

Evaluating Conformity of Municipality Financial Data to Benford's Law: An Exploratory Study

Cheryl L. Prachyl
University of Texas at Arlington

Mary Fischer
The University of Texas at Tyler

Benford's Law provides an expected distribution of leading digits in a set of occurring numbers. The Law has been used to examine the distribution in many different types of data to identify irregularities or evidence of fraud. This paper provides exploratory evidence about the application of Benford's Law in the government sector to analyze reported financial data from municipalities. The results suggest there is a significant nonconformity in reported financial information related to business type activities between those municipalities with unrestricted net position deficit balances (i.e. troubled municipalities) and those municipalities with a positive unrestricted net position balance (non-troubled municipalities).

Keywords: Benford Law, municipalities, troubled financial reporting

INTRODUCTION

Benford's Law provides an expected distribution of leading digits in a set of naturally occurring numbers. One might expect that each of the leading digits, 1 to 9, would occur equally in a large set of numbers. However, the occurrence of each of the leading digits is not evenly distributed. For example, the leading digit will be 1 approximately 30% of the time, 2 about 18% of the time, and the percentage decreasing for each digit down to less than 5% for the leading digit 9 (Fewster 2009). The analysis can also be extended to look at the distribution of the second digits in a set of numbers. The expected distribution has been shown to apply to many different types of data. Benford's Law has been used as an analytical procedure for planning an audit (Nigrini 1996), in evaluating transaction data sets for evidence of fraud (Durtschi, Hillison & Pacini 2004), in evaluating refunds made by call center employees (McGinty 2014), in evaluating survey data to determine its quality (Judge and Schechter, 2009), and other applications.

Benford's Law also has been used to aid in the detection of fraud in accounting data (Durtschi et al. 2004). When used for fraud detection, it is often applied as an internal control mechanism to large sets of transaction data (e.g., sales, purchases, refunds, etc.). In many government situations, fraud results from a lack of internal controls (Greenlee, Fischer, Gordon & Keating, 2007) which is supported by Donelson, Ege and McInnis (2017). They (Donelson et al., 2017) report a significant association between material internal control weakness and financial reporting and disclosure fraud using Benford's Law processes. Recent research suggests municipal financial statement error rates are nearly double those of similarly

sized corporations (Burks, 2015). Other literature finds the control environment and control activities help prevent, deter, and detect fraud in public organizations (Dorminey, Fleming, Kranacher & Riley, 2012).

Another area in which Benford's Law might apply is in governmental accounting recognition activities. Johnson and Weggenmann (2013) examined state government financial data and its consistency with Benford's Law. The data set included three balances taken from the Comprehensive Annual Financial Report (CAFR) of each of the fifty states. The balances chosen were Total General Revenues of the Primary Government, Total Fund Balance of the General Fund, and Total Fund Balance of the Governmental Funds. They had three years of data giving only 150 data points for each balance. Because of the small data set, they suggested the use of Mean Average Deviation as an additional tool to analyze conformity with Benford's Law. Amiram, Bozanic, and Rouen (2015) used Benford's law to evaluate published financial statement data to determine whether individual firm statement information conformed to the Benford's Law distribution. They found that companies that restated financial statements had prior year statements that did not conform to the distribution. After the restatements, they found that the financial statements more closely conformed to the Benford's Law expected distribution.

This research extends the application of Benford's Law in the governmental sector by analyzing data from municipalities. The purpose of this research is to examine the conformity of municipal financial data with Benford's Law and to consider whether Benford's Law might serve as a useful tool in identifying areas where financially troubled municipalities might differ from financially non-troubled municipalities. To accomplish this purpose, a selected subset of financially troubled municipalities is examined separately to identify whether the conformity of those numbers differs from the non-troubled municipalities set. This application of Benford's law may provide additional information to identify characteristics of troubled municipalities so analysts can focus on areas most likely to indicate potential financial failure.

LITERATURE REVIEW

The law of anomalous numbers was first studied and documented by Frank Benford (1938). He found that the first digits of large sets of data were more likely to be 1 than any other number. He identified the logarithmic distribution that shows the expected number of each digit (1 to 9) that should appear as the first digit in large datasets of naturally occurring numbers. Later researchers have used this law in different situations to identify possible fraud or errors in accounting data (Canaghem, 2016; Carlton, 2017; Istrate, 2019).

