Modelling and Forecasting the Conditional Heteroscedasticity With Different Distribution Densities – Frontier Market Evidence

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This paper examines and compares alternative distribution density forecast methodology of three generalised autoregressive conditional heteroscedasticity (GARCH) models. We employed the symmetric GARCH, Glosten Jagannathan and Runkle version of GARCH (GJR-GARCH) and Exponential GARCH methodology to investigate the effect of stock return volatility using Gaussian, Student-t and Generalised Error distribution densities. The evidence reveals that news impact is asymmetric leading to the existence of leverage effect in stock returns. The presence of leverage effect suggests that investors in these markets are to be rewarded for taking up additional leverage risks. This implies that by allocating portfolios, fund managers and /or investors should go beyond the mean-variance analysis and look into information about volatility, information asymmetry, correlation, skewness, and kurtosis. Furthermore, the evidence exhibits reverse volatility asymmetry, contradicting the widely accepted theory of volatility asymmetry. Regarding forecasting evaluation, the results reveal that the symmetric GARCH model coupled with fatter-tail distributions presents a better out-of-sample forecast.

Keywords: leverage effect, GARCH, EGARCH, GJR-GARCH, forecasting volatility, conditional heteroscedasticity, distribution densities

INTRODUCTION

Empirical time series econometrics has shown that stock returns follow non-normal distribution density (Hsu *et al*., 1974; Hagerman, 1978; Lau *et al*., 1990; Kim and Kon, 1994). These studies in turn confirmed that where the kurtosis of time series of stock returns is greater than normal, the distribution is either skewed to the left or to the right and the variance of the stock returns is heteroscedastic (i.e. non-constant variance). This heteroscedasticity in the error variance is described as uncertainty or risk by the financial analyst and it has become important in modern theory of finance. Using Autoregressive Conditional Heteroscedasticity (ARCH), Engle (1982) modelled the time varying variances of United Kingdom inflation. This has become a benchmark econometric tool for modelling economic and financial series over the years. The linear ARCH (q) model is characterized by a long lag length of q in several of its usage. Bollerslev (1986) presents a more malleable lag structure of the ARCH known as the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) to resolve this empirical weakness of the ARCH. Some empirical works have shown that the first order lag length of the GARCH is adequate to model the long memory processes of time varying variance (French et al., 1987; Franses & Van Dijk, 1996). Besides, a study conducted by Black (1976) revealed that variation in stock price has an unequal impact on volatility. This behaviour in financial time series is known as the leverage effect (i.e. large negative returns appear to increase volatility more than positive returns of the same magnitude). The standard GARCH is found inadequate to model the dynamics of this leverage effect. Furthermore, Nelson (1991) and Glosten et al. (1993) respectively presented the Exponential GARCH and Threshold GARCH (also known as GJR after its proponents) to account for this unequal response of volatility.

While there has been extensive research on symmetric and asymmetric GARCH models in the academic literature since the introduction of ARCH/GARCH, GJR-GARCH and EGARCH (Engle, 1982; Bollerslev, 1986; Glosten et al., 1993 and Nelson, 1991), few studies have concentrated on comparing alternative density forecast models Hamilton and Susmel (1994), Lopez (2001), Franses and Ghijsels (1999) and Wilhelmsson (2006) as the obvious ones. The previous studies focused on symmetric GARCH models and none of these studies has explicitly focused on evaluating both symmetric and asymmetric GARCH models with the introduction of symmetric and asymmetric distribution densities. Furthermore, another striking feature of high-frequency financial time series of stock returns is that they are frequently characterized by a fat - tailed distribution. Available literature in finance indicates the fact that, the kurtosis of most financial asset returns is greater than 3 (Simkowitz & Beedles, 1980; Kon, 1984). This suggests that extreme values are much more likely to be observed in stock market returns than the normal distribution**.**

