Housing Bubble in the United States

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This theoretical paper captures the dynamics of post-2000 housing price evolution in the United States by modeling the interaction between credit borrowers and lenders. Through securitization, banks and other financial intermediates were able to supply more credit to potential home buyers. In an infinite time horizon model, we model the financial intermediate's decision making of loans and liquidation as well as borrowers' voluntary default. We employ Bellman equation to describe economic agents' calculation of expected payoffs. We simulate the model and produce dynamics of housing supply, housing demand, housing price and mortgage rate over time.

INTRODUCTION

The housing sector is essential to the American economy. Historical data show that the residential investment averages about 5 percent of GDP and the housing services average about 12 percent, in total the housing sector accounts for about 17 percent of the total GDP. Moreover, housing is an important job-creating sector for construction workers; the boom of related sectors like decoration industry also helps reducing unemployment rate. Housing boom burst in the late 2000s and the subprime mortgage crisis triggered American financial crisis during which many large investment banks and other financial institutions failed.

In the first decade of 21st century, real estate property increasingly attracted fund managers' attention and became an important part of their portfolios in order to provide diversification and risk management. Credit derivatives based on real estate were created. For example, two of the most popular diversification tools in managers' portfolio are financial products related with REIT (Real Estate Investment Trusts) Index and NCREIF (National Council of Real Estate Investment Fiduciaries) Property Index (NPI). In the meanwhile, various securitization derivatives significantly increased the market liquidity. Most famous examples are mortgage-backed securities (MBSs), related credit default swaps (CDSs) and collateralized debt obligations (CDOs). On the one hand they provided investment tools for investors; on the other they reduced the burden of risk-based or regulatory capital that financial intermediates must undertake under the Basel Accord adopted in the G-10 conference, therefore making those financial institutions able to provide more liquidity and lead the investors' capital to fields with higher yields.

House prices have been an essential indicator of the economic trend since years ago, one phenomenon is that many economic bubbles start with a lending boom. Not much analytical work related to this area has been done in literature. Despite of numerous articles explaining bubbles and economic crisis, most of them focus on the stock market sector. They employ models with rational agents or irrational behaviors to provide possible explanations.

The recent economic crisis was triggered by subprime mortgage crisis beginning from year 2008. In this crisis, the houses have more negative effects and influences than in other crises because mortgage-based financial products have firmly connected house prices with other financial indicators. This is an interesting phenomenon. Traditional beliefs regard real estate properties as an effective tool of value preservation. In this sense, effects of houses should be counter-cyclical, and this is why fund managers use them to diversify. However, if real estate-based products have been linked with others, the housing price fall will trigger price fluctuation of other related financial products. And because of the huge amount of capital in real estate sector, the firmer the link, and the greater the influence. This is what Stiglitz (2010) says "contagion of crisis".

This was market participants' analytical false of market and investment diversification. And before the crisis they systematically neglected systematic problems. Historic experience indeed shows that large-scale defaults never happened simultaneously, there can only be a small probability that mortgage-based securities would suffer market value reduction by over 10 percent. But those participants did not fully realize that when market risks are linked together, the value of traditional diversification vanishes. They didn't consider that if housing price fell first, mortgage rate would increase and economy would step into recession, making relative assets more risky and raising the probability of defaults - this would further make housing price fall and would trigger another price-down-rate-up-default-up circle. The most recent rise of housing price started from around year 2000. One cause was low interest and mortgage rate as the government tried to inflate the economy after the internet bubble burst. Most people also believe that one reason was the financial liberalization in late 1990s. Part of the Glass-Steagall Act of 1933, which aims to separate businesses of investment banks, commercial banks and insurance companies, was repealed as the Financial Services Modernization Act of 1999, or the Gramm-Leach-Bliley Act, was passed. Financial intermediates therefore are allowed to securitize the loans they possess by issuing mortgage-backed securities, attracting more capital since investors find it less costly (Bolton and Freixas, 2000). The relaxation of financial institutes' supply constraints allow them to lend more credits to potential home buyers, triggering a potential rise of property price.

Besides the above-mentioned, there are other explanations of this crisis. They include: securitization made the borrowing-lending chain so long that causes serious information asymmetry problem; inconsistent salary incentive mechanism of money managers might encourage shortism and risky behavior; credit rating agencies are effectively employed by the firms they rate, causing wrong incentives which led them to make more credit enhancement tools; incorrect asset pricing models amplify investment risks and worsen financial institutions' risk management; home buyers' speculation led trend-chasing behavior, which led to more serious bubbles; poor due diligence work of credit suppliers created a large number of subprime borrowers; government's inappropriate encouragement of home ownership helped distort public sentiment; globalization and trade deficit created a large pool of investors in emerging economies, whose investments in American housing market and financial products of housing loan securitization inflated housing price, etc.

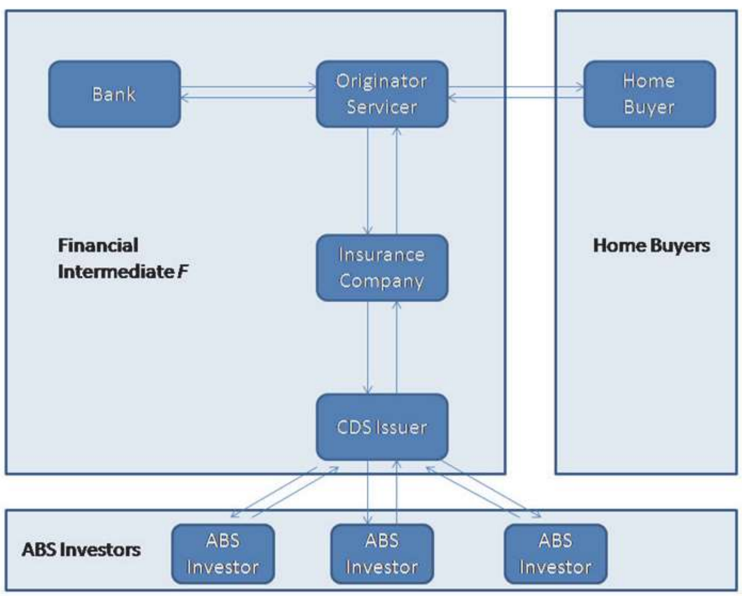
This paper focuses on the initial housing price increase triggered by securitization and tries to endogenize the movement pattern of price increase and crash by analyzing the interaction between potential home buyers with unstable future income flows and financial institutions' lending and liquidation decisions.