Nigrini (1996) applied Benford's Law to individual tax returns to estimate the extent of unplanned taxpayer evasion on tax returns. He specifically evaluated reporting of interest received and interest deducted. He concluded that using Benford's analysis could provide a measure of likely evasion on tax returns.

Wallace (2002) used Benford's Law to assess the quality of data that would be used for other purposes. She evaluated several different data sets to determine whether it was likely there had been errors introduced either intentionally or accidentally. She described the types of data sets that would most likely conform to the Benford's distribution.

Nigrini and Miller (2009) use a Benford's distribution as an analytical tool in auditing to identify anomalies in transaction data. Because the method can be applied to the entire set of transaction data, there is no need to use samples and this may allow auditors to reduce risk from sampling.

Lehman, Watson, and Jones (2007) explain how to use Excel to evaluate transactions by each employee to identify possible fraud. They examined sales returns by each employee to calculate the actual frequency of each leading digit and compared that frequency with what was expected under Benford's Law. The authors use a z-score to identify potentially fraudulent activity. The use of Benford's Law in this application allows the auditors to use a statistical cutoff when examining data for fraud.

Jordan and Clark (2011) and confirmed by Istrate (2019) examined the second digit of corporate earnings announcements to determine whether it was likely that the company had practiced cosmetic earnings management (CEM). CEM occurs when a company's earnings are just below a threshold (e.g.

495 million) and the company manipulates the earnings to increase them above the 500 million thresholds. To determine whether CEM had occurred, they looked at the second digit in the earnings numbers and found there were more zeros than expected and fewer nines than expected, indicating that the numbers had been manipulated to increase above the threshold.

Alali and Romero (2013) analyzed the first two digits of assets, liabilities, revenues, and expenses of publicly traded companies from 2001 through 2010. The data were further divided into different periods. These periods included several subsets related to SOX and two periods related to the financial crisis of 2008. They were looking for evidence of manipulation of the financial data and whether that manipulation differed among the sub-periods studied. While they did find some evidence of manipulation, it was in the opposite direction of what was expected. The evidence of manipulation indicates that further research in this area is warranted.

Grabinski and Paszek (2013) examined financial statement information from European publicly listed companies to determine whether the data were consistent with the Benford's distribution. They found that the data closely followed the expected distribution based on Benford's Law.

Rauch, et.al. (2014) used Benford's Law to evaluate the quality of deficit related data reported by EU members. They first compared social security statistics to the Benford's distribution and found there was close conformity. They postulated the social security statistics were not subject to monitoring in the same way the deficit statistics would be. When they examined the deficit statistics, they found there were significant deviations from the Benford's distribution. They concluded that Benford's Law could be useful in identifying manipulated data.

Amiram, et.al. (2015) analyzed published financial statement data to determine whether the data conformed to the Benford's distribution and whether financial data from firms that had SEC required restatements conformed. They found that when a company was required to restate financial data, the two years prior to the restatement did not conform to the distribution. However, after the restatements, the financial data generally did conform to the distribution. The evidence presented by Amiram, et.al. indicates that the Benford's analysis may indicate firms that have errors or misstatements in their financial statement data.

Johnson and Weggenmann (2013) analyzed balances from the Comprehensive Annual Financial Reports of the fifty U.S. states. They included the Total General Revenues of the Primary Government, Total Fund Balance of the General Fund, and Total Fund Balance of the Governmental Funds. This was one of the first applications of Benford's Law to governmental data. They used the Mean Absolute Deviation (discussed later) to determine whether the data were in conformity with the expectations provided by Benford's Law. The current research extends this application of Benford's Law to governmental data by using the law to analyze conformity of municipal data with the expected values.

DATA ANALYSES

The data for this study consists of information provided by 1949 municipalities that applied to the Governmental Finance Officers Association for the Certificate of Excellence. Data was collected from the Comprehensive Annual Financial Report (CAFR) of each municipality. Troubled municipalities were identified as those with negative unrestricted net position of governmental activities (316 municipalities). The original data set contained 80 variables for each municipality. Several variables were removed because they were unsuited to a Benford's Law analysis. For example, some were percentages, several were items that applied to only a few municipalities, and some were counts of items that were all single digits. After removing these variables, we were left with 40 variables that were included in the analysis.

