

# The Leverage Cycle in the Mortgage Market

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*The recent boom and bust in house prices coincided with a boom and bust in mortgage leverage. At the peak of the boom, regions with the greatest rise in house prices also had the greatest rise in leverage. During the bust, regions with the greatest drop in house prices also had the greatest drop in leverage. During the peak of the housing boom, most highly leveraged borrowers had second mortgages for the portion of the loan with LTV >80%. When house prices began to drop, most second mortgages disappeared.*

## INTRODUCTION AND LITERATURE REVIEW

Most economic models assume that all agents keep all of their promises in all states of the world. In the real world we have several institutions to incentivize agents to keep their promises (Geanakoplos, 1997). Two such institutions are credit scores and collateral. If a borrower defaults on a loan, his credit score will drop, limiting his ability to borrow in the future, and if the loan is collateralized, the lender may confiscate (and sell) the collateral. Therefore, when a lender makes a collateralized loan, he takes into account the value of the collateral in the future.

Suppose a consumer wishes to buy a house for  $\$V$ , obtains a loan for  $\$L$  and makes a  $\$D$  down payment. The leverage ratio is  $Lev \equiv \frac{V}{D}$ , the Loan to Value ratio is  $LTV \equiv \frac{L}{V} = \frac{Lev-1}{Lev}$ . This paper follows the convention in the mortgage literature by referring to LTV as "leverage", this is innocuous because there is a strictly positive 1-1 correspondence between LTV and leverage. Note from these definitions that leverage and loan size are different concepts. Leverage is loan size relative to the price of the asset being purchased. It is possible to have a small loan with high leverage, as well as a large loan with low leverage. Many economic models solve for absolute loan size but few solve for leverage.

The few economic models that have collateralized lending usually assume that leverage is exogenous and fixed. Geanakoplos (2009) argues that it varies over time and develops a competitive equilibrium model in which leverage is endogenous. Moreover, Geanakoplos (2009) shows that greater leverage leads to higher asset prices. Many other authors including Kiyotaki and Moore (1997), and Bernanke, Gertler and Gilchrist (1999), and Zevelev (2017) argue that leverage is important because it makes our economy more fragile by amplifying the impact of small shocks.

Recently there has been discussion about the role of leverage in explaining the housing boom and bust. Mian and Sufi (2010) show that counties which experienced the largest growth in leverage between 2002 -2006 also experienced a large boom and bust in house prices, while counties with the least growth in leverage did not have a bust in house prices. However, Glaeser, Gottlieb and Gyourko (GGG) (2012) argue that there is no convincing evidence that leverage can explain the bulk of the changes in house

prices but admit that definitive judgment on this mechanism cannot be made without better correcting for the endogeneity of borrowers' decisions to apply for mortgages. To better understand the role of leverage in the mortgage market this paper studies how leverage is determined endogenously.

In section 2 I describe how leverage varies over time in the mortgage market. Regions with big booms and busts in house prices (Arizona, California, Florida, Nevada) also had big booms and busts in mortgage leverage. Regions which did not experience a big bust in house prices (Arkansas, Colorado, Indiana, Nebraska) did not experience a big drop in leverage. I also find that during the housing boom most highly leveraged loans included a second "piggyback" mortgage for the portion of the loan with LTV > 80%, however second mortgages sharply dropped (in all regions) after 2007. I argue that one important factor for the drop in leverage during the bust was the decline in optimism among lenders. The decline in optimism may not have been the only reason for the drop in leverage. During the recent housing boom we saw many non-traditional low downpayment, low documentation sub-prime and Alt-A mortgages which quickly disappeared during the bust (Mayer et al 2008). However one can argue that the reason these non-traditional high leverage loans existed in the first place was because of optimistic house price expectations (HPE). This paper does not try to explain how HPE forms, it is only interested in how HPE affects leverage. A factor distinct from HPE that could also explain the decline in leverage is the expected macroeconomic conditions in a region. An unemployed borrower is much more sensitive to negative equity than a borrower who has no problem making his payments.

The model in sections 3 and 4 studies how leverage is determined endogenously in the market for real estate loans. This paper differs from the emerging literature on endogenous leverage by taking into account features that are specific to the real estate market such as Private Mortgage Insurance (PMI), conforming loan limits and recourse. Real estate loans are special because foreclosure is costly for the borrower because he may have children in the school district, or he may really like the house etc and for the lender because a borrower experiencing foreclosure may sabotage the property. In Geanakoplos (2009) leverage is driven by heterogeneity in beliefs about the future and the equilibrium consists of the maximum loan such that there is no default. In this model there is no heterogeneity of beliefs, leverage is driven by the lender's expectation of the future value of the collateral. The lender accepts the possibility of default in some bad states of the world if he is compensated well enough in the good states. Therefore this model has endogenous default in some bad states of the world. Another way this model differs from Geanakoplos (2009) is its emphasis on variables that make borrowers more sensitive to negative equity (e.g. expected macroeconomic conditions in a region). Our model also considers the case when borrowers seek second mortgages to avoid PMI or to meet conforming limits on their first mortgage. I describe the conditions under which lenders are willing to make these loans. Thus the model explains both the size of leverage and the composition of leverage (first vs second loans) over time.

The model predicts:

1. Lenders are willing to make more highly leveraged loans when they are more optimistic about the future value of the collateral.
2. Lenders are willing to make more highly leveraged loans to borrowers who they believe are less likely to default if underwater: primary home buyers (not investors), borrowers in recourse states (especially borrowers who have more assets to lose), borrowers with higher FICO scores, and borrowers with jobs that are not cyclically sensitive (e.g. tenured college professors).
3. Lenders make loans with less leverage if the market interest rate for that type of loan is greater. This is because a higher interest rate will make the loan more difficult for the borrower to repay.
4. Lenders making second loans are much more sensitive to a drop in HPE than lenders making first loans.

The paper is organized as follows. Section 2 describes how leverage varies over time and across space in the data and summarizes the important stylized facts about real estate leverage in the US. Section 3 presents a model that shows how leverage is determined endogenously in an environment with a single lender. Section 4 modifies the previous model to include a second lender. Section 5 summarizes the

predictions of the model. Section 6 discusses directions for future work, especially for testing the predictions of the model. Section 7 concludes.

## REAL ESTATE LEVERAGE IN THE DATA

This paper uses transaction level real estate loan data from DataQuick (DQ). The house price data comes from the FHFA house price indices. I consider six different measures of leverage where  $LTV \equiv \frac{Loan_1}{Price}$  and Combined LTV (CLTV) is defined  $CLTV \equiv \frac{Loan_1 + Loan_2}{Price}$  (CLTV measures a borrower's total leverage):

$LTV80_{i,t}$ : the fraction of all loans in region  $i$  at time  $t$  with  $LTV > 80\%$

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$LTV90_{i,t}$ : the fraction of all loans in region  $i$  at time  $t$  with  $LTV > 90\%$

$CLTV90_{i,t}$ : the fraction of all loans in region  $i$  at time  $t$  with  $CLTV > 90\%$

$LTV99_{i,t}$ : the fraction of all loans in region  $i$  at time  $t$  with  $LTV > 99\%$

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For example, in California in 2006 Q4 we have  $CLTV80 = 69\%$  and  $LTV80 = 10\%$ . This means that 69% of mortgages in California that quarter had  $CLTV > 80\%$  and 10% of mortgages had  $LTV > 80\%$ . Therefore  $\frac{69\% - 10\%}{69\%} = 85.5\%$  of mortgages with  $CLTV > 80\%$  included a second loan.

I start with  $LTV80$  because GSEs require borrowers with mortgages that have  $LTV > 80\%$  to purchase Private Mortgage Insurance (PMI) (DeFusco, 2012). However, there is a big loophole. If a borrower wants a loan with  $LTV > 80\%$ , it is usually cheaper to get two loans, a first loan with  $LTV = 80\%$  and a second “piggyback” loan for the remaining balance (Avery, Brevoort and Canner, 2008). Another reason a borrower may want a second loan is to keep the first-lien loan small enough to be a conforming loan purchased by Fannie Mae or Freddie Mac. GSE conforming requirements such as PMI and loan limits are intended to make the loans they purchase safer, however my model argues that second loans can eliminate these benefits because a borrower's decision to default depends on his total loan size, not just his first loan. This is especially true for loans that included a “silent” second mortgage which the first lender was not aware of. If a borrower with two mortgages defaults and the home is sold then the first lender is paid first, and anything remaining after the first lender is paid goes to the second lender. Therefore second mortgages are much riskier than first mortgages.