In the American housing market, different financial institutions specialize in different services. A mortgage originator, either a mortgage broker or a mortgage banker, works with a borrower to complete a mortgage transaction as the original mortgage lender. Mortgage originators are part of the highly fragmented primary mortgage market. After the transaction, borrowers pay their mortgage loan payments to a mortgage servicer, whose duty includes the acceptance and recording of mortgage payments;

calculating variable interest rates on adjustable rate loans; payment of taxes and insurance from borrower escrow accounts; negotiations of workouts and modifications of mortgage upon default and conducting or supervising the foreclosure process when necessary (FDIC Law, Regulations, Related Acts). CDO issuers, usually insurance companies, either purchase loans from servicers or issue CDSs to servicers as credit protectors. Servicers pay insurance fee to CDO issuers if there is no default in years and receive insurance payment if there will be defaults. If the insurance companies cannot wait for years to receive the insurance payments, it can choose to accept immediate payment from CDS issuers and transfer the future insurance payments to CDS issuers. CDS issuers then use the assets (insurance payments in the future) as collaterals to sell mortgage-backed or asset-backed securities. Market participants also include many other kinds of financial service providers.

In my model, all those financial institutions are simplified into one financial intermediate (this is reasonable if all the service providers merge into a conglomerate, as Gramm-Leach-Bliley Act allows). This intermediate does due diligence like checking income structures of potential home buyers; makes loan lending decisions; collects the mortgage payments; reorganizes the loans and slices the assets into different securities like MBSs to absorb new capital and lends out again. Also this intermediate deals with defaults, renegotiation, foreclosure and liquidation.

FIGURE 1
FINANCIAL INSTITUTES AND MODEL SIMPLIFICATION



We have two main players in our model: credit borrower and financial intermediate. We do not model the behavior of securities investors. The home buyers and also loan borrowers have random income flows in every period; they apply loans and outstanding loans will rollover to next period if they haven't been fully repaid in the current period; they also are allowed to default if they see the floating mortgage rate goes too high for them to afford the rest of mortgage payment. And when they default, they just need to drop off the keys, leave the house to the credit lender and go away. This is so-called nonrecourse debt: the lender can seize the collateral if the borrower defaults, but the lender's recovery is limited to the collateral.

The other player is the financial intermediate which makes lending and liquidation decisions. The financial intermediate considers two effects when it makes lending decisions. On one hand, without pervasive inflation, when house price increases much faster than income, this credit lender has to consider the borrowers' capacity to repay the debt. On the other hand, while house price is high and other lenders are lending, lenders may be more willing to lend because once borrowers are forced to default due to short-term liquidity shock, lenders can foreclose and liquidate the houses, and if home price is greater than principal's value of loans plus interest, the lender will profit. A borrower pays the mortgage period by period. If in one period the borrower has no income under some situation like being unemployed, the financial intermediate may consider to liquidate the borrower's house based on its calculation of payoffs and its expectation of future housing price. If it decides not to liquidate, they may renegotiate the contract and the borrower may delay the payment to the next period. The liquidation and negative income shock push down the price while the lending drives the price up.

In this paper, we assume the financial intermediate begins with an initial capital available to lend in the first period. We build the rationale of the financial intermediate's decision of making loans to potential home buyers over an infinite time horizon. The financial intermediate securitizes the loans to raise new capital and makes loans to more borrowers with unstable future income flows. Currently, we simplify the securitization as a tool to raise capital without cost over time. The financial intermediate calculates the expected payoffs in different scenarios under the realizations of random incomes to decide whether to make loans to a new borrower and whether to liquidate a house if the borrower is short of liquidity in the short run. After clarifying the interaction between the two players and sequence of moves within each period, we compute the financial intermediate's decision rule described by Bellman equations. Then we simulate borrowers' income realization and produce an evolution of housing price as well as other variables.

The ultimate aim of this paper is to generate a housing price evolution path with first-increase-then-crash shape. There are three decisions to make in our model. The first one is the credit lending decision made by the financial intermediate. The decision depends upon a borrower's income flow pattern, current housing price and expected housing price in the next period and further future. The second decision is the financial intermediate's liquidation decision when some borrower's current income realization is not enough to repay that period's mortgage. Here the financial intermediate's goal is to maximize its expected payoffs. The third decision is the home buyers' voluntary default decision. The decision also depends upon above factors and the buyer makes decision to maximize its monetary payoff. Interaction between the financial intermediate's and the home buyers' decisions determines the housing supply and demand and the equilibrium housing price in every period.

Furthermore, the model has a mortgage rate adjustment machine. The readers may find it like a central bank. This machine determines every period's mortgage rate according to an exogenous formula, enlightened from the Taylor rule. As assumed, all the mortgage rates in this model are floating, therefore the future payment of mortgages depends on today's rate, not the rate when the loan contract was made. As rate increases, the future expected payment increases, and vice versa.

Besides the two main players, the financial intermediate and the borrowers, it is useful to include another player, the property developer. Since the home building takes time, we assume that it takes a positive number of periods for the homes to be finished. The developer decides how many homes to build based on its expectation of future housing price and the current mortgage rate, which we assume as the developer's financing cost. We make such an assumption because we see a correlation between borrowing interest rate and mortgage rate, and to make the model simpler and number of variables fewer, we assume the two rates are the same here.

We have an assumption that for any borrower, to own a house gives higher utility than to hold the equivalent value of cash. Therefore, we assume that whenever it's possible to afford a house even through borrowing from the financial intermediate, a borrower will apply loans and buy the house. A borrower may abandon his house under one condition, that is when the mortgage rate goes too high and the discounted value of future mortgage payment is even higher than the current housing price. Once this happen, the borrower will default by leaving the current house to the financial intermediate and then buy a new house. Of course here we assume all houses are homogeneous and the run-away borrower will not face the credit qualification problem when he applies loans again. In fact, we ignore the borrower's behavior after he defaults. We assume there are many districts in the world and each one contains a financial intermediate, borrowers and a developer. Once the borrower defaults, we assume that he goes to another district.

The rest of paper is organized as follows. The second section briefly reviews related literature. The third section models each player's behavior and clarifies the sequence of moves. The fourth section simulates the housing supply, demand and price path. The last section concludes.