To analyze the data, we calculated the mean absolute deviation (MAD) for each variable for both the first digit and second digit. For this calculation, the first digit of each value was extracted (See Table 1). A count of each of the nine digits (1 to 9) was then calculated (Column 1). For each digit, a proportion of the total that began with that digit was calculated (Column 2). Next, the absolute difference between the observed percentage and Benford's Law percentage was calculated (Column 4). Finally, the average of the absolute deviations was calculated. Conformity with Benford's Law was determined based on values

provide by Drake and Nigrini (2000) (See Table 2 for first digit conformity and Table 3 for second digit conformity). The same process was used to extract and analyze the second digit for each variable.

**TABLE 1
EXAMPLE CALCULATION OF MAD FOR DIRECTDEBT IN TROUBLED MUNICIPALITIES**

| | Actual Number (1) | Actual Percentage (2) | Benford's Law Percentage (3) | Absolute Deviation (4) |
|-------------------------|-----------------------|-----------------------------|------------------------------------|---------------------------|
| Count of Number 1 | 82 | 30.5% | 30.1% | 0.004 |
| Count of Number 2 | 44 | 16.4% | 17.61% | 0.013 |
| Count of Number 3 | 35 | 13.03% | 12.49% | 0.005 |
| Count of Number 4 | 30 | 11.2% | 9.69% | 0.015 |
| Count of Number 5 | 22 | 8.2% | 7.92% | 0.003 |
| Count of Number 6 | 16 | 5.9% | 6.70% | 0.007 |
| Count of Number 7 | 18 | 6.7% | 5.80% | 0.009 |
| Count of Number 8 | 12 | 4.5% | 5.12% | 0.007 |
| Count of Number 9 | 10 | 3.7% | 4.58% | 0.009 |
| Total Observations | 269 | | | |
| Mean Absolute Deviation | Acceptable conformity | | | 0.00780 |

**TABLE 2
STANDARDS FOR MEASURING FIRST DIGIT CONFORMITY**

| Mean Absolute Deviation (MAD) | Level of Conformity |
|----------------------------------|----------------------------------|
| Between 0 and .0039 | Close Conformity |
| Between 0.004 and 0.0079 | Acceptable Conformity |
| Between 0.008 and 0.0119 | Marginally acceptable conformity |
| 0.012 or more | Nonconformity |

**TABLE 3
STANDARD FOR MEASURING SECOND DIGIT CONFORMITY**

| Mean Absolute Deviation (MAD) | Level of Conformity |
|----------------------------------|----------------------------------|
| Between 0 and .0079 | Close Conformity |
| Between 0.008 and 0.0119 | Acceptable Conformity |
| Between 0.012 and 0.0159 | Marginally acceptable conformity |
| 0.016 or more | Nonconformity |

Non-troubled Municipalities

For the non-troubled data set that included 1633 municipalities with positive unrestricted net position of governmental activities, all of the variables were at least marginal conformity (based on the Mean Average Deviation) with the Benford's percentages and most were acceptable conformity (Table 4).

TABLE 4
NON-TROUBLED DATA SET FIRST DIGIT (1633 MUNICIPALITIES)

| | |
|----|----------------------------------|
| 2 | Close Conformity |
| 34 | Acceptable Conformity |
| 4 | Marginally acceptable conformity |
| | Nonconformity |

The second digit analysis for the non-troubled data set also indicated that most of the variables conformed to the Benford’s distribution (based on the Mean Average Deviation). For the second digit, only one variable was marginally acceptable, and most were close conformity (Table 5).

TABLE 5
NON-TROUBLES DATA SET SECOND DIGIT (1633 MUNICIPALITIES)

| | |
|----|----------------------------------|
| 32 | Close Conformity |
| 7 | Acceptable Conformity |
| 1 | Marginally acceptable conformity |
| 0 | Nonconformity |

These results indicate that there are no obvious anomalies in the number of each digit (1 – 9) expected and what occurred. An example of an occurrence that might indicate manipulation of the numbers would be a higher number of zeros in the second digit than expected and a lower number of 9s. This might occur because numbers just below a threshold were increased slightly to make the results appear more positive. For example, \$4,950,000 could be increased to \$5,000,000.