Table 1 below describes some basic facts about mortgage leverage across 30 different regions (29 states and DC) during the peak of the housing boom (2007 Q1) and the last quarter of DQ data (2009 Q3):

**TABLE 1**  
**MORTGAGE LTV DISTRIBUTION DURING THE BOOM AND BUST**

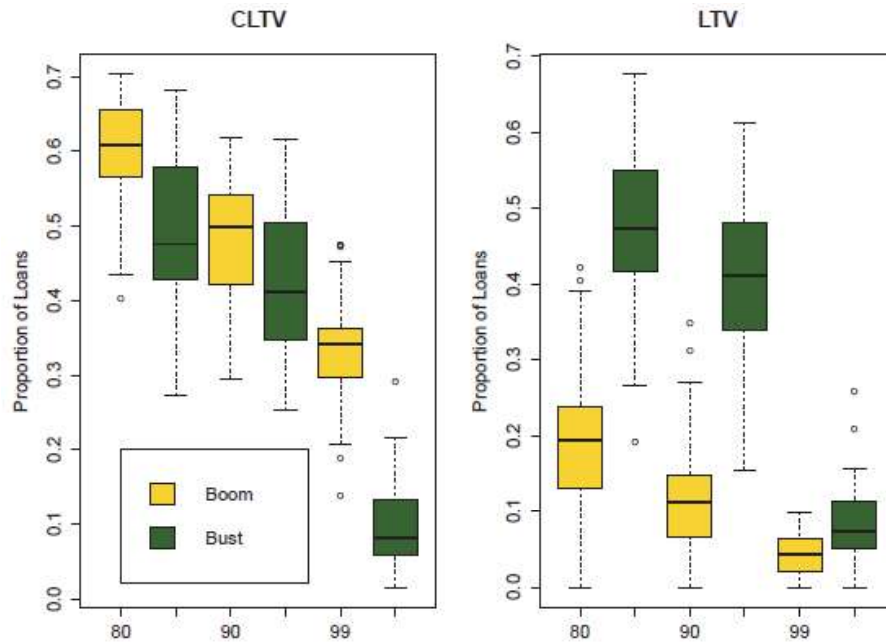
		<i>CLTV</i> 80		<i>LTV</i> 80	
		Peak	Q3 – 2009	Trough	Q3 – 2009
Max		0.7032	0.6825	0.4220	0.6763
Median		0.6094	0.4763	0.1935	0.4740
Min		0.4030	0.2721	0.0000	0.1906
		<i>CLTV</i> 90		<i>LTV</i> 90	
		Peak	Q3 – 2009	Trough	Q3 – 2009
Max		0.6184	0.6169	0.34830	0.6133
Median		0.4991	0.4126	0.11300	0.4109
Min		0.2950	0.2523	0.00000	0.1540
		<i>CLTV</i> 99		<i>LTV</i> 99	
		Peak	Q3 – 2009	Trough	Q3 – 2009
Max		0.4749	0.29140	0.09835	0.25720
Median		0.3409	0.08207	0.04373	0.07321
Min		0.1389	0.01567	0.00000	0.00000

During the peak of the housing boom the median leveraged region in my sample had 60.94% of mortgages with CLTV > 80%, the most leveraged region had 70.32% of such loans, the least leveraged region only had 40.3% of these loans. In 2009 Q3, only 47.6% of loans in the median leveraged region had CLTV > 80%, while the most highly leveraged region had 68% of these loans and the least leveraged region only had 27% of such loans. We see a similar pattern for CLTV 90 and CLTV 99: higher leverage during the boom, a drop in leverage during the bust.

When we consider leverage obtained exclusively through first mortgages (LTV) we see a different pattern. The peak in real estate prices coincided with a trough in LTV and the bust in real estate prices coincided with an increase in LTV. For example during the trough, the median region only had 19% of loans with LTV > 80%, but in 2009 Q3, 47.4% of loans in the median region had LTV > 80%. Basically the right column of Table 1 shows there was a sharp drop in second mortgage lending once house prices began to decline.

The box plots in Figure 1 illustrate how the distribution of the 6 measures of leverage evolved from boom to bust. In the left panel we have the distribution of total leverage (CLTV) and in the right panel leverage on first loans (LTV). Leverage during the boom is in gold and during the bust is in dark green. These box plots show that the drop in second mortgages (i.e. corresponding increase in LTV in the right panel) was much sharper than the drop in total leverage (left panel).

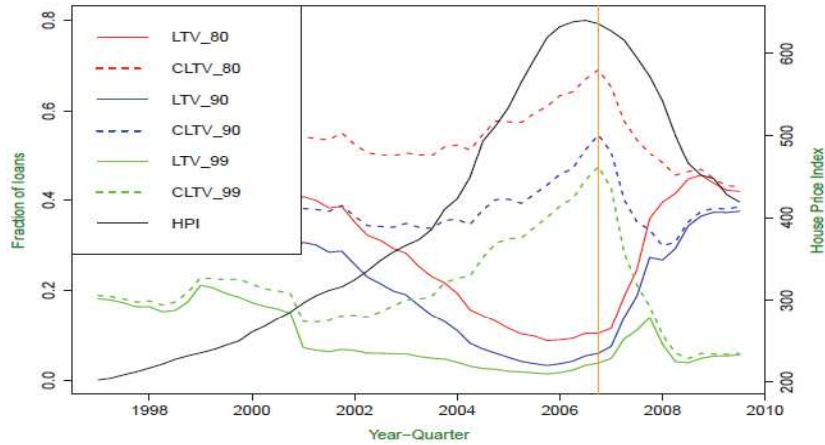
**FIGURE 1**  
**THE DISTRIBUTION OF TOTAL (CLTV) AND FIRST LIEN (LTV) LEVERAGE**



Part of the drop in second mortgages can be explained by a law passed by congress in 2006 that made PMI tax deductible from 2007-2009 (Lewis, 2007). This tax deduction made it more attractive for some borrowers to obtain a large first mortgage and pay PMI rather than obtain a piggyback loan. However this tax deduction does not help high income borrowers because the PMI deduction is reduced by 10 percent for each \$1,000 a filer's income is over the income limit of \$100,000. In addition, borrowers who want mortgages greater than the conforming limit would still want piggyback loans.

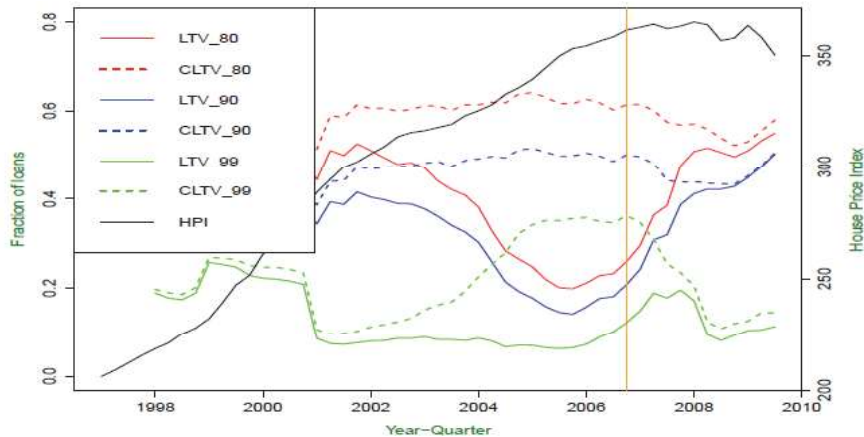
Next we consider leverage in a region with a big boom and bust in house prices. In Figure 2 we have California where the black line denotes the house price index which peaked at 639.75 in 2007 and fell 34% to 419.25 in quarter 3 of 2009. The dashed red line denotes the fraction of all mortgages in California with CLTV > 80%. This peaked at 69% in quarter 3 2006 and fell to 43% in quarter 3 2009. The solid red line denotes the fraction of all mortgages in California with LTV > 80% (this only includes first loans!). Note that the LTV curve moves in the opposite direction compared to the CLTV curve. Slightly before house prices peaked LTV80 reached a low of 8.7% in quarter 3 2005 and then sharply increased as house prices began to fall. In the last quarter of 2008 second mortgages virtually disappear. We see similar patterns in the blue curves (mortgages with LTV > 90%) and the green curves (LTV > 99%). The vertical orange line is at 2006 Q4, the last quarter before PMI became tax deductible.