LITERATURE REVIEW

Related literature at least includes two branches. The first branch is the housing market lending and financing behavior as well as studies of bubbles. Among the great amount of articles on bubbles, economic crisis and asset pricing, few of them study housing market (endnote 1). Most focus on the stock market sector. For example, Scheinkman and Xiong (2003) and Pastor and Veronesi (2003, 2006) studies possible reasons of stock price increases, the former attributes to different beliefs of investors while the latter believes that the uncertainty is the cause.

Other researchers focus on life-cycle models and incomplete markets and housing decisions without aggregate risk. Fernandez-Villaverde and Krueger (2010) study how life-cycle consumption is influenced by consumer durables in an incomplete markets model with production, but limit their focus to equilibriums in which prices, wages and interest rates are constant over time. Kiyotaki, Michaelides, and Nikolov (2011) study a life-cycle model with housing and non-housing production, focusing their analysis on the perfect foresight equilibria of an economy without aggregate risk and an exogenous interest rate. Iacoviello and Pavan (2009) combine aggregate risk, production, and incomplete markets. They study the role of housing and debt for the volatility of the aggregate economy in a model with a single production and single saving technology.

Some researchers study the land market. Kiyotaki and Moore (1995) examine how the amplification and persistence of a negative temporary shock affects the land market. Lustig and Van Nieuwerburgh (2005) employ a two-sector exchange economy model to study the empirical relationships among housing collateral, consumption insurance and risk premia. Bolton and Freixas (2000) build model to compare the costs of different financing methods. Their paper says that with asymmetric information, different financing tools incur different costs, banks choose tools under given conditions. Among the financing channels, internal securitization has a low cost.

One of the most recent and important work is Favilukis, Ludvigson and Van Nieuwer-burgh (2010). Their model's main implications are: 1) house prices relative to measures of fundamental value are volatile; 2) a financial market liberalization drives price-rent ratios up because it drives risk premia down; 3) foreign purchases of U.S. bonds play a central role in lower interest rates but a small role in housing booms; 4) financial market liberalization plus foreign capital leads to a shift in the composition of wealth towards housing.

The second branch is dynamic discrete-choice models, which enlightens me to model the financial intermediate's decision choice. One of the classic papers is Rust (1987) which studies how a manager of bus depot makes decision to replace engines. As mileage goes up because of long-time use of engines, the maintenance cost is higher if not replacing; replacing incurs a large fixed cost but clear the mileage to zero. The manager must compare the tradeoff to make engine replacement decision. Rust constructed a dynamic forward-looking model to examine above what cutoff value the manager will replace.

MODEL

Notations

Timeline and Players

In this model, there are infinite periods: t = 0, 1, 2. There are three players: a financial intermediate F, a number of borrowers and a developer D. The financial intermediate lives in every period. In every period t = 0, 1, 2, ..., a single borrower comes to apply the loans (in another way, a borrower was "born").

We denote the borrower who comes at period t (or "born" at *t*) as borrower *t*. Therefore at *t* = *T*, there have been in total *T* borrowers coming.

Income

At period t, a new borrower t comes with an endowment $I_t^t > 0$. At period t, every borrower born at $t' < t$ receives an uncertain income $I_t^{t'}$. Income is a random variable $I_{t'}^t = I > 0$ with probability $p^{t'}$ and $I_{t'}^t = 0$ with probability $1 - p^{t'}$. Notice that $p^{t'}$ only depends on t', the period that the borrower was born but not t. We make this setting for a reason. In real world, we have borrowers with different financial situations coming to apply loans together. Some of them have more stable jobs and others don't. A rational bank with certain amount of credits available will lend to those with better future income flows first. Since we set in the model that at each period only one borrower was born, the earlier coming borrowers should always have better income realizations, and we let $p^{t_1} > p^{t_2}$ if $t_1 < t_2$ to realize this idea. In fact, borrowers with better credit records and income are always served earlier.

In sum, at period t, every borrower receives $I_t^{t'}$. The superscript indicates the period at which the borrower was born and the subscript indicates the current period. For example, I_6^4 is the period 6's income received by the borrower who applied loans at period 4. Before period 6, this borrower had endowment I_4^4 at period 4 and received income I_5^4 at period 5.

Housing Price, Mortgage Rate and Loans

At period *t*, there is a sequence of moves by players. During the period, when each player's decision-making depends on the housing price, they use H_t. H_t is the housing price determined by housing supply and demand at period $t - 1$. We assume that at each period, $H_t > I_t^t$. It says that for the buyer born at period *t*, his endowment is not enough to afford a house, therefore he must go to apply loans from the financial intermediate. The amount of loans he applies at period t is $H_t - I_t^t$, if approved, the amount he needs to repay at $t + 1$ is

$$L_{t+1}^t = (1 + r_t)(H_t - I_t^t) \quad (1)$$

where the superscript of L_{t+1}^t is the period at which he was born. The subscript $t + 1$ indicates that it is the amount that still remains to be repaid at $t + 1$ by the borrower t. For example, L_5^3 is the amount of outstanding loan that the borrower 3 (that is, this borrower comes to apply loans at period 3) has to repay at period 5.

In the formula, the r_t is the mortgage rate at period t. As described in the introduction part, it is determined by an adjustment machine (or the central bank) at every period.

Timeline

As we mentioned above, there are infinite periods in our model: t = 0, 1, 2, ... During each period t, players' moves have a sequence as follows. We will explain each move in the rest of the paper.

