index drops, investors tend to sell off their risky assets, such as corporate bonds. The effect is supposed to be stronger for long maturity bonds because they have a longer durations than do shorter ones. NIKKEI225 is provided by Yahoo Finance.

Risk-free rate

Han and Zhou (2015) suggest that the slope factor is negatively related to the risk-free rate. A higher riskfree rate increases the risk-neutral drift of the firm value process, therefore reducing the probability of default and tightening the credit spread. The effect is stronger with longer maturity, because over a longer horizon, firm value is expected to increase more under a higher interest rate, making firm default less likely. Thus, an increase in the risk-free rate lowers the long-term corporate bond spread,leading to a flatter credit spread. The risk-free rate is provided by the Ministry of Finance.

Ted Spread

Ted spread, the difference between the three month LIBOR and three month Treasury Discount Bills, is considered a proxy for market liquidity. When liquidity of money market become dries up and the credit risks of banks appear in the front,the LIBOR rate will increase while the interest rate of the Treasury Discount Bills will drop due to flight-to-quality. Ted spread is expected to widen. The effect is stronger with longer maturity because the flight-to-quality is recognized more in longer maturity bonds than in shorter ones. The three month LIBOR and Treasury Discount Bills are provided by Bloomberg.

NikkeiVol and **EquityVol**

Han and Zhou (2015) also suggest that credit slope is positively related to equity volatility. A higher volatility increases the probability of default of firms and raises the level of credit spread.This effect is stronger with longer maturity than with shorter maturity because options Vega increase with maturity. Collin-Dufresne et al (2001) suggest using the best available substitute of the firm implied volatility (VIX index) because most of the firms we investigate lack publicly traded options. We also use the historical-firm-specific volatility. NikkeiVIX is taken from Bloomberg. EquityiVol is constructed by calculating the standard deviation of the firm equity return taken from Yahoo Finance.

RatesVol

Kim and Stock (2014) suggest that if a firm has a high level of interest rate volatility and therefore a high level of debt volatility, the firm is more likely to reach a critical value for default, thereby resulting in a high probability of default. Thus, interest rate volatility should be priced in corporate yield spreads. The effect is stronger for longer maturity bonds than for shorter ones.

Next we estimate the following logit regression model[7] where $p_t^{f_p}$ is the smoothed probability of regime 1 of DNSRS(fp) model, Z is macro variables and α_0 and α_1 are unknown parameters.

$$p_t^{f_p} = \frac{1}{1+\exp\left(-\alpha_0 - \alpha_1 Z_{t-1}\right)} \tag{35}$$

TABLE 2
THE RATIO OF THE FIRMS WHOSE ESTIMATED PARAMETERS ARE STATISTICALLY SIGNIFICANT

	Nikkei	JGB 2year	Nikkei VIX	Rates Vol	Ted Spread	Equity Vol
Significance level (5%)	75.0%	55.4%	55.4%	53.6%	60.7%	58.9%
Significance level(10%)	89.3%	73.2%	66.1%	67.9%	73.2%	69.6%

Note: Table 2 shows the ratio of the firms whose estimated t values of logit regression model are statistically significant at significance levels 5% and 10% against the whole sample.

Table 2 shows the ratio of the firms whose estimated parameters are statistically significant against the whole sample and indicates the main driver of regime shifts. Nikkei, Ted Spread and Equity Vol are selected for the drivers of the transition probability as these variables shows high value of the ratio.

Examination of parameter estimates of DNSRS(tvp) model

This subsection examines the parameter estimates of the DNSRS(tvp) model. We estimate this model with time-varying transition probability driven by the macro variables selected in the previous subsection.

Table 3 indicates the following. First coefficients p_1 and q_1 of the logit function show the variety of the size and sensitivity of transition probability to macro variables. The median of coefficient p_1 of transition probability shows the positive value which implies that the improvement of economic sentiment, caused by the NIKKEI stock index, keeps the credit curve flat, while the deterioration of the economic condition keeps the credit curve steep. On the other hand, the median of coefficient p_1 of Ted spread indicates a negative value, which implies that the flight-to-quality makes the credit curve steeper. The median of coefficient p_1 of equity volatility shows a negative value, which implies that firm-specific equity volatility makes the credit curve steeper, as the effect of longer maturity is stronger than that of the short maturity. The above results indicate that there can be an economic linkage between the regime switch of the term structure of credit spreads and macro-economic variables.