**FIGURE 2**  
**CLTV AND LTV IN CALIFORNIA**



Next we consider leverage in a region that did not experience a bust in house prices. In Figure 3 we have Colorado where the solid black line denotes the house price index which peaked at 360 in 2006 and has remained flat since then. The dashed red line, the fraction of loans with  $CLTV > 80\%$ , is flat at approximately 60%. However the solid red line, the fraction of loans with  $LTV > 80\%$ , is U-shaped moving inversely relative to the national house prices. Even though house prices never dropped in Colorado we still saw a drop in second mortgages. We see a similar pattern in the blue curve ( $LTV > 90\%$ ). However we still saw a drop in the most highly levered mortgages depicted by the green curve ( $LTV > 99\%$ ). Although we did not see a drop in total leverage as measured by  $CLTV_{80}$  and  $CLTV_{90}$ , we still saw a big change in the composition of leverage. After 2006 second mortgages began to decline, and after 2007 no cash mortgages ( $CLTV_{99}$ ) almost disappeared.

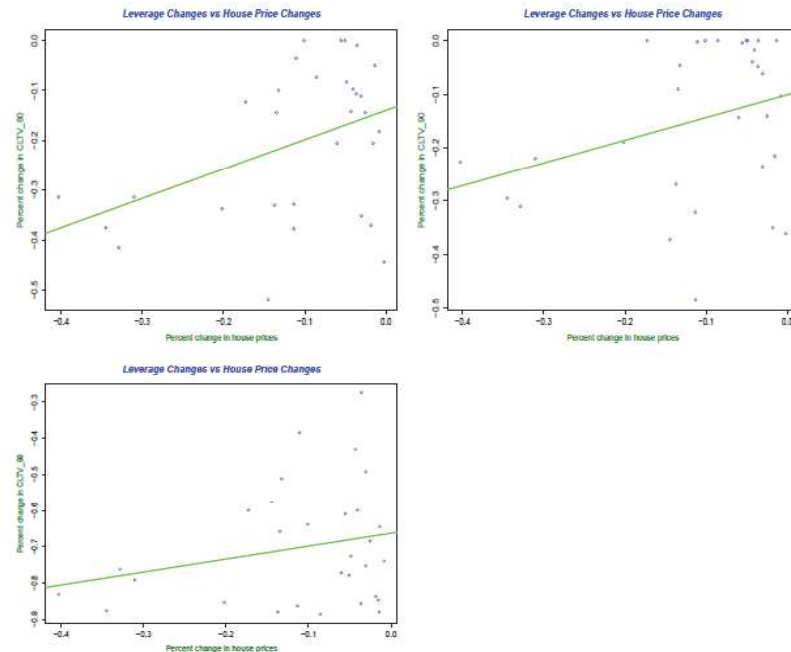
**FIGURE 3**  
**CLTV AND LTV IN COLORADO**



It is also interesting to point out that the leverage cycle in the Real Estate market is different from the leverage cycle in other asset markets. Figure 1 in Geanakoplos (2009) plots the average margin offered for all securities purchased by the hedge fund Ellington Capital Management. For example, during the Russian financial crisis in Q3 1998 there was a big drop in leverage for financial securities but there was no change in leverage in the Real Estate market. This figure also shows that leverage for financial securities is generally much more volatile than Real Estate leverage.

Next we consider how leverage was related to house price growth across 30 different regions. Figure 4 plots the percent change in CLTV (from peak until quarter 3 2009) against the percent change in house prices over the same period along with a line estimated by OLS. This is done for CLTV80, CLTV90 and CLTV99:

**FIGURE 4**  
**THE RELATIONSHIP BETWEEN LEVERAGE GROWTH AND HOUSE PRICE GROWTH**



The regression estimates are summarized in Table 2 where  $\% \Delta hp$  denotes the percent change in house prices. In all three plots we see a positive relationship between percent change in house prices and percent change in leverage i.e. regions that saw a larger drop in house prices also saw a larger drop in highly leveraged loans. For example, the first column in Table 2 shows that a 1% drop in house prices is associated with a 0.59% drop in the proportion of loans with LTV > 80% in a region.

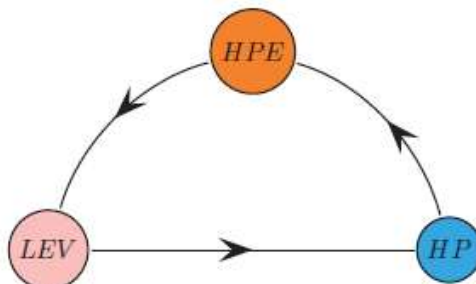
**TABLE 2**  
**LEVERAGE GROWTH AND HOUSE PRICE GROWTH**

Table 2			
	$\% \Delta CLTV_{80}$	$\% \Delta CLTV_{90}$	$\% \Delta CLTV_{99}$
(Intercept)	-0.14*** (0.04)	-0.10*** (0.04)	-0.66*** (0.04)
$\% \Delta hp$	0.59** (0.24)	0.43* (0.24)	0.36 (0.28)
R <sup>2</sup>	0.17	0.10	0.06
Adj. R <sup>2</sup>	0.15	0.07	0.02
Num. obs.	31	31	30

\*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$

It is important to point out that we cannot interpret the coefficients in Table 2 as causal because there is a nasty simultaneity between  $\% \Delta CLTV$  and  $\% \Delta hp$ . The issues are summarized in the following diagram:

**FIGURE 5**  
**ENDOGENEITY BETWEEN HOUSE PRICE EXPECTATIONS LEVERAGE AND HOUSE PRICES**



Observe that the supply of loans by lenders depends on house price expectations (HPE). The model will show that higher HPE today should lead to higher leverage. However, Geanakoplos (2009) shows that greater leverage today should lead to higher house prices (HP). Similarly, Zevelev (2018) shows that even an expansion of after-purchase mortgage leverage raises house prices. Moreover, Ghysels et al. (2012) show that there is short-run predictability in house prices therefore an increase in house prices today should also increase house price expectations. A careful test of the predictions of the model requires a variable that affects house price expectations and does not affect leverage. One idea from Mian and Sufi (2011) is to use housing supply elasticity at the MSA level as an instrument for house prices. This is discussed in the future work section.

Another big challenge for understanding leverage over time is separating supply from demand. The drop in highly leveraged loans may have come from nervous lenders who wanted borrowers to have more skin in the game, or from borrowers demanding fewer high leverage loans. To analyze the role of demand I am tempted to use the HMDA data which has all mortgage applications in the US. However I cannot compute leverage from the HMDA data because it only contains loan size but not house price. If I use the HMDA data that has been merged with the DQ data I can only compute leverage for loans that have been approved. Therefore I am unable to observe leverage for loans that have been rejected. Moreover it is not possible to observe potential borrowers who did not bother to apply for mortgages during the bust because they expected they would get rejected. There is evidence that the composition of borrowers changed over the housing boom and bust. Mayer, Pence and Sherlund (2009) report that the median CLTV for subprime purchase loans rose from 90% in 2003 to 100% in 2005. Much of this subprime lending disappeared after the housing bust.

Below is a summary of stylized facts about leverage in the housing market:

1. The proportion of no cash loans (CLTV > 99%) began growing in 2003, almost doubled when they peaked in 2007, and quickly fell to 2003 levels in 2008-2009.
2. Regions that experienced bigger drops in house prices saw bigger drops in leverage.
3. CLTV and LTV moved in opposite directions. During the housing boom CLTV peaked and LTV reached a trough (presumably as borrowers sought second loans to avoid PMI and conforming limits). After the housing boom second loans began to disappear.

### MODEL WITH A SINGLE LENDER

This section constructs a model that will explain why:

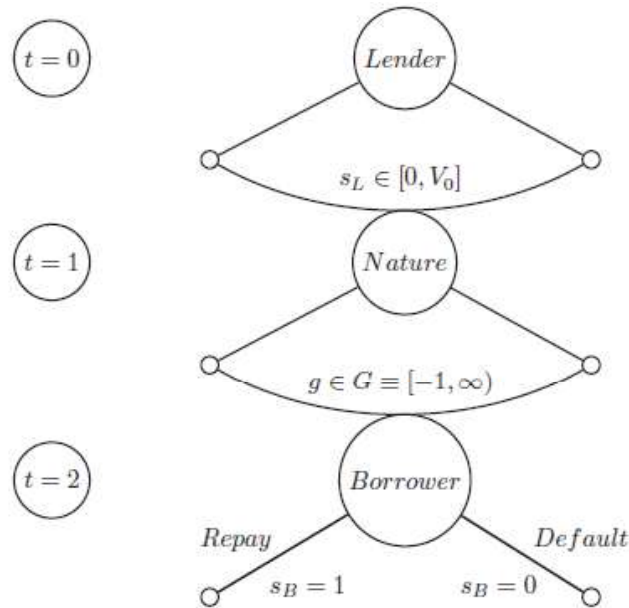
1. Lenders make loans with higher leverage when they are more optimistic about the future value of the collateral.