Nevertheless, in this study we show that this gap can be filled by introducing rigorous alternative density distribution methodology to symmetric and asymmetric GARCH models. The performance of GARCH (1, 1), GJR-GARCH (1, 1) and EGARCH (1, 1) models are compared with the introduction of different distribution densities (Gaussian, Student-t and GED). The study is thus motivated by recognising the importance of accurate volatility measurement and forecast in a wide range of financial applications such as asset pricing, option pricing as well as portfolio selection. Furthermore, the paper contributes to the academic literature in three ways. First, we demonstrate that this gap can be filled by a rigorous density forecast models comparison methodology. Second, the performance of GARCH-type models are compared with the introduction of normal and non-normal distribution densities for modelling and forecasting the conditional volatility. This addresses the methodological issue as to which GARCH-type model couple with distribution density variant better estimates and forecasts stock returns volatility. Third, we use high frequency stock data from the Stock Exchange of Mauritius (SEM) and Zambia Stock Exchange (ZSE) Composite Indices to facilitate meaningful comparison of the forecast results.

Studies into economic and financial time series have long recognized that stock returns exhibit heavy-tailed distribution probability. One main motivation for this heavy-tailed feature is that the conditional variance may be non-constant. Although excess kurtosis of stock returns can successfully be removed by GARCH model, it cannot cope with the skewness of the distribution of stock market returns. Thus, forecast estimates from GARCH can be expected to be biased for a skewed time series. Stock market returns distribution has tails that are heavier than implied by the GARCH process with Gaussian. Therefore, by modelling financial time series such as stock returns, a researcher must assume not only Gaussian white noise but also independently identical distribution (*i.i.d*) white noise process with a heavy-tailed distribution. Standard GARCH models assume that the error distribution is Gaussian. However, evidence shows that the error exhibits non-normal distribution densities. Wilhelmsson (2006) showed that allowing for a leptokurtic error distribution leads to significant improvements in variance forecasts compared to using the Gaussian distribution. Nelson (1991) found that assuming a generalised error distribution better modelled the conditional variance than using normal distribution. The choice of the underlying distribution for the error term is crucial if the volatility model is used in risk modelling. It was anticipated that the problems posed by skewness and kurtosis could produce residuals of conditional heteroscedasticity models that could be condensed when appropriate distribution density was used. Most recent econometric studies have shown the development of other non-linear models which consider the skewed distribution, for example, the exponential GARCH (EGARCH) model, introduced by Nelson (1991). Thus, choosing the appropriate distribution density that can model and forecast the first and second moments is important, hence, our motivation to investigate conditional heteroscedasticity with the introduction of different distribution densities.

The remainder of the study is organised as follows: the next section details the empirical models. Data description and methodology used in this study are explained in section three. The fourth section presents results and analyses and the conclusions are presented in the final section.

Empirical Models

Two moments (i.e. mean and variance) equations are used to define the ARCH/GARCH models. The return process, r_t, was taken into account by the mean equation which was made up of the conditional mean, μ, which might encompass terms of autoregressive(*AR*) and moving average(*MA*) and error term, ε_t, that followed a conditional normal distribution with mean of zero and variance, σ^2. Furthermore, the information set available to investors up to time t_{-1} is represented by, Ω_{t-1}, thus,

$$r_t = \mu + \varepsilon_t \tag{1}$$

where

$$\varepsilon_t \big| \Omega_{t-1} \approx N\left(0, \sigma_t^2\right) \quad \sigma_t^2 = h_t \tag{2}$$

The conditional variance h_t was modelled using symmetric and asymmetric GARCH models with the introduction of three different distribution densities (i.e. Gaussian, Student-t and GED).

ARCH Model

Engle (1982) seminal work suggested to model time varying conditional heteroscedasticity using past error term to estimate the series variance as follows:

$$\boldsymbol{h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2} \tag{3}$$

GARCH Model

Bollerslev (1986) proposed the GARCH model which suggests that time varying heteroscedasticity was a function of both past innovations and past conditional variance (i.e. past volatility).The GARCH model signifies an infinite order ARCH model expressed as:

$$\boldsymbol{h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}} \tag{4}$$

where α_0, α_i and β_j are non-negative constants.