Troubled Municipalities

To separate Troubled Municipalities, we selected those with negative unrestricted net position of the governmental activities. Unrestricted net position resources are usable for any municipal purpose and it is not uncommon for an unrestricted net position balance to be a deficit. A deficit unrestricted net position balance does not necessarily mean that the municipality is on the brink of bankruptcy. Instead, the municipality could be paying long-term liabilities on a pay-as-you-go basis rather than accumulating resources each year as payments come due. Although fiscal defalcation can be the issue, common transactions that result in an unrestricted net position deficit include judgements, claims, and pension costs. Whatever the cause, additional information and investigation is warranted.

There were 316 municipalities that met this criterion. The same Benford’s analysis was performed on this subset of observations. For this subset, there were thirty-three of the forty variables which did not meet the Benford’s Law distribution of Mean Absolute Deviation (Table 6). No variables were in Close Conformity and only seven of the forty were Acceptable Conformity.

TABLE 6
TROUBLED DATA SET FIRST DIGIT DATA SET FIRST DIGIT (316 MUNICIPALITIES)

| Number of Variables | Level of Conformity |
|----------------------------|----------------------------------|
| 0 | Close Conformity |
| 7 | Acceptable Conformity |
| 14 | Marginally acceptable conformity |
| 19 | Nonconformity |

The second digit analysis indicated that twenty-nine of the variables were Marginally Acceptable or Nonconforming. Only eleven were Acceptable Conformity and none was Close Conformity (Table 7).

TABLE 7
TROUBLED DATA SET SECOND DIGIT (316 MUNICIPALITIES)

| Number of Variables | Level of Conformity |
|----------------------------|----------------------------------|
| 0 | Close Conformity |
| 11 | Acceptable Conformity |
| 21 | Marginally acceptable conformity |
| 8 | Nonconformity |

Table 8 shows a side by side comparison of the analysis of conformity for all variables.

TABLE 8
MEAN ABOSOLUTE DEVIATION (MAD) BOTH DATA SETS BY ACTIVITY TYPER

| Variable | Non-troubled MAD First Digit | Troubled MAD First Digit | Non-troubled MAD Second Digit | Troubled MAD Second Digit |
|---|---|---|--|--|
| Governmental Activities | | | | |
| Gaunresnet | Close Conformity | Marginal | Close | Marginal |
| Gacapgrts | Acceptable | Nonconformity | Close | Nonconformity |
| Gadirectexp | Acceptable | Marginal | Close | Acceptable |
| Gafees | Acceptable | Marginal | Close | Marginal |
| Gainvcap | Acceptable | Acceptable | Close | Marginal |
| Gaopgrts | Acceptable | Marginal | Close | Acceptable |
| Garesnet | Acceptable | Acceptable | Close | Marginal |
| Directdebt | Acceptable | Acceptable | Close | Marginal |
| Business Type Activities Variables | | | | |
| Btadirectexp | Acceptable | Nonconformity | Close | Marginal |
| Btafees | Acceptable | Nonconformity | Close | Acceptable |
| Btainvcap | Acceptable | Nonconformity | Close | Acceptable |
| Btaunresnet | Acceptable | Nonconformity | Close | Marginal |
| Btachangenet | Marginal | Marginal | Close | Marginal |

| Enterprise Fund Variables | | | | |
|---|------------|---------------|------------|---------------|
| Efchangenet | Acceptable | Nonconformity | Acceptable | Marginal |
| Efcurassets | Acceptable | Nonconformity | Acceptable | Acceptable |
| Efcurliab | Acceptable | Marginal | Close | Marginal |
| Efopinc | Acceptable | Marginal | Close | Marginal |
| General Fund and Other Governmental Fund Variables | | | | |
| Gfcash | Acceptable | Nonconformity | Close | Marginal |
| Gfintgovre | Acceptable | Marginal | Acceptable | Acceptable |
| Gfnonfb | Acceptable | Nonconformity | Marginal | Nonconformity |
| Gfofsu | Acceptable | Marginal | Close | Acceptable |
| Gfrevenue | Acceptable | Acceptable | Acceptable | Marginal |
| Gftotassets | Acceptable | Nonconformity | Close | Acceptable |
| Gfunassignedfb | Acceptable | Acceptable | Close | Marginal |
| Ogfofsu | Acceptable | Acceptable | Close | Nonconformity |
| Ogfrevenue | Acceptable | Marginal | Close | Marginal |
| Ogftotalfb | Acceptable | Nonconformity | Close | Marginal |
| Ogftotassets | Acceptable | Marginal | Close | Marginal |
| Ogflexpend | Marginal | Marginal | Close | Nonconformity |
| Other Variables | | | | |
| Opebcovpay | Acceptable | Marginal | Close | Nonconformity |
| Ovrlapdebt | Acceptable | Nonconformity | Close | Marginal |
| Persaal | Acceptable | Nonconformity | Acceptable | Acceptable |
| Population | Acceptable | Nonconformity | Close | Nonconformity |
| Gengovtotexp | Marginal | Acceptable | Close | Marginal |
| TaxLevy | Marginal | Nonconformity | Acceptable | Acceptable |
| Gengovdebsvc | Acceptable | Nonconformity | Close | Marginal |
| Total Municipal Equity | Acceptable | Nonconformity | Close | Marginal |
| Properteav | Acceptable | Nonconformity | Close | Nonconformity |