2. Lenders making second mortgages are more sensitive to a drop in house price expectations than first mortgages.

The game tree in Figure 6 describes the dynamics of this game. In period  $t = 0$  a borrower comes to a bank looking for a loan to buy a home that costs  $V_0$ . The lender chooses the size of the loan to give the borrower  $s_L \in S_L \equiv [0, V_0]$ . If  $s_L = 0$  the lender doesn't lend the borrower any money, if  $s_L = V_0$  the lender gives the borrower a 100% loan. (In reality there were some loans with LTV > 100% during the housing boom because some loans included closing fees etc.) For now assume that the borrower will accept any offer the lender gives him. Later sections consider the case in which the borrower can afford to make a downpayment of at most  $\lambda$  and will purchase the biggest home he can with this downpayment.

**FIGURE 6  
GAME TREE**



After the loan is originated, Nature rolls the dice at  $t = 1$  and chooses the house price growth rate  $g \in G \equiv [-1, \infty)$  so that the price of the house becomes  $V_1 = (1 + g)V_0$ . For example, the case  $g = 0$  corresponds to no house price growth,  $g = -0.5$  corresponds to a 50% drop in house prices etc. The CDF of  $g$  is denoted  $F_g$ . The model could be further generalized to include income shocks by letting Nature choose income  $y$  along with the house price growth rate  $g$  according to the joint CDF  $F_{g,y}$ .

At  $t = 2$ , the borrower observes  $s_L$  and  $g$  and decides either to repay or to default  $s_B \in S_B \equiv \{f: S_L \times G \rightarrow \{Repay, Default\}\}$ . If the borrower repays  $s_B(s_L, g) = 1$ , else  $s_B(s_L, g) = 0$ . We will see that negative equity is a necessary but not sufficient for default in this model. This is consistent with empirical evidence by Foote, Gerardi and Willen (2008) who use Massachusetts data.

In this model a loan is a contract in which the borrower receives  $s_L$  at  $t = 0$  and if he repays  $s_L(1 + r) = s_L R$  at  $t = 2$  he can keep the house, if he defaults on the loan he gives up the house and loses his equity. The lender receives  $s_L(1 + r)$  if the borrower repays and  $V_0(1 + g)$  if the borrower defaults.

House price growth is assumed to be an exogenous random variable determined by Nature. The market interest rate is also exogenous. However leverage is endogenous. It is important to note that solving for leverage is different from solving for absolute loan size because leverage is about *relative* loan size.

The normal form of this game is  $G \equiv \{S_i, u^i\}_{i \in I}$  where the players are  $I \equiv \{B, L, N\}$ . B denotes Borrower, L denotes Lender, and N denotes Nature. The objective functions  $u^i$  are described below

corresponding to the various cases considered. First, the simplest possible model is solved and layers are added gradually.

### Deterministic Price Costless Default

For now let  $g$  be deterministic, i.e. known at  $t = 0$ . The objective function of the lender is:

$$u^L(s_B, s_L, g) = \begin{cases} \text{repayment} - \text{loan} = s_L R - s_L = s_L r, & \text{if repaid } s_B = 1 \\ \text{collateral} - \text{loan} = V_1 - s_L = V_0(1 + g) - s_L, & \text{if defaulted } s_B = 0 \end{cases} \quad (1)$$

This can be rewritten  $u^L(s_B, s_L, g) = s_B \times (s_L r) + (1 - s_B) \times (V_0(1 + g) - s_L)$ .

The objective function of the borrower is:

$$u^B(s_B, s_L, g) = \begin{cases} \text{collateral} - \text{repayment} - \text{downpayment} = V_1 - s_L R - (V_0 - s_L) = V_0 g - s_L r, & \text{if repaid } s_B = 1 \\ -\text{downpayment} = -(V_0 - s_L), & \text{if defaulted } s_B = 0 \end{cases} \quad (2)$$

This can be rewritten  $u^B(s_B, s_L, g) = s_B \times (V_0 g - s_L r) + (1 - s_B) \times (-(V_0 - s_L))$ .

Using backwards induction at ( $t = 2$ ) the borrower observes  $s_L$  and decides to repay if:

$$u^B(s_B = 1, s_L, g) \geq u^B(s_B = 0, s_L, g) \Leftrightarrow s_L \leq V_0 \frac{(1+g)}{(1+r)}. \quad (3)$$

The borrower's strategy is  $s_B(s_L, g) = 1 \left\{ s_L \leq V_0 \frac{(1+g)}{(1+r)} \right\}$ . In this setting the borrower will repay as long as the house price growth is greater than or equal to the LTV times the interest rate i.e.  $(1 + g) \geq \left( \frac{s_L}{V_0} \right) \times (1 + r)$ . If  $LTV = 100\%$ , the borrower will repay whenever house price growth is at least as high as the interest rate.

At ( $t = 0$ ) the lender knows that  $s_B(s_L, g)$  will be the borrower's strategy and solves:

$$\max_{s_L \in [0, V_0]} u^L(s_B(s_L), s_L, g) = 1 \left\{ s_L \leq V_0 \frac{(1+g)}{(1+r)} \right\} \times (s_L r) + 1 \left\{ s_L > V_0 \frac{(1+g)}{(1+r)} \right\} \times (V_0(1 + g) - s_L) \quad (4)$$

Since this objective function is piece-wise continuous the problem is solved over the two regions where it is continuous.

For  $g < r$ , region 1 is  $R_1 \equiv \left[ 0, \frac{V_0(1+g)}{(1+r)} \right]$ , and  $R_2 \equiv \left( \frac{V_0(1+g)}{(1+r)}, V_0 \right]$ . For  $s_L \in R_1$  the borrower will repay and  $s_L^* = \frac{V_0(1+g)}{(1+r)}$ . For  $s_L \in R_2$  there is no maximum as the borrower wants to choose the smallest loan possible ( $\arg \sup = \frac{V_0(1+g)}{(1+r)}$ ).

For  $g \geq r$ , the borrower will always repay because  $\frac{V_0(1+g)}{(1+r)} \geq V_0$  and the lender's loan  $s_L \leq V_0$ . Thus the lender chooses the biggest loan possible  $s_L^* = V_0$ .

The Subgame Perfect Nash Equilibrium (SPNE) is summarized below:

Case 1 ( $g < r$ ): the SPNE strategies are  $s_L^* = \frac{V_0(1+g)}{(1+r)}$  and  $s_B^*(s_L, g) = 1 \left\{ s_L \leq V_0 \frac{(1+g)}{(1+r)} \right\}$ .

The equilibrium path is  $s_L^* = \frac{V_0(1+g)}{(1+r)}$  and  $s_B^*(s_L^*, g) = 1$ .

The equilibrium payoffs are  $u^L = \frac{V_0(1+g)}{(1+r)} r$  and  $u^B = \frac{V_0(g-r)}{(1+r)}$ .

The equilibrium leverage is  $LTV^* \equiv \frac{s_L^*}{V_0} = \frac{(1+g)}{(1+r)}$ .

Case 2 ( $g \geq r$ ): the SPNE strategies are  $s_L^* = V_0$  and  $s_B^*(s_L, g) = 1 \left\{ s_L \leq V_0 \frac{(1+g)}{(1+r)} \right\}$ .