TABLE 3
PARAMETER ESTIMATES OF DNSRS (tvp) MODEL

	Nikkei			Ted Spread			Equity Vol		
Parameter	Median	First quantile	Third quantile	Median	First quantile	Third quantile	Median	First quantile	Third quantile
p_0	2.85	0.27	9.86	2.62	0.43	9.23	4.08	1.71	9.16
p_1	0.31	−1.42	2.77	−0.20	2.10	2.74	−0.97	−3.90	−0.02
q_0	3.00	0.68	7.32	2.73	0.56	6.85	2.19	0.18	3.90
q_1	−0.11	−3.76	0.80	0.29	−0.77	3.57	0.01	−1.03	1.70

Note We calculate the median, the first quantile, the third quantile of the estimated parameters of DNSRS(tvp) model

Model evaluation

This subsection investigates the goodness of fit of the sample data for the DNS, DNSRS(fp), and DNSRS(tvp). We calculated the estimation error of the credit spreads of individual firms and the ratio of the number of firms that have a smaller estimated error than do the DNS. Finally, we compared the model based on the AIC. Table 4 shows the RMSE of the term structure of credit spreads according to time to maturity. The median of the whole sample is used as a base point. Additionally, the number of firms in which RMSE is lower than that of the DNS model is summed and then divided by the whole sample. The larger the ratio becomes, the greater the number of firms with superior predictive power against the DNS.

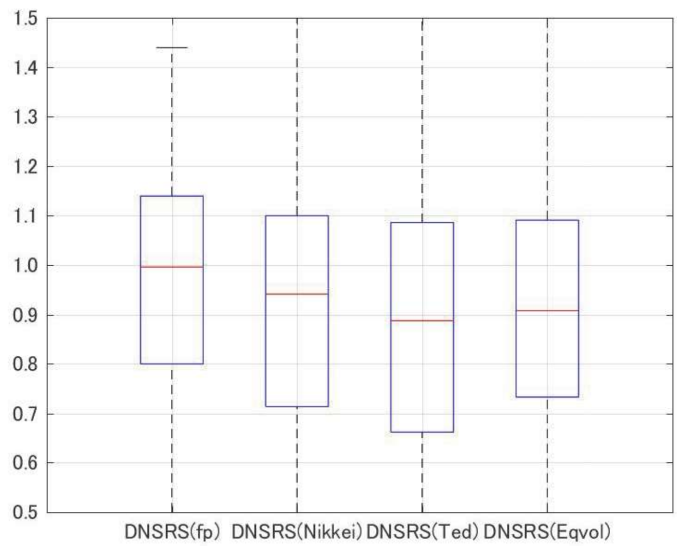
TABLE 4
RMSE BY MODEL

	DNS	DNSRS (fp)		DNSRS(tvp)					
				Nikkei		Ted Spread		Equity Vol	
Time to maturity	RMSE	RMSE	Ratio	RMSE	Ratio	RMSE	Ratio	RMSE	Ratio
1	1.57	1.65	39.3%	1.59	42.9%	1.62	57.1%	1.64	41.1%
3	1.72	1.76	60.7%	1.77	53.6%	1.73	51.8%	1.72	58.9%
5	2.33	2.18	69.9%	2.35	66.1%	2.30	64.3%	2.37	66.1%
7	2.00	2.00	55.4%	1.96	51.8%	1.97	55.4%	1.99	50.0%
8	1.59	1.63	60.7%	1.57	75.0%	1.51	75.0%	1.55	69.6%

Note: We calculate the RMSE and AIC of the term structure model. The median of the whole sample is used as a base point. Additionally, the number of firms for which RMSE is lower than that of the DNS model is summed and divided by the whole sample. The larger the ratio becomes, the greater the number of firms with superior predictive power against the DNS.