The analysis indicates that there is a difference in conformity with Benford's Law between many of the values in the CAFRs of Troubled Municipalities and the CAFRs of Non-troubled Municipalities. Four of the variables for which there is a difference in the Benford's distribution are related to business type activities. Twelve of the variables are related to the general fund or other governmental funds. The business type activities are more like for-profit entities and might be more subject to manipulation of the numbers to maintain performance evaluation targets.

The largest Mean Absolute Deviation (MAD) occurred in the POPULATION variable. A review of the cities with negative unrestricted net position of the governmental activities shows seven cities of more than 1 million population, sixteen with more than 500,000 population, sixty-three cities with population between 100,000 and 499,999, and two-hundred twenty-nine cities with population of less than 100,000.

While the number of large troubled large cities is small, the proportion of large cities that are in the Troubled category is high. Seven of nine (77.8%) cities with over 1,000,000 population had a negative unrestricted net position but only 229 of 1,684 (13.6%) of cities with less than 100,000 fell into this category. Large cities are more likely to provide more extensive public services such as transportation, utilities, parks, pensions, etc. It is not surprising that they would be more likely to have a negative unrestricted net position.

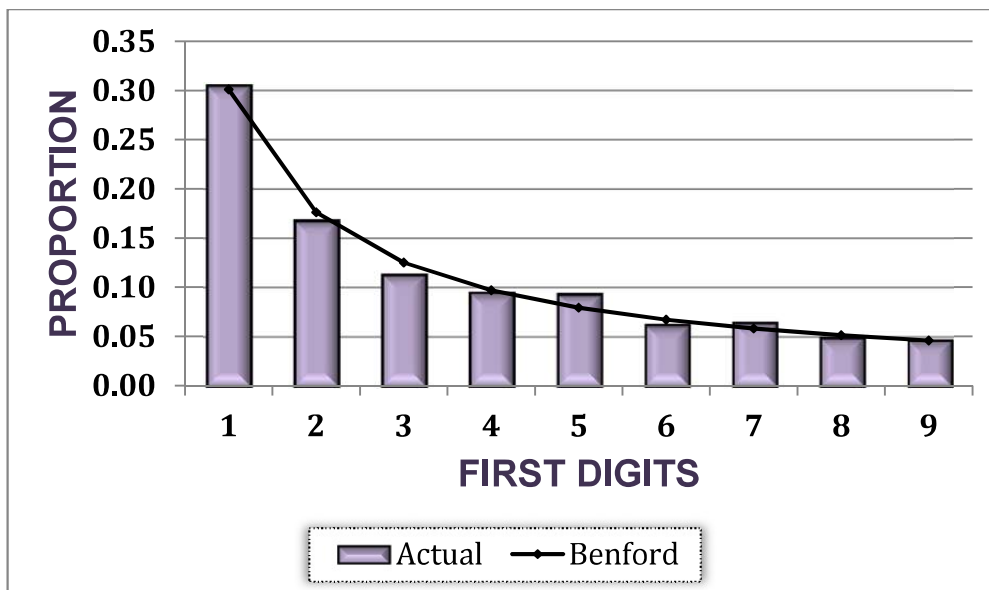
Nonconformity was identified in several of the variables related to business type activities (BTAINVCAP, BTAUNRESNET, BTADIRECTEXP, and BTAFEES) as well as total current assets of the enterprise funds (EFCRRASSETS). These variables involve services that generally are to some extent self-funded through user fees or other charges for services. The services are managed in a manner like for-profit entities but still are supported by taxpayer fees for service. These include such activities as utilities, garages, parks, etc. Because most of the variables related to business type activities are non-conforming, investigation of the business-type activities would be an important part of the analysis of a municipality when evaluating its long-term viability.