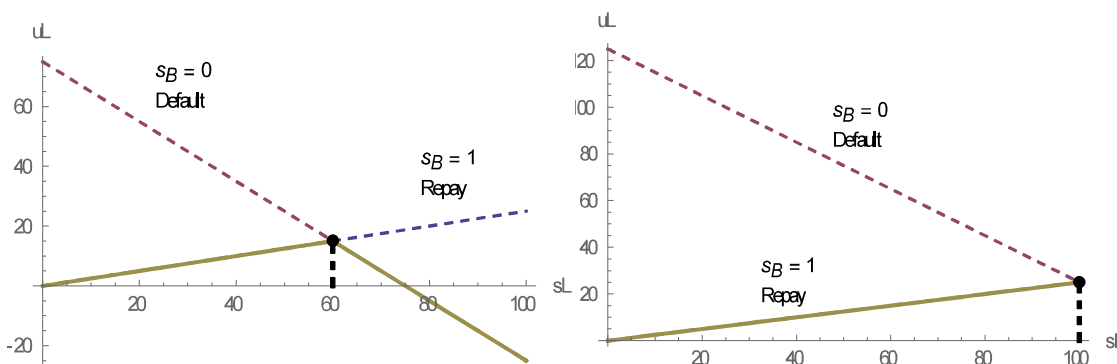
The equilibrium path is  $s_L^* = V_0$  and  $s_B^*(s_L^*, g) = 1$ .

The equilibrium payoffs are  $u^L = V_0 r$  and  $u^B = V_0(g - r)$ .

The equilibrium leverage is  $LTV^* \equiv \frac{s_L^*}{V_0} = 1$ .

The intuition behind the SPNE is that the lender chooses the biggest loan without default, and the borrower repays. Figure 7 below plots the lender's objective function for two different house price growth scenarios where  $V_0 = 100$  and  $r = 0.25$ .

**FIGURE 7  
LENDER'S OBJECTIVE FUNCTION**



In the left plot,  $g = -0.25$ , so  $g < r$ , and  $s_L^* = \frac{V_0(1+g)}{(1+r)} = 60$ . In the right plot,  $g=0.25$ , so the house price grows at the same rate as the interest rate  $g = r$ , and  $s_L^* = V_0 = 100$ . The plot will look exactly the same as the third plot for any  $g \geq r$  because the lender will never lend the borrower more than the value of the house  $V_0$ .

Combining the two cases, the equilibrium leverage is  $LTV^* = \min \left\{ 1, \frac{1+g}{1+r} \right\}$ , therefore the comparative statics are:

1.  $\frac{\partial LTV^*}{\partial g} \geq 0$ , lenders make higher LTV loans if they expect higher house price growth
2.  $\frac{\partial LTV^*}{\partial r} \leq 0$ , a higher interest rate reduces the probability of repayment, hence reduces LTV.

In this model the only thing the borrower cares about is the capital gain he receives from selling the house. This ignores the dividend stream many borrowers receive from home-ownership. We relax this assumption in section 3.3.

### Deterministic Price Costless Default and Liquidity

The model above assumed the borrower will accept a loan of any size from the lender. In reality most borrowers are constrained and have a limited amount of cash they can put up as a down payment on a house. I now assume that when a borrower applies for a loan he can afford a down-payment of at most  $\lambda > 0$ . He is considering houses priced  $V \in [V_0, \bar{V}_0]$  and will buy the most expensive house he can afford. This borrower needs a loan of at least  $s_L \geq V_0 - \lambda$ .

Case 1 ( $g < r$ ): above it was shown that  $s_L^* = \frac{V_0(1+g)}{(1+r)}$ . The borrower can afford the house if:

$V_0 - s_L^* \leq \lambda \Leftrightarrow V_0 \leq \lambda \frac{(1+r)}{(r-g)}$ . Given down-payment  $\lambda$  the most expensive home the borrower can afford is  $\lambda \frac{(1+r)}{(r-g)}$ . Assume the borrower will buy the biggest house he can afford  $V_0^* = \lambda \frac{(1+r)}{(r-g)}$ . Thus  $s_L^* = \frac{V_0^*(1+g)}{(1+r)} = \lambda \frac{(1+g)}{(r-g)}$ .

Case 2 ( $g \geq r$ ): above it was shown that  $s_L^* = V_0$ . In this case the lender is optimistic enough about the value of the collateral that the borrower can get any loan desired. He doesn't even have to pay  $\lambda$ . The borrower will buy a house that costs  $V_0^* = \overline{V_0}$ .

Note that the equilibrium leverage is exactly the same in this liquidity constrained model. The only difference liquidity makes is the size of the house the borrower buys.

### Deterministic Price Costly Default

In the model in section 3.1 default was costless to both the borrower and lender, consequently a borrower would always default if he had negative equity. Recent work by Bhutta, Dokko, and Shan (2010) shows that many borrowers with negative equity choose not to default and the median borrower does not strategically default until equity falls to -62% of their home's value.

This model is essentially the same as the model in section 3.1 except the cost to the borrower of defaulting is  $B_j \geq 0$ , and the cost to the lender is  $L_k \geq 0$ .  $B_j$  captures the cost of damage to the borrower's credit score, stigma, and possibly moving out of the current school district etc. In states with recourse  $B_j$  should be higher according to Ghent and Kudlyak (2011). There is also evidence from Gao and Li (2012) that out of town second house buyers (distant speculators) are more willing to default once they have negative equity compared to buyers of primary residences. This makes sense because an investor who defaults won't lose the roof over his head. Thus investors should have lower  $B_j$ . We can think of  $B_j$  as a measure of the borrower's tolerance for negative equity. In principle  $B_j$  should also depend on a borrower's employment status. There are some borrowers who may have high tolerance for negative equity as long as they can afford the monthly payments. Corbae and Quintin (2011) call a default when borrowers can afford to repay a "voluntary default".  $L_k$  captures possible damage to the house from a foreclosure (e.g. sabotage from a disgruntled borrower or debt overhang).

For now assume  $B_j$  is common knowledge between the borrower and lender. However in reality  $B_j$  is privately known by the borrower and not by the lender. The borrower's FICO credit score can be seen as a (noisy) signal of his  $B_j$ . Some borrowers who are investors may pretend to be primary home buyers so they appear to have a higher  $B_j$  and thus receive a loan on more favorable terms. This model can be generalized further to include a collection of J borrowers each with a different  $B_j$  which is privately known. From the perspective of the lender this asymmetric information can lead to adverse selection and result in credit rationing.

With costly default the objective functions are:

$$u^L(s_B, s_L, g) = s_B \times (s_L r) + (1 - s_B) \times (V_0(1 + g) - s_L - L_k). \quad (5)$$

$$u^B(s_B, s_L, g) = s_B \times (V_0 g - s_L r) + (1 - s_B) \times (-(V_0 - s_L) - B_j) \quad (6)$$

At ( $t = 2$ ) the borrower observes  $s_L$  (and  $g$ ) and decides to repay if:

$$u^B(s_B = 1, s_L, g) \geq u^B(s_B = 0, s_L, g) \Leftrightarrow s_L \leq \frac{V_0(1+g)+B_j}{(1+r)}. \quad (7)$$

The borrower's strategy (best-response) is  $s_B(s_L, g) = 1 \left\{ s_L \leq \frac{V_0(1+g)+B_j}{(1+r)} \right\}$ . Since  $s_L \leq V_0$ , the borrower will repay loans of any size if  $\frac{V_0(1+g)+B_j}{(1+r)} \geq V_0 \Leftrightarrow r \leq g + \frac{B_j}{V_0}$ .

At ( $t = 0$ ) the lender knows that this will be the borrower's strategy and solves:

$$\max_{s_L \in [0, V_0]} u^L(s_B(s_L), s_L, g) = \mathbb{1}\left\{s_L \leq \frac{V_0(1+g)+B_j}{(1+r)}\right\} \times (s_L r) + \mathbb{1}\left\{s_L > \frac{V_0(1+g)+B_j}{(1+r)}\right\} \times (V_0(1+g) - s_L - L_k) \quad (8)$$

The SPNE strategies are  $s_L^* = \min\left\{V_0, \frac{V_0(1+g)+B_j}{(1+r)}\right\}$  and  $s_B^*(s_L, g) = \mathbb{1}\left\{s_L \leq \frac{V_0(1+g)+B_j}{(1+r)}\right\}$ . (More precisely if  $r > g + \frac{B_j}{V_0}$  then  $s_L^* = \frac{V_0(1+g)+B_j}{(1+r)}$ , else  $s_L^* = V_0$ .) The equilibrium path is  $s_L^* = \min\left\{V_0, \frac{V_0(1+g)+B_j}{(1+r)}\right\}$  and  $s_B^*(s_L, g) = 1$ . The equilibrium payoffs are  $u^L = \min\left\{V_0, \frac{V_0(1+g)+B_j}{(1+r)}\right\} \times r$  and  $u^B = V_0 g - \min\left\{V_0, \frac{V_0(1+g)+B_j}{(1+r)}\right\} \times r$ . The equilibrium leverage is  $LTV^* \equiv \frac{s_L^*}{V_0} = \min\left\{1, \frac{(1+g)}{(1+r)} + \frac{B_j}{(1+r)V_0}\right\}$ .