At the end of period $t - 1$, H_t was determined then keeps sticky within the entire period t

At the beginning of period *t*, adjustment machine randomly decides whether to adjust the mortgage rate r_t

Financial intermediate *F* makes a 1) liquidation rule (to earlier borrowers), a function of H_t and outstanding loans of a borrower and 2) lending rule (to the new borrower), a function of H_t

Income $I_t^{t'}$ of each borrower born at $t' < t$ realizes

Every borrower independently decides whether to voluntarily default according to r_t and $L_t^{t'}$, if borrower t''s income $I_t^{t'} = 0$ and he doesn't default, he pays $I_t^{t'}$, *F* can't liquidate

For borrowers with $I_t^{t'} = 0$, *F* decides whether to liquidate their houses according to the liquidation rule if the borrower t' doesn't voluntarily default

A new borrower t comes/born

F decides whether to make loans to the new borrower according to the lending rule

Housing demand H_t^D determined by new loans

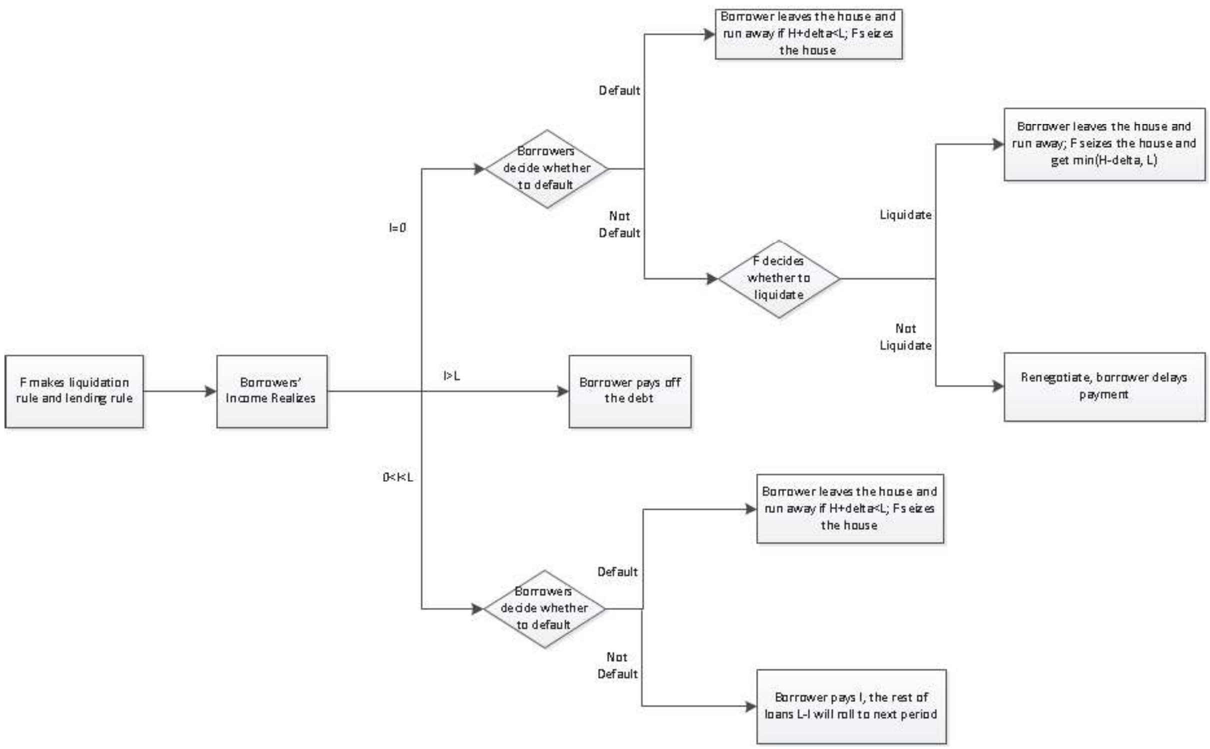
Developers decide whether to build new homes. Homes that were built are complete for sell, together with liquidations and defaults, determining housing supply H_t^S

Supply and demand determine the new housing price $H_{t+1} = f(H_t^D, H_t^S)$

Sequential Game

The interaction of liquidation and voluntary default between borrowers and the financial intermediate is a sequential game shown as follows. The readers should find the sequence clearer if the above timeline is confusing.

FIGURE 2
INTERACTION OF LIQUIDATION AND VOLUNTARY DEFAULT



Borrowers

Characteristics

Some of the borrowers' characteristics have been mentioned in the introduction part. To summarize, when a new borrower comes, she goes to F to make initial loans for home purchase. Her income flow is unstable: she may have no income in some periods under situation like being unemployed. Her loans have to be repaid until fully paid off and she has to pay every period if she has positive income. Mortgage rate of loans is floating, so if the rate goes up, she has a higher expected future payment to make. She may voluntarily default her loan payment when the mortgage rate goes too high and the discounted value of future mortgage payment is even higher than the current housing price. Once this happens, the borrower will default by leaving the current house to *F* and then buy a new house.

Rollover of Loans

Once at some period, the borrower's realized income is not enough to pay off the outstanding loans, the loans will roll to the next period. The process of rollover of loans is as follows.

As mentioned before, the borrower t applies loans at t for the amount of $H_t - I_t^t$, if approved, he needs to repay at $t + 1$ this amount

$L^t_{t+1} = (1 + r_t)(H_t - I^t_t)$ (2)

At $t + 1$, borrower t repays his debt, wholly or partially, with his realized income I^t_{t+1}. We assume mandatory repayment, that is if $I^t_{t+1} > 0$, the borrower must repay unless he defaults. We assume that the borrower is not allowed to delay the payment to future. Actually he has no incentive to hold the cash and delay the payment because loans will be greater in the next period because of the multiplication of the mortgage rate.

If the realized income $I^t_{t+1} = 0$, the borrower is unable to repay, but he can choose whether to default. If he chooses not to default, *F* can choose whether to liquidate his house. If he defaults, then he runs away and *F* seizes the house.

When $I^t_{t+1} > 0$, there are two situations. If $I^t_{t+1} \geq L^t_{t+1}$, the borrower will have no obligation from $t + 2$. Otherwise, if $I^t_{t+1} < L^t_{t+1}$, the borrower still has to repay $(1 + r_{t+1})(L^t_{t+1} - I^t_{t+1})$ from period $t + 2$. Therefore, loans that borrower t has to repay at $t + 2$ equals

$L^t_{t+2} = \max[(1 + r_{t+1})(\ L^t_{t+1} - I^t_{t+1}),0]$ (3)

Similarly, loans that borrower t has to repay at t' + 1 ($t' > t$) equals

$L^t_{t'+1} = \max[(1 + r_{t'})(\ L^t_{t'} - I^t_{t'}),0]$ (4)

Voluntary Default

At period t, every borrower $t' < t$ has to consider whether to default voluntarily. A borrower may want to default because he sees that the future payment $L^{t'}_t$ is too high and even higher than the current housing price. If that happens, his better option is to default and apply loans to buy a new house. Someone may question about the expected mortgage rate. If the borrower expected the rate to go down and the outstanding loans might fall, would he choose not to default? The answer is no. Even he expects a lower future mortgage rate, he will still be better off by buying a new house because the new loan will be smaller than $L^{t'}_t$ and it would be also reduced if future rate goes down. Of course, default must have some costs even it's a nonrecourse debt. The costs include moving cost and possible credit record harm. Therefore we model the default as follows. If

$H_t + \delta_b < L^{t'}_t$ 0

When a borrower decides to default, the housing supply H^S_t increases by 1.