The General Fund also had several variables that were not in conformity. These included cash, total assets, non-spendable fund balance, and assigned fund balance (GFCASH, GFTOTASSETS, GFNONFB, and GFASSIGNEDFB).

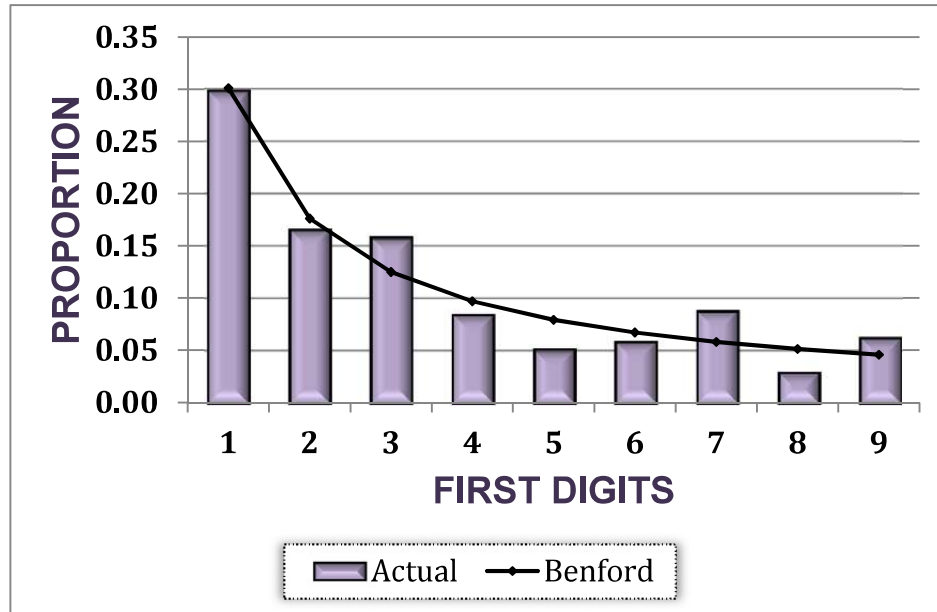
Other areas of nonconformity were in the accrued liability for employer pension plans (PERSAAL) and in the actuarial accrued liability for OPEB plans (OPEBAAL). Since pensions and post-employment benefits are troubling for many entities, both government and for-profit, this is a variable that should be evaluated carefully when looking at the long-term financial health of an entity.

If you look at graphs of the variables, you can see which numbers differ most from the expected values. Below are charts of first digits for Btadirectexp for both the Troubled Municipalities and Non-troubled Municipalities. You can see that the Non-troubled Municipality values closely conform to the Benford's distribution. The Troubled Municipality values do not conform as closely. One area of possible concern is that the Troubled Municipalities have more nines as first digits than expected. Since Btadirectexp is an expense, some municipalities may try to keep the values below the next level to make the financials look better. For example, rather than rounding the expenditures to 10,000,000, they might round down and leave the expenditures at 9,999,000. This type of "rounding" anomaly is one that might be discovered by using Benford's analysis to evaluate financial data.

FIGURE 1
BTADIRECTEXP NON-TROUBLED



**FIGURE 2
BTADIRECTEXP TROUBLED**



SUMMARY AND CONCLUSIONS

This study used Benford’s Law to analyze municipal data from the Comprehensive Annual Financial Reports of 1949 municipalities. Of the total data set, 316 municipalities were classified as Troubled because they had negative unrestricted net position of governmental activities. The Mean Average Deviation (MAD) of these entities was then compared with the MAD of the non-troubled data set. In several groups of variables, the Troubled Municipality entities’ data did not conform to Benford’s Law expectations while the Non-troubled Municipality data set’s MAD did conform.

The differences between the two data sets can provide areas that should be carefully investigated when determining the long-term financial outlook for a specific municipality. The primary areas where the troubled municipalities were nonconforming were in business type activities, general fund balances, and pension and other post-employment benefit obligations.

The primary purpose of this exploratory study was to determine whether differences between Troubled Municipalities and Non-troubled Municipalities could be identified using a Benford’s Law analysis. Future research may try to identify patterns in the specific ways that the two data sets differ. Another area for consideration would be to identify specific areas within a municipality’s numbers that could help to identify potential financial difficulties or fraud.