In the case with costless default, we had 100% LTV loans only if  $g \geq r$ . With costly default  $B_j > 0$  we can have 100% LTV loans if  $g \geq r - \frac{B_j}{V_0}$ . The intuition is that when default is more costly, borrowers are less likely to default so lenders are willing to lend more.

The equilibrium leverage is  $LTV^* = \min\left\{1, \frac{(1+g)}{(1+r)} + \frac{B_j}{(1+r)V_0}\right\}$  and the comparative statics are:

1.  $\frac{\partial LTV^*}{\partial g} \geq 0$ , lenders make higher LTV loans if they expect higher house price growth
2.  $\frac{\partial LTV^*}{\partial r} \leq 0$ , a higher interest rate reduces the probability of repayment, hence reduces LTV.
3.  $\frac{\partial LTV^*}{\partial B_j} \geq 0$ , the more costly default is for the borrower, the greater the leverage.

NOTE: this model assumes the cost of default is fixed  $B_j$  and unrelated to the loan size or house price. This assumption implies the additional comparative static  $\frac{\partial LTV^*}{\partial V_0} \leq 0$ . If the default cost is assumed to be proportional to the house price (or loan size)  $B_j V_0$ , the decision rule would be  $s_B(s_L, g) = \mathbb{1}\left\{s_L \leq \frac{V_0(1+g+B_j)}{(1+r)}\right\}$  and leverage would not depend on  $V_0$ . Alternatively if default had both a fixed cost and variable cost component, this model predicts that lenders would allow less leverage on bigger loans. A model with fixed cost of default predicts that a borrower seeking to buy a home that costs \$200k could get a higher LTV loan than an otherwise identical borrower seeking to buy a home that costs \$900k.

Observe that if default is costless for the borrower ( $B_j = 0$ ) then we have the exact same leverage as in the model with costless default in section 3.1. Also note that the lender's cost of default  $L_k$  does not affect leverage in this model. This is because house price growth is deterministic so the lender gives the largest loan such that the borrower does not default. We will see that once we have stochastic house price growth the borrower will default in some bad states of the world and the leverage will depend on  $L_k$ .

### Stochastic Prices Costly Default

Nature chooses the house price growth  $g = g_1$  with probability  $p_1 \in [0, 1]$ , and  $g = g_2$  with probability  $p_2 = 1 - p_1$ , such that  $g_1 < g_2$ . The corresponding house price will be

$$V_1 = \begin{cases} V_0(1 + g_1), & \text{with probability } p_1 \\ V_0(1 + g_2), & \text{with probability } p_2 = 1 - p_1 \end{cases} \quad (9)$$

After Nature chooses  $g$  the ex-post utilities are:

$$u^L(s_B, s_L, g) = s_B \times (s_L r) + (1 - s_B) \times (V_0(1 + g) - s_L - L_k). \quad (10)$$

$$u^B(s_B, s_L, g) = s_B \times (V_0 g - s_L r) + (1 - s_B) \times (-(V_0 - s_L) - B_j) \quad (11)$$

At (t = 2) the borrower observes  $s_L$  (and  $g$ ) and decides to repay if:

$$u^B(s_B = 1, s_L, g) \geq u^B(s_B = 0, s_L, g) \Leftrightarrow s_L \leq \frac{V_0(1+g)+B_j}{(1+r)}.$$

The borrower's strategy (best-response) is  $s_B(s_L, g) = 1 \left\{ s_L \leq \frac{V_0(1+g)+B_j}{(1+r)} \right\}$ .

At (t = 0) the lender knows that this will be the borrower's strategy and solves:

$$u^L(s_B(s_L), s_L, g) = \max_{s_L \in [0, V_0]} p_1 \left( 1 \left\{ s_L \leq \frac{V_0(1+g_1)+B_j}{(1+r)} \right\} \times (s_L r) + 1 \left\{ s_L > \frac{V_0(1+g_1)+B_j}{(1+r)} \right\} \times (V_0(1+g_1) - s_L - L_k) \right) + p_2 \left( 1 \left\{ s_L \leq \frac{V_0(1+g_2)+B_j}{(1+r)} \right\} \times (s_L r) + 1 \left\{ s_L > \frac{V_0(1+g_2)+B_j}{(1+r)} \right\} \times (V_0(1+g_2) - s_L - L_k) \right) \quad (12)$$

This section assumes  $V_0 \geq \frac{V_0(1+g_2)+B_j}{(1+r)}$  so the biggest possible loan is  $s_L = V_0$ . The equilibrium is solved for in the online appendix and summarized below:

Case 1  $\left( \frac{p_2 - \frac{1}{1+r}}{1-p_2} < \frac{L_k+B_j}{V_0(g_2-g_1)} \right)$  “low-leverage equilibrium”

The SPNE strategies are  $s_L^* = \frac{V_0(1+g_1)+B_j}{(1+r)}$  and  $s_B^*(s_L, g) = 1 \left\{ s_L \leq \frac{V_0(1+g)+B_j}{(1+r)} \right\}$ .

The equilibrium path is  $s_L^* = \frac{V_0(1+g_1)+B_j}{(1+r)}$  and  $s_B^*(s_L, g) = 1$ .

The equilibrium leverage is  $LTV^* \equiv \frac{s_L^*}{V_0} = \frac{(1+g_1)}{(1+r)} + \frac{B_j}{(1+r)V_0}$ .

Case 2  $\left( \frac{p_2 - \frac{1}{1+r}}{1-p_2} = \frac{L_k+B_j}{V_0(g_2-g_1)} \right)$  “multiple equilibria”

The SPNE strategies are  $s_L^* = \left\{ \frac{V_0(1+g_1)+B_j}{(1+r)}, \frac{V_0(1+g_2)+B_j}{(1+r)} \right\}$  and  $s_B^*(s_L, g) = 1 \left\{ s_L \leq \frac{V_0(1+g)+B_j}{(1+r)} \right\}$ .

The low LTV equilibrium path is  $s_L^* = \frac{V_0(1+g_1)+B_j}{(1+r)}$  and  $s_B^*(s_L, g) = 1$ .

The high LTV path is  $s_L^* = \frac{V_0(1+g_2)+B_j}{(1+r)}$  and  $s_B^*(s_L, g_1) = 0$  and  $s_B^*(s_L, g_2) = 1$ .

The equilibrium leverage is  $LTV^* \equiv \frac{s_L^*}{V_0} \in \left\{ \frac{(1+g_1)}{(1+r)} + \frac{B_j}{(1+r)V_0}, \frac{(1+g_2)}{(1+r)} + \frac{B_j}{(1+r)V_0} \right\}$ .

Case 3  $\left( \frac{p_2 - \frac{1}{1+r}}{1-p_2} > \frac{L_k+B_j}{V_0(g_2-g_1)} \right)$  “high-leverage equilibrium”

The SPNE strategies are  $s_L^* = \frac{V_0(1+g_2)+B_j}{(1+r)}$  and  $s_B^*(s_L, g) = 1 \left\{ s_L \leq \frac{V_0(1+g)+B_j}{(1+r)} \right\}$ .

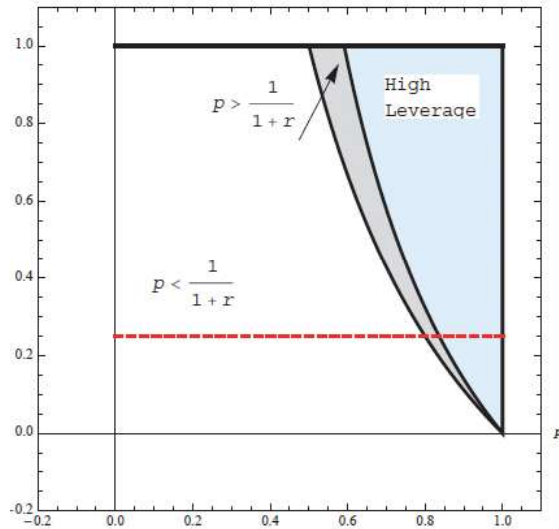
The equilibrium path is  $s_L^* = \frac{V_0(1+g_2)+B_j}{(1+r)}$  and  $s_B^*(s_L, g_1) = 0$  and  $s_B^*(s_L, g_2) = 1$ .