Exponential GARCH (EGARCH) Model

Nelson (1991) introduced the exponential GARCH model to capture the asymmetric (or 'directional') response of volatility. Nelson and Cao (1992) argue that the imposition of non-negativity constraints on the parameters; α_i and β_j in the linear GARCH model are too restrictive, while in the EGARCH model there is no such restriction. The conditional variance, h_t, in the EGARCH model is an asymmetric function of lagged disturbances as follows:

$$ln(h_t) = \alpha_0 + \alpha\left(|z_{t-1}| - E(|z_{t-1}|)\right) + \gamma z_{t-1} + \beta\, ln(h_{t-1}) \tag{5}$$

Since the log of the conditional variance is modelled, the leverage effect is exponential, rather than quadratic and even if the parameters are negative, the conditional variance will be positive. For $\gamma < 0$ means that negative shocks will have a bigger impact on expected volatility than positive shocks of the same magnitude. This is often referred to in the literature as the leverage effect. The EGARCH model allows positive and negative shocks to have a distinct impact on volatility. It also allows large shocks to have a superior impact on volatility than the standard GARCH model.

The GJR-GARCH Model

The GJR-GARCH model was presented by Glosten, Jagannathan and Runkle (1993). The GJR augments the standard GARCH with an additional ARCH term conditional on the sign of the past innovation and is expressed as:

$$h_t = \alpha_0 + \sum_{i=1}^{q}\left(\alpha_i \varepsilon_{t-i}^2 + \lambda_i \varepsilon_{t-i}^2 I_{t-1}\right) + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{6}$$

where λ_I measures the asymmetric (or leverage) effect and I_t is a dummy variable which is equal to 1 when $\varepsilon_t$is negative. In the GJR (1, 1) model, good news,$\varepsilon_{t-1} > 0$ and bad news,$\varepsilon_{t-1} < 0$, possess differential effects on the conditional variance. Good news has an impact ofα_1, while bad news has an impact of$\alpha_1 + \lambda_1$. If$\lambda_1 > 0$, bad news increases volatility and this in turn means that there is a leverage effect for the AR (1)-order. If$\lambda_1 \neq 0$, the news impact is asymmetric.

DATA AND METHODS

Data Description

The daily stock price indices data which are used in this research are obtained from Standard & Poor/International Finance Corporation Emerging Market Database (S&P/IFC EMDB). This source is used largely because it is a very organized and comprehensive source of stock price data, providing readily accessible and reliable data on emerging equity markets than most other sources. For example, S&P/IFC EMDB was the first database, from 1975, to track comprehensive information and statistics on emerging stock market indices. The S&P/IFC Global indices, used in this study, do not impose restrictions on foreign ownership and include a sufficient number of stocks in individual market indices without imposing float or artificial industry-composition models of markets. Besides, the S&P/IFC database is attractive because it has been adjusted for all capital changes as well as the effects of corporate restructuring such as merger, acquisition, and spin offs/demerger as well as being free from data backfilling and survivorship bias.

The daily return, r_t consists of transformed daily closing index price, P_t measured in local currency. Our measurements include the Stock Exchange of Mauritius (SEM) Index (SEMI) and Zambia Stock Exchange Overall Index (ZSEI). The stock price indices are transformed into natural log returns in order to obtain a stationary series as:

$$r_t = ln\left({}^{p_t}/_{p_{t-1}}\right) * 100 \tag{7}$$

where r_t is the market return at time t, p_t and p_{t-1} are the contemporaneous and one period lagged equity price indices, respectively. Natural lognormal is preferred as it computes continuous compound returns.

Tables 1 below provides further details of the data used in this research, including the types of the stock indices used, the time period of the data for each market (and hence sample observations), and currency of denomination. The indices used in this study are the benchmark indices in their respective stock markets.