This study indicates that Benford’s Law may be a useful tool in analyzing governmental financial data as well as analyzing for-profit financial data. Further research may provide additional ways this type of analysis can be used to evaluate the financial statements of governmental entities including counties, schools, and special districts.

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APPENDIX 1

| Governmental Activities Variables | |
|---|--|
| Gaunresnet | Unrestricted net position of the governmental activities |
| Gacapgrts | Total capital grants and contributions program revenues of the governmental activities |
| Gadirectexp | Total direct program costs of the governmental activities |
| Gafees | Total fees and charges program revenues of the governmental activities |
| Gainvcap | Net investment in capital assets of the governmental activities |
| Gaopgrts | Total operating grants and contributions program revenues of the governmental activities |
| Garesnet | Total restricted net position of the governmental activities |
| Directdebt | The general bonded debt of the governmental activities |
| Business Type Activities Variables | |
| Btadirectexp | Total direct program costs of the business-type activities |
| Btafees | Total fees and charges program revenues of the business-type activities |
| Btainvcap | Net investment in capital assets of the business-type activities |
| Btaunresnet | Unrestricted net position of the business-type activities |
| Btachangenet | Total change in net position of the business-type activities |
| Enterprise Fund Variables | |
| Efchangenet | Total change in net position for enterprise funds |
| Efcurreasets | Total current assets of enterprise funds |
| Efcurreliab | Total current liabilities of enterprise funds |
| Efopinc | Total operating income of enterprise funds |
| General Fund and Other Governmental Fund Variables | |
| Gfcash | Total cash and investments of the government's general fund |
| Gfintgovre | The total of all intergovernmental revenues reported in the government's general fund |
| Gfnonfb | Total nonspendable fund balance of the government's general fund |
| Gfofsu | The net amount of other financing sources and uses reported in the government's general fund |
| Gfrevenue | The total revenues of the government's general fund |
| Gftotassets | Total assets of the government's general fund |
| Gfunassignedfb | Total unassigned fund balance of the government's general fund |
| Ogfofsu | The net amount of other financing sources and uses reported in governmental funds other than the government's general fund |
| Ogfrevenue | The total revenues of governmental funds other than the government's general fund |
| Ogftotalfb | Total fund balance of governmental funds other than the general fund |
| Ogftotassets | Total assets of governmental funds other than the general fund |
| Ogffexpend | The total expenditures of governmental funds other than the government's general fund |
| Other Variables | |
| Opebcovpay | The covered payroll for one of the government's single or agent multiple-employer OPEB plans |
| Ovrlapdebt | The overlapping debt of the government from the appropriate statistical table |
| Persaal | The actuarial accrued liability for one of the government's single or agent multiple-employer pension plans |

| | |
|-------------------------------|--|
| Population | The government's population. |
| Gengovtotexp | Total governmental funds expenditures from the appropriate statistical table |
| TaxLevy | The total amount of the government's property tax levy for the current year |
| Gengovdebsvc | Total debt service expenditures of the governmental funds from the appropriate statistical table |
| Total Municipal Equity | Total Equity of the entity. |
| Properteav | The total estimated actual value of all property (i.e., real, and personal property) taxable by the government |

APPENDIX 2

To extract only the first digit of any number in Excel, use the *Left* string function. The form of the formula is *=Left(cell reference, number of digits)*. This function returns the leftmost digit of the string.

To extract the second digit, use the *Mid* string function. This formula would be *=Mid(cell reference, place of desired digit, number of digits)*. The place of the desired digit is where it occurs in the string (1st place, 2nd place, etc.). The number of digits is how many digits you want. In this case, it would only 1 since we are getting the 2nd digit. If you wanted to analyze the first two digits, you could use 2 for the number of digits and the function would return the first two digits of the string.

Next, the *Countif* function is used to determine the number of each digit that occurs in the data set. The form of the function is *=Countif(range,criteria)*. For example, to count the number of the digit 3 in the range C31:C55, the function would be *=Countif(C31:C55,3)*.

Once the number of occurrences of each digit has been determined, the percentage can be calculated. Take the number of digit 1s that occur and divide by the total number of instances. That percentage can then be compared to the Benford's percentage as discussed in the body of the paper.