All above processes (income realization, loans making, rollover of loans and default) happen every period for every borrower with positive outstanding loans.

Financial Intermediate

Characteristics

Characteristics of financial intermediate have been described in the introduction part. In sum, at each period t, the financial intermediate decides whether to offer loans to a newly born borrower t; collects mortgage payments in the future; once borrowers with $I^{t'}_t = 0$ fail to repay, F decides whether to liquidate (takes house without future payback of outstanding loans).

In real world, this conglomerate also has the function of absorbing investment into housing market. The financial intermediate slices loans into MBSs and sells them to investors, quickly recover cash and offer more loans to new borrowers. There is a tradeoff in this securitization; it's between the time of recovering future payments and the amount of payments you recover. For example, a financial intermediate lends \$1 million; without MBSs, it may recover \$2 million in 10 years if there is no default, every year it receives 200 thousand dollars; but with MBSs, it is allowed to recover \$1.5 million in merely 1 year.

Liquidation Rule

When the borrower t''s $I^{t'}_t = 0$ and the borrowers doesn't default, the financial intermediate decides whether to liquidate or allow the borrower to delay mortgage payment (renegotiation). As the timing part has clarified, the financial intermediate makes a liquidation rule and a lending rule before borrowers move. Since the borrowers make default decision before the financial intermediate moves, therefore the financial intermediate's liquidation rule should consider this when calculating expected payoffs. We can picture that in real world, every time after the central bank adjusted the interest rate and the market adjusted mortgage rate accordingly, the bank made a documents telling its employees how to deal with the credit borrowers who will come. For liquidation, the document should have liquidation criteria based on the borrowers' outstanding loans and expected housing price.

We model this as follows. After liquidation, F receives $\min(H_t - \delta_F, L_t^{t'})$ where δ_F is F's liquidation cost (including foreclosure cost and so on). The reason of the minimum operator is that if $H_t > L_t^{t'}$, the borrower t' will sell house before F liquidates and pay $L_t^{t'}$; if $H_t < L_t^{t'}$, the borrower would have defaulted.

The readers should notice that F may not want to liquidate when H_t is much higher than outstanding loans $L_t^{t'}$, because if F waits, the loans of next period will be greater: $L_{t+1}^{t'}|_{I_t^{t'}=0} = (1+r_t)L_t^{t'} > L_t^{t'}$. For a similar reason, F may not liquidate if it expects housing price to increase (endnote 2).

Before income realization of any borrower $t' < t$, F makes a liquidation rule $D\big(H_t, E(H_{t+1}), L_t^{t'}\big) =$ 1 (liquidate) or 0 (not liquidate) to decide whether to liquidate given $I_t^{t'} = 0$ under the conditions of housing price and a borrower's outstanding loans. Remember that H_t is determined by H_{t-1}^D and H_{t-1}^S in the last period $t-1$.

We assume a shortism-like martingale expectation $E(H_{t+1}) = H_t$. It's not necessarily rational but reasonable. Therefore the decision rule becomes $D\big(H_t, L_t^{t'}\big)$. At period t, forward-looking F makes the following analysis for every borrower t' with $L_t^{t'} \geq 0$ given r_t, considering its future value function:

$$V\big(H_t, L_t^{t'}\big)|_{r_t,p^{t'}} = p^{t'}\left\{\left\{\min\big(L_t^{t'}, I_t^{t'}\big) + \beta_F E\left[V\big(H_{t+1}, L_{t+1}^{t'}\big)|_{I_t^{t'}=I}\right]\right\}\left(1_{H_t+\delta_b>L_t^{t'}}\right) + H_t\left(1 - 1_{H_t+\delta_b>L_t^{t'}}\right)\right\} + \big(1-p^{t'}\big)\{\max_{D(H_t,L_t^{t'})\in\{0,1\}}\{D(H_t,L_t^{t'})\min\big(H_t-\delta_F, L_t^{t'}\big), (1 - D\big(H_t,L_t^{t'}\big))\beta_F E\left[V\big(H_{t+1}, L_{t+1}^{t'}\big)|_{I_t^{t'}=0}\right]\}(1_{H_t+\delta_b>L_t^{t'}}) + H_t(1 - 1_{H_t+\delta_b>L_t^{t'}})\} \quad (6)$$

where

$$L_{t+1}^{t'}|_{I_t^{t'}=I} = (1+r_t)\max(L_t^{t'} - I_t^{t'}, 0) \quad (7)$$

and

$$L_{t+1}^{t'}|_{I_t^{t'}=0} = (1+r_t)\big(L_t^{t'} - I_t^{t'}\big) = (1+r_t)L_t^{t'} \quad (8)$$

Let us explain this Bellman equation. $V\big(H_t, L_t^{t'}\big)|_{r_t,p^{t'}}$ is the value function at period t as F is calculating based on current housing price (because of martingale expectation, it's also the expected price) H_t and the borrower t''s outstanding loans $L_t^{t'}$. The calculation is under the mortgage rate at period t, r_t, and the probability of positive income realization, $p^{t'}$.

On the right hand side, the first part,

$$p^{t'}\left\{\left\{\min\big(L_t^{t'}, I_t^{t'}\big) + \beta_F E\left[V\big(H_{t+1}, L_{t+1}^{t'}\big)|_{I_t^{t'}=I}\right]\right\}\left(1_{H_t+\delta_b>L_t^{t'}}\right) + H_t\left(1 - 1_{H_t+\delta_b>L_t^{t'}}\right)\right\} \quad (9)$$

is the financial intermediate's payoffs when the borrower t''s income realization $I_t^{t'} > 0$.

There are two situations under positive income.