The equilibrium leverage is  $LTV^* \equiv \frac{s_L^*}{V_0} = \frac{(1+g_2)}{(1+r)} + \frac{B_j}{(1+r)V_0}$ .

Note that the set of parameters such that case 2 holds has Lebesgue measure zero, therefore generically we will not see multiple equilibria. We see a high LTV equilibrium when the benefits of leverage sufficiently outweigh the costs. The only time there is default along the equilibrium path is when a high LTV loan is taken and a bad economic shock  $g = g_1$  is realized.

The model is analyzed numerically for the case:  $V_0 = 100, g_1 = -.25, g_2 = .2, B_j = 5, L_k = 0, r = .25$ . Figure 8 illustrates how different combinations of the free parameters  $p_2$  &  $r$  correspond to different equilibria. In times when  $p_2$  &  $r$  are in the blue region, the lender will choose the high LTV loan. When  $p_2$  &  $r$  are on the black line at the intersection of the grey region and blue region, the lender will be indifferent between the high and low leverage loans, so we have two equilibria. In times when  $p_2$  &  $r$  are outside the blue region, the lender will choose the low LTV loan.

**FIGURE 8**



The equilibrium leverage is  $LTV^* = \frac{(1+g_1)1\{A\}+g_21\{A^c\}}{(1+r)} + \frac{B_j}{(1+r)V_0}$  where the event  $A \equiv \left\{ parameters: \frac{p_2 - \frac{1}{1+r}}{1-p_2} \leq \frac{L_k+B_j}{V_0(g_2-g_1)} \right\}$ . The comparative statics are:

1.  $\frac{\partial LTV^*}{\partial g_i} \geq 0$ , lenders make higher LTV loans if they expect higher house price growth
2.  $\frac{\partial LTV^*}{\partial r} \leq 0$ , a higher interest rate reduces the probability of repayment, hence reduces LTV.
3.  $\frac{\partial LTV^*}{\partial B_j} \geq 0$ , the more costly default is for the borrower, the greater the leverage.
4.  $\frac{\partial LTV^*}{\partial L_k} \leq 0$ , the more costly default is for the lender, the lower the leverage.
5.  $\frac{\partial LTV^*}{\partial p_2} \geq 0$ , lenders make higher LTV loans if they expect higher house price growth.

A big challenge in testing the predictions of this model is to find a measure of lender's house price expectations that is exogenous to LTV.

## TWO LENDERS AND PMI

The market in which a borrower obtains a mortgage from a lender is called the primary market. Lenders often sell these mortgages in the secondary market. Two of the biggest purchasers of mortgages in the secondary market, Fannie Mae and Freddie Mac, have certain requirements a loan must meet. One requirement is that if LTV is greater than 80% i.e.  $s_L > 0.8V_0$ , then the borrower must purchase Private Mortgage Insurance (PMI). However, if a borrower wants a loan with LTV greater than 80% he can avoid paying PMI by going to another lender and obtaining a second "piggyback" mortgage which is junior to the first mortgage. Unfortunately a second mortgage is more risky for the lender because if the borrower

defaults the first mortgage lender gets paid first. I will denote the first loan  $s_{L,1}$  the second loan  $s_{L,2}$ . The total loan is  $s_L \equiv s_{L,1} + s_{L,2}$ .

It is possible that the second lender chooses not to give the borrower a loan (because it is too risky) and the first lender will give  $s_{L,1} > 0.8V_0$  in which case the borrower is forced to pay PMI. I assume that the borrower will avoid PMI whenever possible (i.e. he wants to save money). If the borrower defaults, Lender 1 receives  $\min\{V_0(1+g), s_{L,1}\}$ , Lender 2 receives whatever is left after Lender 1 is paid  $V_0(1+g) - \min\{V_0(1+g), s_{L,1}\}$ .

In this two lender model the total loan size and leverage is exactly the same as in the one lender model, the only difference will be in the composition of the loan. In optimistic times the second lender will wish to lend and the borrower can avoid PMI. In bad times the second lender will choose not to lend and a borrower seeking a loan with  $LTV > 80\%$  will need to pay PMI.

### Deterministic Price Costless Default and PMI

The borrower's objective function is the same as before:

$$u^B(s_B, s_L, g) = s_B \times (V_0g - s_Lr) + (1 - s_B) \times (-(V_0 - s_L)) \quad (13)$$

So his decision rule is  $s_B(s_{L,1}, s_{L,2}, g) = 1 \left\{ s_{L,1} + s_{L,2} \leq V_0 \frac{(1+g)}{(1+r)} \right\}$ .

Lender 1's objective function is:  $u^{L,1} = s_B \times (s_{L,1}r) + (1 - s_B) \times (\min\{V_0(1+g), s_{L,1}\} - s_{L,1})$ .

Lender 2's objective function is:  $u^{L,2} = s_B \times (s_{L,2}r) + (1 - s_B) \times (V_1 - \min\{V_1, s_{L,1}\} - s_{L,2})$ .

Case 1 ( $g \geq r$ ): the borrower repays loans of any size.

The SPNE capital structure  $s_{L,1}^* = 0.8V_0$  and  $s_{L,2}^* = 0.2V_0$ .

Case 2 ( $g < r$ ): the borrower repays if  $s_{L,1} + s_{L,2} \leq V_0 \frac{(1+g)}{(1+r)}$ .

The SPNE capital structure  $s_{L,1}^* = \min\left\{0.8V_0, V_0 \frac{(1+g)}{(1+r)}\right\}$  and  $s_{L,2}^* = V_0 \frac{(1+g)}{(1+r)} - s_{L,1}^*$ .

In a deterministic world where there is no default, the borrower will never borrow more than  $0.8V_0$  from the first lender because the second lender will always be willing to lend the balance.

### Stochastic Prices Costly Default PMI

The borrower's objective function is:

$$u^B(s_B, s_L, g) = s_B \times (V_0g - s_Lr) + (1 - s_B) \times (-(V_0 - s_L) - B_j) \quad (14)$$

The borrower's decision rule is  $s_B(s_{L,1}, s_{L,2}, g) = 1 \left\{ s_{L,1} + s_{L,2} \leq \frac{V_0(1+g)+B_j}{(1+r)} \right\}$ .

Lender 1's objective function is:  $u^{L,1} = s_B \times (s_{L,1}r) + (1 - s_B) \times (\min\{V_1, s_{L,1}\} - s_{L,1} - L_k)$ .

Lender 2's objective function is:  $u^{L,2} = s_B \times (s_{L,2}r) + (1 - s_B) \times (V_1 - \min\{V_1, s_{L,1}\} - s_{L,2} - L_k)$ .

In the 1-lender model the equilibrium:

$$\text{Case 1 } \left( \frac{p_2 - \frac{1}{1+r}}{1-p_2} < \frac{L_k + B_j}{V_0(g_2 - g_1)} \right) s_L^* = \frac{V_0(1+g_1)+B_j}{(1+r)}$$

$$\text{Case 2 } \left( \frac{p_2 - \frac{1}{1+r}}{1-p_2} = \frac{L_k + B_j}{V_0(g_2 - g_1)} \right) s_L^* = \left\{ \frac{V_0(1+g_1)+B_j}{(1+r)}, \frac{V_0(1+g_2)+B_j}{(1+r)} \right\}$$



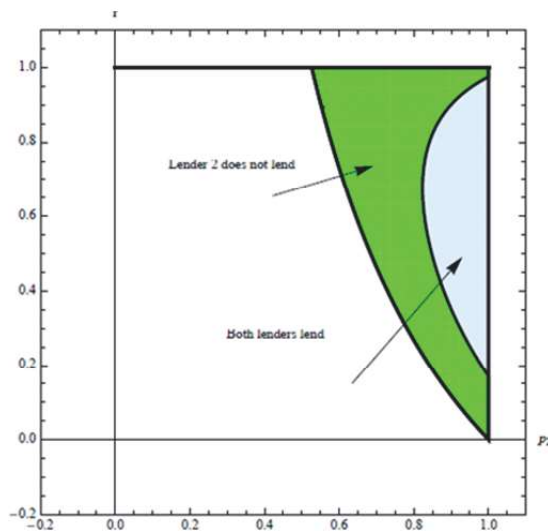
$$\text{Case 3} \left( \frac{p_2 - \frac{1}{1+r}}{1-p_2} > \frac{L_k + B_j}{V_0(g_2 - g_1)} \right) s_L^* = \frac{V_0(1+g_2) + B_j}{(1+r)}$$

In the low leverage case (Case 1) if  $s_L^* = \frac{V_0(1+g_1) + B_j}{(1+r)} > .8V_0$  then both borrowers will lend so  $s_{L,1}^* = .8V_0$  and  $s_{L,2}^* = s_L^* - .8V_0$ . In this equilibrium, the borrower repays the loan in both states of the world.