TABLE 1
STOCK MARKET DATA PROFILE

Country	Method of compiling data	Index Name	Period of data	No of Observ.	Currency	Source of Data
Mauritius	Weighted index market capitalization	SEM All Share Index	1997 – 2014	4691	Rupee	S&P/IFC EMDB
Zambia	Weighted index market capitalization	ZSE All Share Index	1997 – 2014	4691	Kwacha	S&P/IFC EMDB

TABLE 2
DESCRIPTIVE STATISTICS FOR DAILY RETURNS

	Mean	Std. Dev.	Skewness	Kurtosis	J. Bera	Q-Stat(100)
Mauritius	0.0375	0.9933	2.0482	105.9492	(2074852)***	(34)**
Zambia	0.0879	1.8235	1.4849	41.9360	(298041)***	(177)**

J. Bera is the Jarque-Bera test for normality, Q-stat refers to Ljung-Box test for autocorrelation.
*** denotes statistical significance at 1%. ** denotes statistical significance at 5%

The descriptive statistics in Table 2 indicate that both markets produce positive mean returns. However, the daily mean return for Zambia is higher than that of Mauritius. Furthermore, the non-conditional variance as measured by the standard deviation for Zambia is higher than that of Mauritius. The returns distribution for both indices is positively skewed. The null hypothesis for skewness that conforms to a normal distribution with coefficients of zero is rejected by both indices. The returns for both indices exhibit fat tail distribution as seen in the significant kurtosis well above the normal value of 3. The high and significant value of Jacque-Bera test for normality decisively rejects the hypothesis of a normal distribution. Ljung-Box Q test statistic (Q-Stat) rejects the null hypothesis of no autocorrelation at 1 and 5 per cent levels for all numbers of lags (100) considered. The preceding statistics legitimize the use of autoregressive conditional heteroscedasticity models.

The statistical results indicate that both indices display similar characteristics. For instance, they both have positive mean returns, are positively skewed, found to display non-normal distribution and exhibit autoregression. These stylized features are similar to the existing empirical literature from the developing markets (Kim, 2003; Ng, 2000) and developed markets (Fama, 1976; Kim & Kon, 1994). Furthermore, as return series revealed high value of kurtosis, it can be expected that a fatter-tailed distribution density, such as the Student-t or GED should provide a more accurate results than the Gaussian (Normal) distribution.

Empirical Methods

The GARCH models are estimated using maximum likelihood estimation (MLE) process. This allowed the mean and variance processes to be jointly estimated. The MLE has numerous ideal characteristics in estimating parameters and this included sufficiency, (i.e. complete information about the parameter of importance contained in its MLE estimator); consistency (true parameter value that generated the data recovered asymptotically, i.e. data of sufficiently large samples) and efficiency (lowest possible variance of parameter estimates to achieve asymptotically). Moreover, several methods of inference in statistics and econometrics were developed based on MLE, such as chi-square test, modelling of random effects, inference with missing data and model selection criteria such as Akaike information criterion and Schwarz criterion.

Gaussian

The Gaussian, also known as the normal distribution, is the widely used model when estimating GARCH models. For a stochastic process, the log-likelihood function for the normal distribution is calculated as:

$$L_{gaussian} = -{}^{1}/_{2} \sum_{t=1}^{T} (ln[2\pi] + ln[\sigma_t^2] + z_t) \tag{8}$$

where T is the number of observations.

Student-T Distribution

For a student-t distribution, the log-likelihood is computed as:

$$L_{stu-t} = ln\left(\Gamma\left[\frac{v+1}{2}\right]\right) - ln\left(\Gamma\frac{v}{2}\right) - {}^{1}/_{2}\, ln(\pi[v-2]) - {}^{1}/_{2} \sum_{t=1}^{T}\left(ln\,\sigma_t^2 + [1+v]\,ln\left[1+\frac{z_t^2}{v-2}\right]\right) \tag{9}$$

where v is degrees of freedom, 2<v=∞and $\Gamma(\bullet)$ is the gamma function.