The first situation is when the borrower doesn't default, which is what the indicator $1_{H_t+\delta_b>L_t^{t'}}$ means. Then when $I_t^{t'} > L_t^{t'}$, the borrower t' pays off his debt and F receives $L_t^{t'}$; when $I_t^{t'} < L_t^{t'}$, the

borrower t' pays $I_t^{t'}$ and F receives $I_t^{t'}$. So F will receive $\min\left(L_t^{t'}, I_t^{t'}\right)$ at current period t. Moreover, under positive income, no default and no liquidation, the whole analysis will be the same at the next period $t+1$, therefore we have $\beta_F E\left[V\left(H_{t+1}, L_{t+1}^{t'}\right)|_{I_t^{t'}=I}\right]$ where β_F is the discount factor.
The other situation is when the borrower defaults, which is $1 - 1_{H_t+\delta_b > L_t^{t'}}$. If this happens, F will receive $\min\left(H_t, L_t^{t'}\right)$ as if $H_t > L_t^{t'}$, the borrower would sell the house, pay the outstanding loans and run away with the rest. However, the borrower's default has indicated that $H_t < H_t + \delta_b < L_t^{t'}$, so F will receive H_t.
The second part on the right hand side looks even more complicated, but the default part is just the same as the first part as we have explained. Putting $\left(1 - p^{t'}\right)$ and $H_t(1 - 1_{H_t+\delta_b > L_t^{t'}})$ aside, we now focus on the liquidation part given that the borrower t' does not default:

$$max_{D(H_t, L_t^{t'}) \in \{0,1\}}\{ D(H_t, L_t^{t'}) \min\left(H_t - \delta_F, L_t^{t'}\right), (1 - D\left(H_t, L_t^{t'}\right))\beta_F E\left[V\left(H_{t+1}, L_{t+1}^{t'}\right)|_{I_t^{t'}=0}\right]\} \quad (10)$$

The max operator means that F must choose between liquidation or not to maximize its payoff. If F liquidates, it receives $\min\left(H_t - \delta_F, L_t^{t'}\right)$ as we discussed before. If F doesn't liquidate, it receives nothing at the current period t as the income realization $I_t^{t'} = 0$; but it still has future value of $\beta_F E\left[V\left(H_{t+1}, L_{t+1}^{t'}\right)|_{I_t^{t'}=0}\right]$ where β_F is again the discount factor.

Lending Rule

As explained in the last part, the bank should have a "document". For lending, this document should have different credit lending limit for borrowers with different credit history and financial situations.
When new borrower t comes with $I_t^t > 0$, F decides whether to provide credit to him. Let F's lending decision as $d(H_t) = 1$ (lend) or 0 (not). Notice the lending decision rule is based on the current housing price which is also the expected housing price because of the martingale expectation assumption. If F decides not to lend, its payoff is 0; if it lends, the payoff is the discounted expected value of the value function at period $t+1$ minus loans

$$\beta_F E[V(H_{t+1}, L_{t+1}^t)] - (H_t - I_t^t) \quad (11)$$

Thus F 's value function as a function of $d(H_t)$ is

$$\mathrm{W}(H_t) = \max_{d(H_t) \in \{0,1\}}\{d(H_t)[\beta_F E[V(H_{t+1}, L_{t+1}^t)] - (H_t - I_t^t)], 0\} \quad (12)$$

where

$$L_{t+1}^t = (1 + r_t)(H_t - I_t^t) \quad (13)$$

Mortgage Rate

We assume there was a mortgage rate adjustment machine, or the central bank, which decides when and how to adjust the mortgage rate r_t. We assume the machine does not adjust the mortgage rate every period, at which period it adjusts is random and not predictable to the market. This assumption is also reasonable because otherwise if the adjustment is predictable, the market should have responded to it before announcement.

We model the adjustment as follows. The machine adjusts once every T_c periods, T_c is random. In real world, the Federal Reserve's target is the inflation rate, or CPI. When CPI goes up and the central bank decides to make adjustment to the interest rate, it will make the rate higher. Taylor rule says the rate should be set as a function of desired, current and historical inflation

$$r_t = f(H_t, H_{t-1}, H_{t-2}, \dots, H_1) \quad (14)$$

Here we assume a close correlation between housing price and inflation and a close correlation between mortgage rate and interest rate.

Property Developer

The property developer is a home builder. Reasonably, it builds more houses when r_t (endnote 3) is low and when expected future housing price is high. Again, here we assume martingale expectation $\mathrm{E}(H_{t+1}) = H_t$.

In real world, the home building takes time. Therefore the home builder' behavior does not follow rational expectation as its information sources are more or less limited. We assume the home building takes T_d periods to accomplish and when houses were completed, housing supply thereafter increases by Q_t.

Now we model the developer's rationale. At period t, the developer maximizes its profit of

$$\max_{Q_t} E\left(H_{t+T_d}\right)Q_t - (1 + r_t)c(Q_t) \tag{15}$$

where the cost of building $\mathrm{c}(Q_t)$ can be specified as $b_1 Q_t^2 - b_2 Q_t + b_3$.

Solve out the maximization problem, the housing supply increase is

$$Q_t = \frac{1}{2b_1}\frac{H_t}{1+r_t} + \frac{b_2}{2b_1} \tag{16}$$

To make the simulation easier, we assume the developer has a capacity constraint. We assume that it provides a fixed number of houses when $Q_t > \eta$ and builds nothing otherwise. When it builds at period t, housing supply in the future $H_{t+T_d}^S$ changes.

Intuition of Housing Price Evolution

Now we summarize and try to capture the reason of the housing price moving path. At first, securitization makes new loans possible and allow the financial institutions to recover new capital quickly; then new borrowers' demand pushes up housing price; higher price makes the developer build new homes; increasing price drives inflation higher, at some point, central bank raises mortgage rate, increasing borrowers' future loans; when new homes are complete, supply increase pushes down housing price; borrowers voluntarily default because of increasing loans and lower housing price; default leads to increasing housing supply, pushing down housing price; besides, worse economy gives worse income realizations (if $I_t^{t'}$ random); liquidation and foreclosure increase, increasing housing supply and pushing down housing price; lower housing price further induces more default; the circle finally make the bubble burst.

SIMULATION RESULTS

In our simulation, we make some specifications. They are not perfect so far and we will make improvement in the future work. The mortgage rate adjustment function is set as

$$r_t = \phi\sqrt{H_t} \tag{17}$$

Fixed amount of new homes are built when

$$\frac{H_t}{1+r_t} > \eta \tag{18}$$

where η is a threshold. $p^{t'}$s are set the same for all t to save simulation time.

