# Examining RADR as a Valuation Method in Capital Budgeting

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*The risk adjusted discount rate (RADR) method is used as a valuation tool to assess projects that involve multi-period uncertain cash flows. Little research has been conducted to examine the validity of the RADR method. Extensive literature has been developed to assess the value of the multi-period project within the capital asset pricing model (CAPM) framework. Most were normative, so that their implementation was virtually impossible. In this paper the general valuation method, RADR, is examined to see if it is appropriate to discount a series of uncertain cash flows at the risk-adjusted discount rate.*

## INTRODUCTION

Capital budgeting decisions involve finding the present value of a series of uncertain cash flows from a project to determine if the project is worthwhile. The net present value (NPV) approach discounts the expected cash flows at a risk adjusted discount rate (RADR), commensurate with the risk of the cash flows, to find the value of the project. The NPV approach is a primary tool for evaluating multi-period cash flows from the project because it is operational and compatible with the principle of value maximization. The certainty equivalent (CE) approach is frequently presented in finance and accounting text books as an alternative method to the RADR. The CE approach involves finding certainty equivalents of a series of uncertain cash flows over time and discounting them at the risk-free discount rate. When the project's NPV or CE value is positive, the project is acceptable because the project will increase the value of the firm.

Meyer (1976) explained two other approaches for valuing a series of uncertain cash flows. The first method involves finding the utility of each period's uncertain cash flow and discounting them back to the present at some discount rate. This approach is equivalent to the case of the multi-attribute utility function which is decomposed into a series of individual utilities if the cash flow attributes are mutually utility independent and the sum of the individual coefficients, $k_i$, equals 1 (Keeney & Raiffa (1976), p. 292). Meyer (1976) presented the second approach in which certainty equivalent of each individual cash flow streams are discounted at the risk-free (or certainty) rate to find their utilities and to calculate their weighted average utilities using the probabilities of the individual streams of cash flow. Although these two methods appear to be sound in theory, they have not been used in the literature as a valuation approach due to the lack of their operational feasibility.

The purpose of this paper is to examine the RADR method as a tool for analyzing a project with multi-period uncertain cash flows and to determine whether the RADR approach is correct for capital budgeting analysis. Although the RADR approach uses almost exclusively the rate of return that is derived within the framework of a single period equilibrium analysis, there is no adequate justification for using this equilibrium rate for discounting multi-period uncertain cash flows. If the RADR approach unduly penalizes all distant cash flows (for t = 2… T), it will undervalue multi-period projects and, consequently, lead to a wrong decision in which profitable projects are rejected.

Sharpe (1964) and Treynor (1961) developed a schedule of market equilibrium return on risky assets within the confines of a single period and two parameter model, which was later extended by Mossin (1966), Lintner (1965) and many other researchers. Ross (1976) added an additional dimension to the theory of capital asset pricing by developing the Arbitrage Pricing Theory (APT) in which the return on any risky asset is expressed as a linear combination of various factors that affect asset return. Although these market equilibrium returns are derived within the framework of a single period, they have been used exclusively for discounting multi-period cash flows for capital projects. It is probably because these required returns have a firm theoretical foundation, and there exist no better substitute for the RADR.

Bogue and Roll (1974) state that it would be incorrect to use a single-period risk-adjusted discount rate to discount multi-period cash flows, unless a secondary market exists for the project at the end of the first period. When there is a secondary market, the project with multi-period cash flows can be reduced to a single period project in which only the cash flow and the market value of the project at the end of period 1 are needed to find the value of the project. They presented a more complex procedure to analyze the multiperiod uncertain cash flows in a dynamic programming fashion within the capital asset pricing model (CAPM) framework. They conclude that, in addition to systematic risk in the CAPM sense, the uncertainty associated with the risk-free rate and covariance risk of the intermediate value of the project must be considered explicitly in order to assess the value of the project with multi-period uncertain cash flows.

Fama (1980) and Constantinides (1980) specified the condition under which a constant discount rate can be used to discount multi-period uncertain cash flows within the CAPM framework. Brennan (1973), Fama (1977), Myers and Turnbull (1977), and Bhattacharya (1978), developed multiperiod models for evaluating projects as an extension of one-period or a continuous-time CAPM. While these researchers provide additional insights into valuation of a multi-period project within the context of the CAPM framework, the sequential application of the single-period model used in the discounting of stochastic cash flows becomes computationally very complex and theoretical so that it is of little practical use.