Considering the median of RMSE by time to maturity, there is no such difference between DNS(indep), DNSRS(fp), and DNSRS(tvp). The ratio of the number of DNSRS(fp) and DNSRS(tvp) firms that have a lower estimation error than approximates 40 to 60%. As time to maturity increases, the number of firms with a lower estimation error increases. The estimation error of the DNSRS(tvp) model is small for the time to maturity of eight years. The Figure 6 shows that the box plot of three DNSRS(tvp)/DNS fall below 1. This indicates that the number of firms with superior sample fit is greater for the DNSRS(tvp) model than the DNS(fp) model. Table 5 shows the ratio of the number of firms of the DNSRS(fp)and DNSRS(tvp) models with lower AIC to DNS ratios. The larger the ratio becomes, the better the goodness of fit of the model. The ratio of DNSRS(fp)/DNS of 100% shows that the whole sample of DNS indicates a smaller AIC. This result demonstrates that the DNSRS(fp) model has superior goodness of fit compared to the DNS from Equity Vol, NIKKEI225, and Ted Spread in that order. Comparing DNSRS(fp) with DNSRS(tvp), the ratio(60%) of DNSRS(tvp) relative to the DNSRS(fp) model indicates that the DNSRS(tvp) model has better goodness of fit in terms of information criteria.

FIGURE 6
BOX PLOT RATIO OF RMSE (TIME TO MATURITY OF EIGHT YEARS)



Note1 This figure shows the box plots of the RMSE ratios for the DNSRS(fp) and DNSRS(tvp) (DNSRS(Nikkei),DNSRS(Ted) and DNSRS(Eqvol),respectively) models relative to the RMSE of the DNS model, firm by firm.
Note2 Time **to maturity of eight years**

TABLE 5
RATIO OF THE NUMBER OF FIRMS OF AIC

DNSRS (fp) / DNS	DNSRS (tvp) / DNS			DNSRS (tvp) / DNSRS (fp)		
	NIKKEI225	Ted Spread	Equity Vol	NIKKEI225	Ted Spread	Equity Vol
100%	90%	87%	91%	61%	56%	65%

Note At the bottom of Table 5, we calculate the ratio of the number of firms of the DNSRS model with lower AIC to that of the DNS model. The larger the ratio becomes, the better the goodness of fit of the model. For example, the ratio of DNSRS(fp)/DNS of 100 shows that the whole sample of DNS shows a smaller AIC.

The above results lead to the conclusion that the term structure model with time-varying regime shifts demonstrates strong in-sample performance. This result implies that the DNSRS(tvp) model captures the dynamics of credit spread in an uncertain economic environment. The stock market index and liquidity indicator perform as driving factors of the transition matrix.

CONCLUDING REMARKS

This study develops a regime-switching extension of the dynamic Nelson-Siegel model and demonstrates its estimation methodology. The models are estimated using Japanese corporate bond spread term structure data on an individual firm basis. The conclusion is summarized as follows. First, the regime of the DNSRS model suggests a "slope" shift among the driving factors of credit spreads. The estimation results indicate that the estimated regime probability is closely linked to business and market sentiment. Second, the term structure model with regime shifts has a superior in-sample fit compared to that without regime shifts. Third, the regime-switching model is extended by adopting a time-varying transition probability matrix that is driven by the variables of the structural model and leading macroeconomic indicators. The overall fit is improved by incorporating a time-varying transition probability matrix. Our results imply the importance of incorporating regime shifts into modeling the term structure of credit spreads. Our study contributes to further understanding the determinants of the term structure of the credit spreads. The new findings and practical usefulness of this study contribute to and can be applied to credit default swaps, which have non-linear time series properties.

ACKNOWLEDGEMENTS

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ENDNOTES

1. Alexander and Kaeck (2008) and Pavlova et al (2015) propose a regime-switching model for credit spreads and credit default swaps with single maturity.
2. The B-spline model is used to construct the zero coupon yield of corporate bonds instead of the bootstrap method because it fits better for the whole sample period from April 1997 to December 2011.
3. The AR matrix and variance of the term structure factor do not depend on regimes.
4. $\boldsymbol{P}^{Z}$ is applied to permanent structural change, while $\mathbf{P}^{Z}$ is used when structural change is driven by the macroeconomic environment by incorporating macroeconomic variables into the model.
5. The Production Industrial Production or Consumer Price Indices, or accounting variables, such as debt-to -equity ratio, are excluded because they are published on a monthly or quarterly basis and it is difficult to match the date of credit spreads and announcement date of these variables.
6. Alenxander and Kaeck (2008) estimate the relationship between the regime probability and macro variables using a logit function
7. Alenxander and Kaeck (2008) estimate the regime probability and macro variables

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