***F*'s Value Function**

Solving decision rules follows standard Bellman equation numerical computation. We do have some interesting and intuitive findings. One of them is that when loan-price ratio is high enough, the financial intermediate tends not to liquidate. Mathematically, an increase of loans increases $\beta_F \mathrm{E}[\mathrm{V}\left(H_{t+1}, (1 + r_t)L_t^{t'}\right)|_{I_t^{t'}=0}]$ (endnote 4) but not $\min\left(H_t - \delta, L_{it}^{t'}\right)$ since the latter takes a form of minimum. The intuition is that when the outstanding loan is high enough (possibly because the loan compounds period by period and rises fast when the borrower's realized income keeps being zero for several successive periods), to liquidate the house only gives F a given one time payoff but not to liquidate gives F right to receive the borrower's income of every period in the future which is probably more than what it can get from liquidation.

The figure shows how different values of loans and housing price are associated with corresponding$\mathrm{E}[\mathrm{V}(H_{t+1}, L_{t+1}^{t'})]$. It shows that when loans increase to a high level, the value function stops increasing as the curve becomes flat. Intuitively, for an ordinary home buyer, owing one billion dollars and owing one trillion dollars to the bank gives the bank no difference because the amount this borrower owes is too large for him to repay in the future.

Voluntary Default and Liquidation

In the following figure we show in what combination of outstanding loan and housing price will a borrower defaults and in what combination will a borrower be liquidated by the financial intermediate.

The upper area is the set of combinations of default and the middle area is the set of liquidation. It says if the outstanding loans are much higher than housing price, the borrower will default. Readers may find the dark color line is about 45 degree. It's not a perfect 45 degree line (see the upper end is not (loan 20, housing price 15), the housing price is a little more than 15) because of the existence of δ_b, the default costs. When outstanding loans are high but not higher than housing price, the borrower will not default but if his income realization becomes zero, the financial intermediate will implement liquidation. When outstanding loans are low, neither default nor liquidation will happen.

FIGURE 3
VALUE FUNCTION WITH PARAMETERS
$\boldsymbol{p^t = 0.5, \beta_F = 0.9, r_1 = 0.2\ , \delta_F = 0.5, \delta_b = 4, T_d = 2, T_c = 15, I = 2, \eta = 1,\ \phi = 0.4}$

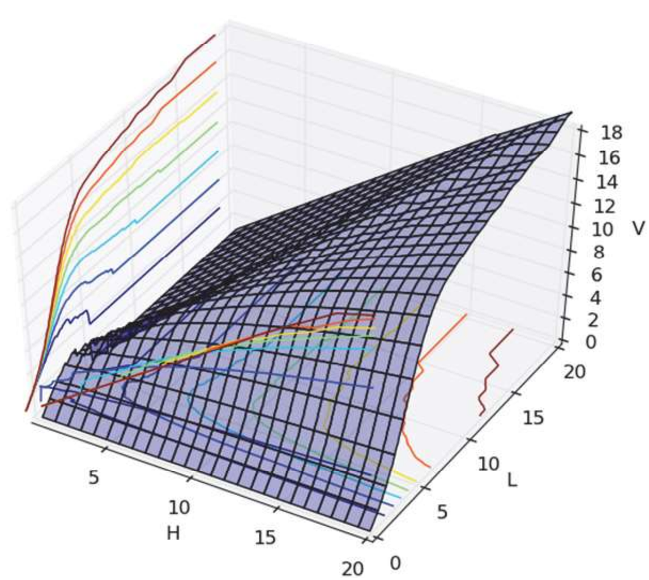
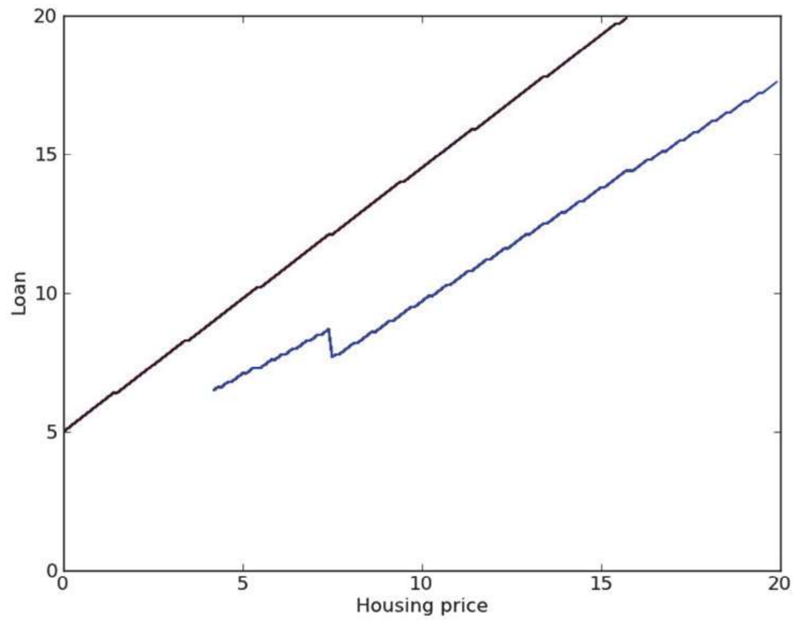


FIGURE 4

$$p^t = 0.5, \beta_F = 0.9, r_1 = 0.2, \delta_F = 0.5, \delta_b = 4, T_d = 2, T_c = 15, I = 2, \eta = 1, \phi = 0.4$$



Housing Supply and Demand, Mortgage Rate and Housing Price

Figure 5 and 6 show the simulated path of housing supply and demand, mortgage rate and housing price under two sets of parameters. When housing price goes up, it’s because the supply increase is less than demand increase, and vice versa. Instead of letting mortgage rate adjustment completely random, we set a fixed time period for the adjustment machine to change the mortgage rate in the simulation.