## AN EXAMINATION OF THE RADR APPROACH

The net present value (NPV) approach involves discounting multi-period uncertain cash flows at the risk adjusted discount rate (RADR). Given a stream of stochastic cash flows $\widetilde{X} = (\widetilde{x}_1,...,\widetilde{x}_n)$, the general form of the net present value of $\widetilde{X}$ is:

$$NPV\ (\widetilde{X}) = \sum_{t=0}^{n} \frac{E(\widetilde{x}_t)}{\prod_{j=1}^{t}(1+r_f+\theta_{tj})} \qquad \text{.........(1)}$$

where $x_0$ is the cost of the project, $r_f$ is a risk-free discount rate, $\theta_{tj}$ is a risk premium which depends on the probability distribution of the cash flow at t and its covariance with the market. Equation (1) implies that the risk of the cash flow increases exponentially as the timing of the cash flow is further away from the present. Brealey and Myers justified the RADR approach by stating:

> *"Any risk-adjusted discount rate automatically recognizes the fact that more distant cash flows have more risk....the reason is that the discount rate compensates for the risk borne per period."* Brealey and Myers (1991, p. 196)

The risk premium associated with a cash flow should depend upon the probability distribution (uncertainty) of the cash flow involved, but not on the timing of the cash flow. Consider a vector of stochastic cash flows, $\widetilde{X} = (\widetilde{x}_1,...,\widetilde{x}_n)$, which are generated from throwing a die at the end of each period for 'n' periods. The probability distribution of each cash flow is independent and identical in the sense that each period is the same and not affected by any other period cash flows. The risk premium should depend on the uncertainty associated with the cash flow at a given point in time, but not on the timing of the cash flow. The beta risk also depends upon the standard deviation of the return (or the distribution of the cash flow at year end) involved and its correlation with the market portfolio return, given the standard deviation of the market portfolio. It has nothing to do with the time interval of the return data.

When estimating cash flows of a capital project, one may conjecture that a distant cash flow could be more uncertain compared to earlier cash flow and therefore should be considered riskier. The NPV approach explicitly assumes that the risk of a cash flow increases exponentially and automatically as the timing of a cash flow moves away from the present time. However, there exists no financial or economic theory that explains how the risk associated with a future uncertain cash flow changes over time. It is very difficult to justify the change of risk premium in a manner that is implied in the NPV method.

The RADR approach also conflicts with the CE approach. Robicheck and Myers (1976) stated that the CE approach not only conflicts with the RADR approach but is superior to the RADR approach. The net present value of the project by the CE approach can be presented as:

$$NPV(CE) = \sum_{t=0}^{n} \frac{\alpha_t * E(\widetilde{x}_t)}{(1+r_f)^t} \qquad \text{.....(2)}$$

where $\alpha_t$ is the certainty equivalent adjustment factor at time t, which varies from 0 to 1. Consider a project which offers a cash flow $\widetilde{x}_t$ at time t. If both the certainty equivalent and the risk-adjusted discount rate approaches are correct, the present values obtained by RADR and CE methods should be identical:

$$\frac{E(\widetilde{x}_t)}{(1+r_f+\theta_t)^t} = \frac{\alpha_t * E(\widetilde{x}_t)}{(1+r_f)^t} \quad \text{for t = 1...N} \qquad \text{.....(3)}$$

From equation (3), the certainty equivalent adjustment factor $\alpha_t$ can be defined as follows:

$$\alpha_t = \left[\frac{1+r_f}{1+r_f+\theta_t}\right]^t \quad \text{for all t} \qquad \text{.....(4)}$$

Equation (4) implies that, if $\theta_t$ is positive, $\alpha_t$ is exponentially decreased as t increases so that $\alpha_1 > \alpha_2 > ... > \alpha_n$. Chen (1967) stated that the RADR approach not only is correct, but also is not contradictory to the CE method if the risk of future cash flows increases at a constant rate. The risk of cash flows increases exponentially in the NPV approach. Equation (4) shows that the RADR approach is correct only if the risk of future cash flows increases exponentially over time. If each individual cash flow is independently and identically distributed, all $\alpha_t$ should remain the same, assuming an individual's utility function does not change over time.