Generalised Error Distribution (GED)

Skewness and kurtosis are very important in applied finance such as asset pricing, option pricing, and portfolio selection. Thus, choosing the appropriate distribution density that can model these two moments is important, hence, the GED log-likelihood function of a normalised random error is computed as:

$$L_{GED} = \sum_{t=1}^{T}\left(ln\left[{}^{v}/_{\lambda_v}\right] - 0.5\left|\frac{z_t}{\lambda_v}\right|^{v} - [1+v^{-1}]\,ln\,2 - ln\,\Gamma\left[{}^{1}/_{v}\right]0.5\,ln[\sigma_t^2]\right) \tag{10}$$

where $\lambda_v = \sqrt{\frac{\Gamma\left({}^{1}/_{v2^{-2/v}}\right)}{\Gamma\left({}^{3}/_{v}\right)}}$ (11)

Goodness-of- Fit Diagnostics

The order of the GARCH process can be identified by computing Q-statistic from the squared residuals and the Engle (1982) LM test is used to test for the ARCH effect in the residuals. The GARCH models in this study are compared by using various goodness-of-fit diagnostics such as Akaike information criterion, Schwarz Bayesian information criterion and log-likelihood.

Forecast Evaluation

The one-step-ahead forecast of the conditional variance for the GARCH, EGARCH and GJR is obtained by updating equations (4), (5) and (6) by one period as,

$$\boldsymbol{h_{t+1} = \alpha_0 + \alpha_1\varepsilon_t^2 + \beta_1 h_t} \tag{12}$$

$$ln(h_{t+1}) = \alpha_0 + \alpha_1 g(Z_t) + \beta_1\, ln(h_t) \tag{13}$$

$$h_{t+1} = \alpha_0 + \alpha_1\varepsilon_t^2 + \lambda_1\varepsilon_t^2 I_t + \beta_1 h_t \tag{14}$$

Similarly, *j*-step-ahead forecast on the conditional variance can be obtained by updating equations (12), (13) and (14) by *j*-periods as,

$$h_{t+j} = \alpha_0 + \alpha_1 \varepsilon_{t+j-1}^2 + \beta_1 h_{t+j-1} \tag{15}$$

$$ln(h_{t+j}) = \alpha_0 + \alpha_1 g(Z_{t+j-1}) + \beta_1\, ln(h_{t+j-1}) \tag{16}$$

$$h_{t+j} = \alpha_0 + \alpha_1 \varepsilon_{t+j-1}^2 + \lambda_1 \varepsilon_{t+j-1}^2 I_{t+j-1} + \beta_1 h_{t+j-1} \tag{17}$$

However, it is rather difficult to obtain the *j*-step-ahead forecasts than the one-period-ahead forecasts assumed in this study although it is possible to obtain the *j*-step-ahead forecasts of the conditional heteroscedasticity recursively.

In order to evaluate the forecasting performance of the GARCH, EGARCH and GJR models, forecasting tests encompassing different distribution densities are performed. The model that minimises the loss function under these evaluation criteria is preferred. To measure the performance of the asymmetric GARCH models in forecasting the conditional variance, we compute four statistical measures of fit as follows;

Mean Absolute Error (MAE) – This is represented as:

$$MAE = \frac{1}{h}\sum_{t=s}^{s+h} |\hat{\sigma}_t^2 - \sigma_t^2| \tag{18}$$

where h is the number of steps ahead (i.e. number of forecast data points, where h is equal to 1, representing one step ahead), *s* the sample size, $\hat{\sigma}^2$ is the forecasted variance and σ^2 is the conditional variance computed from equations (4), (5) and (6).

Root Mean Square Error (RMSE) is represented as:

$$RMSE = \sqrt{\frac{1}{h}\sum_{t=s}^{s+h} (\hat{\sigma}_t^2 - \sigma_t^2)^2} \tag{19}$$

The Mean Absolute Percentage Error (MAPE) is represented as:

$$MAPE = \frac{1}{h}\sum_{t=s}^{s+h} \left| (\hat{\sigma}_t^2 - \sigma_t^2) \Big/ \sigma_t^2 \right| \tag{20}$$

Theil Inequality Coefficient (TIC) is represented as:

$$TIC = \frac{\sqrt{MSE}}{\sqrt{1/h \sum_{t=s}^{s+h} \sigma^2 + 1/h \sum_{t=s}^{s+h} \hat{\sigma}^2}} \tag{21}$$

To calculate daily forecast and in order to assess the forecasting performance of each model, we simply split the respective time series in half between the in-sample period, $t = 1, \ldots, T$ and the out-of-sample period, $t = T, \ldots, h$. We further estimate each model over the first part of the sample and then apply these results to forecast the conditional variance (volatility) over the second part of the sample period.