FIGURE 5

$p^t = 0.7, \beta_F = 0.9, r_1 = 0.2\ , \delta_F = 1.5, \delta_b = 1, T_d = 2, T_c = 15, I = 2, \eta = 1, \phi = 0.4$

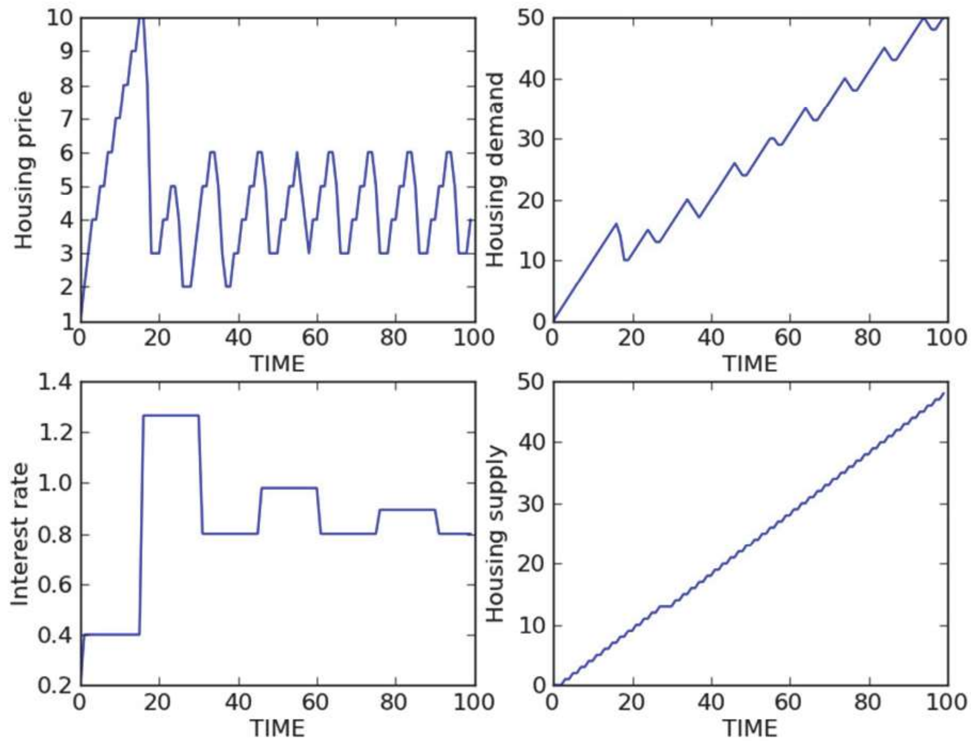
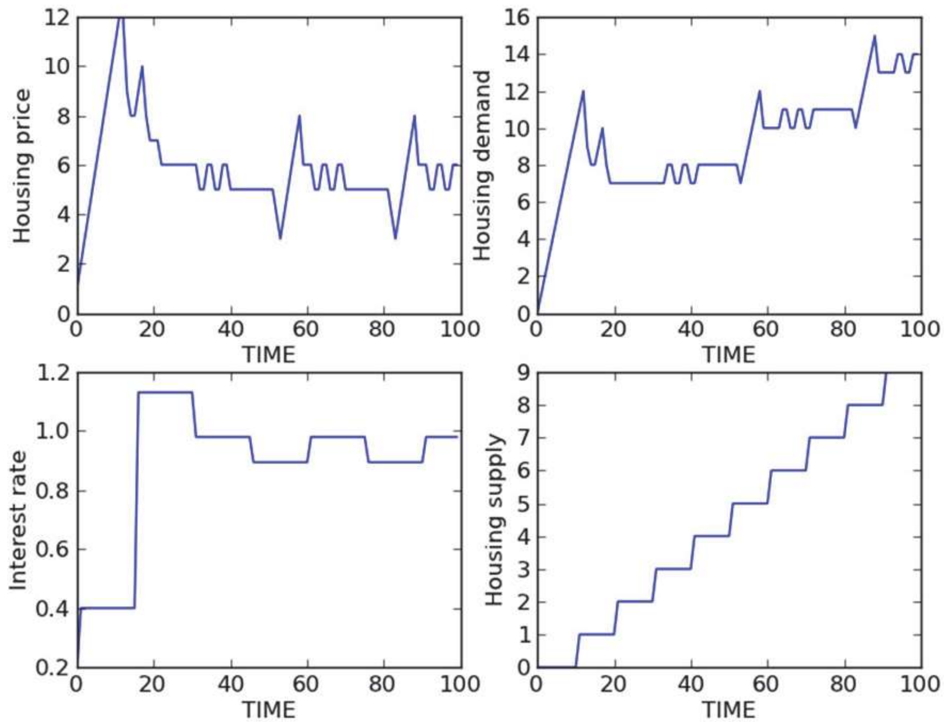


FIGURE 6

$$p^t = 0.7, \beta_F = 0.9, r_1 = 0.2, \delta_F = 1.5, \delta_b = 1, T_d = 20, T_c = 15, I = 2, \eta = 1, \phi = 0.4$$



CONCLUDING REMARKS

This theoretical paper captures the dynamics of post-2000 housing price evolution in the United States by modeling the interaction between credit borrowers and lenders. Through securitization, banks and other financial intermediates were able to supply more credit to potential home buyers. In an infinite time horizon model, we model the financial intermediate's decision making of loans and liquidation as well as borrowers' voluntary default. We employ Bellman equation to describe economic agents' calculation of expected payoffs. We simulate the model and produce dynamics of housing supply, housing demand, housing price and mortgage rate over time. Results match the empirical evidence.

We find several things to improve in the future work. The first one is the mortgage rate adjustment. Here we assume a close correlation between CPI and the housing price. The historical data does not clearly show that correlation. During year 2005-2006, the Federal Reserve raised the interest rate not only because of the increasing housing price. At that time, energy price and food price were increasing even more and they are two main reasons for the inflation. Therefore, it seems more persuasive to set the interest rate adjustment as an external shock. Secondly, we haven't fit our model with housing supply, demand and price data, which makes our work not much convincing. We may calibrate our model in later work.

ENDNOTES

1. For example, Storesletten, Telmer, and Yaron (2007) and Gomes and Michaelides (2008) model the production side of the economy but focus on single-sector economies without real estate sector.
2. A further question is, why doesn't F hold houses by itself if it expects increasing housing price. Literature gives answers to this question. Possible reasons include maintenance cost; borrowers would have more utility by holding the house than banks, therefore banks can charge them more, etc.

3. We assume r_t is also the developer' financing cost as we have assumed a close correlation between mortgage rate and interest rate.
4. Readers can derive the equation and find it equivalent to $\beta_F \mathrm{E}[\mathrm{V}(H_{t+1}, L_{t+1}^{t'})|_{I_t^{t'}=0}]$

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