The difference in valuation between the RADR and CE approaches is calculated and shown in Appendix A. This example assumes that the RADR for a project is 12 percent, the risk-free return 8 percent, and that the annuity payments of $1,000 are made at the end of each period. The "NPV-CE" and the "NPV-RADR" columns indicate the present values of annuity payments of $1,000 as the life of the project changes. As the project life increases, both the percentage and absolute sizes of "NPV difference" increases, *ceteris paribus*. In other words, the long-term project is penalized more heavily in the valuation process when the RADR method is used for valuation.

If an investment project is traded at the present value of cash flows discounted at the RADR, an arbitrage opportunity exists in the capital market. Since the certainty equivalent value of each cash flow can be considered as a forward price at time t, a significant size of an arbitrage profit can be realized if the project is purchased at the present value of future cash flows discounted at the RADR, and sold at the forward price, assuming no transaction cost. For example, if an investor purchases a 10 year project at $5650.22 as shown in Appendix A, there will be an arbitrage profit of $820.21. The longer the maturity of the investment project, the larger will be the arbitrage profit. It is also true that the arbitrage opportunity from the riskier cash flow will grow more and faster. Therefore, the price of the project should increase until there is no arbitrage profit possible.

## CONCLUDING REMARKS

This paper examined if the RADR should be used to find the net present value of the project. The result of the study indicates that the net present value approach is seriously flawed as a valuation tool. The arguments against the RADR approach revolve around penalizing heavily for the risk associated with the cash flow in later years. Consequently, longer term projects are penalized more heavily than the project of shorter duration.

The certainty equivalent method determines the risk of each cash flow separately from the discount rate. Although there exist no financial theory that provide a market-determined ‘certainty equivalent (CE) adjustment factor’ within the utility theoretic framework, the CAPM can be used to find market determined CE adjustment factors. The results of this paper indicate that the CE approach is more appropriate for valuing a multi-period project.

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**APPENDIX A**

**VALUE DIFFERENCES BETWEEN THE RADR AND THE CE METHODS**

| Year | NPV-CE | NPV-RADR | Amount undervalued by RADR | Percent of Error |
|---|---|---|---|---|
| 1 | 892.86 | 892.86 | 0.00 | 0.00 |
| 2 | 1719.58 | 1690.05 | 29.53 | 1.72 |
| 3 | 2485.06 | 2401.83 | 83.23 | 3.35 |
| 4 | 3193.84 | 3037.35 | 156.49 | 4.90 |
| 5 | 3850.11 | 3604.78 | 245.34 | 6.37 |
| 6 | 4457.78 | 4111.41 | 346.37 | 7.77 |
| 7 | 5020.43 | 4563.76 | 456.67 | 9.10 |
| 8 | 5541.40 | 4967.64 | 573.76 | 10.35 |
| 9 | 6023.78 | 5328.25 | 695.53 | 11.55 |
| 10 | 6470.44 | 5650.22 | 820.21 | 12.68 |
| 11 | 6884.00 | 5937.70 | 946.30 | 13.75 |
| 12 | 7266.93 | 6194.37 | 1072.56 | 14.76 |
| 13 | 7621.50 | 6423.55 | 1197.95 | 15.72 |
| 14 | 7949.80 | 6628.17 | 1321.63 | 16.62 |
| 15 | 8253.78 | 6810.86 | 1442.92 | 17.48 |
| 16 | 8535.25 | 6973.99 | 1561.26 | 18.29 |
| 17 | 8795.87 | 7119.63 | 1676.23 | 19.06 |
| 18 | 9037.18 | 7249.67 | 1787.51 | 19.78 |
| 19 | 9260.61 | 7365.78 | 1894.84 | 20.46 |
| 20 | 9467.50 | 7469.44 | 1998.06 | 21.10 |
| 25 | 10293.53 | 7843.14 | 2450.40 | 23.81 |
| 30 | 10855.72 | 8055.18 | 2800.54 | 25.80 |
| 40 | 11498.73 | 8243.78 | 3254.96 | 28.31 |
| 50 | 11796.57 | 8304.50 | 3492.08 | 29.60 |
| 100 | 12048.09 | 8333.23 | 3714.86 | 30.83 |

*Note: Assumed payments of $1,000 at the end of each year with RADR=12% and risk-free return of 8%.