EMPIRICAL RESULTS AND ANALYSES

We present and analyse our results of the estimated models in this section. Tables 3, 4 and 5 presents the results for the estimated parameters of GARCH, EGARCH and GJR models respectively, while some useful in-sample diagnostics statistics are reported in Tables 6, 7 and 8.

TABLE 3
ESTIMATED STATISTICS-COMPARATIVE DISTRIBUTION DENSITY GARCH MODEL

	ZAMBIA			MAURITIUS		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
μ	0.0727 (4.4826)***	0.0046 (1.009)	9.67e-11 (0.0133)	0.0259 (2.1379)**	0.0002 (0.0779)	-0.0001 (-2.6726)***
α_0	0.0115 (2.5220)**	0.3298 (0.0095)	0.1563 (6.5970)***	1.1734 (7.2198)***	0.1836 (1.1770)	0.3679 (10.3038)***
α_1	0.0527 (6.6816)***	209.7568 (0.0095)	0.3695 (6.1454)***	0.1255 (2.1844)**	0.3848 (1.1904)	0.5844 (4.9929)***
β_1	0.9470 (123.6829)***	0.7500 (106.6598)***	0.7157 (34.7710)***	-0.0215 (-1.9523)*	-0.0011 (-11.2356)***	-0.0015 (-1.4820)
$\alpha_1 + \beta_1$	0.9997	210.5068	1.0852	0.1040	0.3837	0.5829

TABLE 4
ESTIMATED STATISTICS-COMPARATIVE DISTRIBUTION DENSITY EGARCH MODEL

	ZAMBIA			MAURITIUS		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
μ	0.1273 (4.7320)***	0.0041 (0.6834)	-2.16e-10 (-0.0934)	0.1690 (3.4864)***	-8.78e-05 (-0.0483)	1.43e-12 (0.0718)
α_0	-0.0833 (-7.0104)***	-0.0966 (-5.8802)***	1.5855 (7.2004)***	-0.0144 (-0.7772)	-0.0705 (-20.1090)***	-0.2719 (-23.6302)***
α_1	-0.0009 (-0.0464)	-1.3129 (-4.5911)***	-0.2393 (-2.6575)***	0.0169 (0.7614)	-0.0342 (-8.8811)***	-0.0982 (-17.2642)***
β_1	0.9886 (264.1718)***	0.9830 (387.0410)***	0.1551 (2.7030)***	0.9786 (41.9297)***	0.9887 (1500)***	0.9239 (284.6561)***
γ	0.1623 (7.6070)***	5.7280 (6.3579)***	1.2500 (7.1070)***	0.0408 (1.1508)	0.0357 (8.8484)***	0.0805 (15.6321)***

TABLE 5
ESTIMATED STATISTICS-COMPARATIVE DISTRIBUTION DENSITY GJR-GARCH MODEL

	ZAMBIA			MAURITIUS		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
μ	0.0729 (4.6603)***	0.0039 (0.8864)	-9.99E-08 (-0.0005)	0.0031 (0.1410)	-2.67e-06 (-0.0051)	-6.51E-05 (-0.0312)
α_0	0.0115 (2.5154)**	0.0425 (0.0311)	0.1523 (6.4637)***	1.4091 (185.2814)***	1.92e- 05 (0.4624)	0.1641 (16.4233)***
α_1	0.0528 (3.9805)***	46.2667 (0.0309)	0.2647 (5.0581)***	0.0275 (4.6513)***	14.1048 (0.6962)	0.0704 (6.6356)***
β_1	0.9472 (123.5484)***	0.7490 (110.4265)***	0.7407 (38.2404)***	-0.0323 (-9.9151)***	0.0013 (13.3519)***	0.2268 (6.2263)***
λ_1	-0.0005 (-0.0238)***	65.7271 (0.0309)	0.2671 (3.0797)***	0.0722 (5.1227)***	28.9304 (0.7006)	0.2533 (5.2553)***

Tables 3, 4 and 5 report the results estimated for GARCH, EGARCH and GJR-GARCH with three different distribution densities, while tables 6, 7 and 8 presents some useful in-sample diagnostics statistics.