In the high leverage case if  $s_L^* = \frac{V_0(1+g_2) + B_j}{(1+r)} > .8V_0$ , then the second lender needs to decide if he wants to lend i.e. if the expected utility from lending exceeds his reservation utility  $E[u^{L,2}] > \bar{u}_2$ . Conversely if  $E[u^{L,2}] < \bar{u}_2$ , i.e. the second lender decides it is too risky to make a piggyback loan, then the borrower has no choice but to get a large loan from the first lender and pay PMI. This corresponds to parameters that satisfy: (1) Case 3, (2)  $s_L^* > 0.8V_0$ , and (3)  $E[u^{L,2}] < \bar{u}_2$ .

The following figure illustrates how various combinations of  $p_2$  and  $r$  correspond to different equilibria. Let  $V_0 = 100, B_j = 10, L_k = 0, g_1 = -.1, g_2 = .8$ .

**FIGURE 9**



The intuition behind this figure is that for any given interest rate  $r$ , the second lender needs the probability of repayment  $p_2$  to be especially high for him to participate because his loan is riskier than the first lender's loan. When  $r$  and  $p_2$  are in the blue region both lenders lend,  $s_{L,1}^* = .8V_0$  and  $s_{L,2}^* = s_L^* - .8V_0$ . When  $r$  and  $p_2$  are in the green region, the second lender's participation constraint ( $E[u^{L,2}] > \bar{u}_2$ ) does not hold so the borrower obtains the entire loan from the first lender.

## MODEL SUMMARY

The predictions of the model are summarized below:

1. Lenders are willing to make more highly leveraged loans when they are more optimistic about the future value of the collateral.
2. Lenders are willing to make more highly leveraged loans to borrowers who they believe are less likely to default if underwater. Primary home buyers (not investors), borrowers who live in recourse states (especially if borrowers have more assets to lose), borrowers with higher FICO scores, and if borrowers have jobs that are not cyclically sensitive (e.g. tenured college professors).

3. Lenders make loans with less leverage if the market interest rate for that type of loan is greater. This is because a higher interest rate will make the loan more difficult for the borrower to repay.
4. Lenders making second loans are much more sensitive to a drop in house price expectations than lenders making first loans.

For example, the model in section 3.3 gave the following equilibrium leverage:  $LTV^* = \frac{(1+g)}{(1+r)} + \frac{B_j}{(1+r)V_0}$ . During ebullient times lenders are optimistic about house price growth, so they believe  $g$  is big and are willing to give loans with large LTVs. In pessimistic times lenders believe  $g$  is small and give loans with smaller LTVs. In this model the parameter  $B_j$  captures all the variables that can make a borrower more or less sensitive to negative equity. For example a borrower who has a very cyclically sensitive job and therefore has a high probability of becoming unemployed should have a smaller  $B_j$  than a borrower with a safer job. The model predicts that the borrower with the risky job would get a loan with less leverage than the borrower with the safer job.

## FUTURE WORK

Currently, the geographic regions in the data section are very coarsely defined (30 regions: 29 states and DC). It is possible that within a given state (or MSA or city) different neighborhoods experienced different cycles in both house prices and leverage. In future work researchers can use neighborhood level data from DataQuick as in Ferreira and Gyourko (2011) to obtain a more precise picture. To test the predictions of this model a researcher would need to cleanly identify the impact of house price expectations on leverage. This is a difficult task. The first equation that one could estimate is:

$$CLTV90_{i,t} = \beta_0 + \beta_1 HPE_{i,t} + \beta_2 Macro_{i,t} + c_i + \delta_t + u_{i,t} \quad (15)$$

Where the outcome variable  $CLTV90_{i,t}$  is a measure of leverage for loans originated in location  $i$  at time  $t$ ,  $HPE_{i,t}$  is a measure of house price expectations in region  $i$  at time  $t$ ,  $Macro_{i,t}$  is a vector of macroeconomic variables that correspond to  $B_j$  in the model.  $c_i$  and  $\delta_t$  are state and time fixed effects. The researcher would need to find a good proxy for house price expectations in the data. One possibility is to use evidence from Ghysels et al (2012) that house prices are predictable in the short run and define  $HPE_{i,t} \equiv \frac{HPI_{i,t} - HPI_{i,t-1}}{HPI_{i,t-1}}$ , where  $HPI_{i,t}$  is the FHFA house price index for region  $i$  in quarter  $t$ .

The model predicts  $\beta_1 > 0$  ( $\frac{\partial LTV^*}{\partial g_i} > 0$ ). If  $Macro_{i,t}$  is a measure of expected employment in region  $i$ , then higher values of this variable would imply that borrowers would have more tolerance for negative equity so the model predicts  $\beta_2 > 0$  ( $\frac{\partial LTV^*}{\partial B_j} > 0$ ).

Unfortunately the estimates from this equation are not reliable for inference because it is deeply endogenous. The first issue is the simultaneity between leverage and house price expectations. My model predicts that higher house price expectations should lead to higher leverage. Geanakoplos (2009) shows that greater leverage today should increase house prices (and therefore house price expectations). To break this endogeneity a researcher would need to find a variable that affects house price expectations without affecting leverage. Mian and Sufi (2011) offer a solution: "...proper identification of the effect of house prices on borrowing requires an exogenous source of variation in house price growth. We use two different instruments for house price growth, one based on across metropolitan statistical area (MSA) variation and another based on within-MSA variation. The across-MSA specification uses housing supply elasticity at the MSA level as an instrument for house prices. MSAs with elastic housing supply should experience only modest increase in house prices in response to large shifts in the demand for housing because housing supply can be expanded relatively easily. In contrast, inelastic housing supply MSAs

should experience large house price changes in response to the same housing demand shock (Edward Glaeser, Joseph Gyourko, and Albert Saiz 2008).”

Following their logic a researcher can estimate the following IV specification:

$$CLTV90_{i,t} = \beta_0 + \beta_1 HPE_{i,t} + \beta_2 Macro_{i,t} + c_i + \delta_t + u_{i,t} \quad (16)$$

$$HPE_{i,t} = \gamma_0 + \gamma_1 elasticity_{i,1997} + \gamma_2 Macro_{i,t} + v_{i,t} \quad (17)$$

However this is still problematic because the composition of borrowers changed significantly over the recent cycle.

Another interesting way to study leverage would be to look at the mortgage rate sheets published daily by banks. These rate sheets have a menu of mortgages offered by the bank, the interest rate for a loan of a given leverage (LTV) and a given FICO score. It would be interesting to see how the different banks adjusted their leverage requirements in different regions over time. The company LOANSIFTER collects these rate sheets.

## CONCLUSION

The recent boom and bust in real estate prices coincided with a boom and bust in leverage. Most regions saw a rise in no cash loans (CLTV > 99%) between 2002 and 2006 Q3, followed by a very sharp drop. At the peak of the housing boom, regions with the greatest increase in house prices also had the greatest increase in leverage. During the housing bust, regions with the greatest drop in house prices also had the greatest drop in leverage. This paper also documents a change in the composition of leverage. During the peak of the housing boom, most highly leveraged borrowers had second mortgages for the portion of the loan with LTV > 80%. However, when house prices began to drop, most second mortgages disappeared, even for very highly leveraged loans.

The model in this paper solves for leverage endogenously and shows that the key drivers of leverage are the lender's expectations of the future value of the collateral and negative income shocks to the borrower which make him less willing to tolerate negative equity. The model also predicts that lenders making second loans are much more sensitive to drops in HPE.

There are many challenges to testing the predictions of the model. The first is the simultaneity of HPE and leverage. Mian and Sufi (2011) suggest a way to partially alleviate this by using housing supply elasticity as an instrument for HPE. Another difficulty in testing the model is that the composition of borrowers changed significantly over the recent cycle. These issues should be addressed in future work.

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