The statistics reported in the tables above show that the use of EGARCH and GJR with normal and non-normal distribution appears justified to model the asymmetric characteristics of both indices. All the asymmetric coefficients (with the exception of student-t for both asymmetric model and Gaussian EGARCH for Mauritius) are statistically significant at standard levels for both indices. The evidence also shows that news impact is asymmetric in both stock markets as the asymmetric coefficients for all densities are unequal to zero.

In Mauritius, the sum of the lagged error (α) and the lagged conditional variance (β) of the symmetrical GARCH model for both indices is far from the expected value of 1 regardless of the distribution density. While in Zambia, as typical of GARCH model, the sum of α and β for the Gaussian distribution is close to 1 and that of non-normal distributions is greater than 1. This implies that the current shocks to the conditional variance will have less impact on future volatility in Mauritius, while in Zambia the impact of current shocks to the conditional variance on future volatility is either persistent or explosive.

For both markets, the coefficient estimates of the EGARCH were generally positive and statistically significant at standard levels, suggesting that positive instead of negative shocks implied a higher next period conditional variance of the same sign. This means that negative shocks would have no greater effects on volatility than positive shocks as expected. As an alternative, positive shocks would have a greater effect on volatility as the asymmetric term, is greater than zero for all density distributions. This evidence invalidated the EGARCH proposition that bad news has greater impact on volatility than good news. In line with Wan et al. (2014) research, the evidence in Zambia and Mauritius showed that both markets exhibit a reverse volatility asymmetry, contradicting the widely accepted theory of volatility asymmetry (i.e. negative returns induce a higher volatility than positive returns). In both markets, the evidence demonstrated that return volatilities react more intensely to positive returns than their reaction to negative returns. Mainly, this reverse volatility asymmetry is attributed to higher trading volume associated with momentum stocks (i.e. price rising stocks) as investors from Zambia and Mauritius (as typical of most African investors) were known to rush for such stocks than their contrarian counterparts and this leads to the arousal of higher volatility for positive returns than negative returns. Therefore, positive return-volatility correlation was observed in both stock markets.

The leverage effect term,$\boldsymbol{\lambda}$ in the GJR for Zambia had the satisfactory sign for student-t and GED while the Gaussian coefficient was negative and significant. Furthermore, in Mauritius, as typical of GJR model, the leverage effect term has the correct sign and was statistically significant with GED and Gaussian while the Student-t produced insignificant coefficient**.** The evidence so far showed that in both markets, negative shocks would generally have a greater impact on future volatility than positive shocks of the same magnitude, confirming the existence of leverage effect. The presence of leverage effect suggested that investors in these markets were to be rewarded for taking up additional leverage risks. Therefore, investors and fund managers should go beyond the simple mean-variance approach when allocating portfolios for these markets. Instead, they should explore information about volatility, information asymmetry, correlation, skewness and kurtosis. Required rate of return is expected to be high in these markets due to compensation for additional leverage risk which places additional burden on indigenous companies seeking to raise finance from the domestic capital markets.

Generally, the estimated parameters for both indices of the asymmetric GARCH model indicated that the ARCH (α_1) and GARCH (β_1) terms were statistically significant at standard levels. For example, the estimated coefficients for the GARCH (β_1) term in the asymmetric models for both indices produced statistically significant results at 1 per cent level. Furthermore, the ARCH (α_1) estimates in the EGARCH are significant with non-normal distribution in both markets, while the GJR produces significant ARCH (α_1) with Gaussian and GED. This provided sufficient justification of the use of asymmetric GARCH models.

TABLE 6
DIAGNOSTICS STATISTICS-COMPARATIVE DISTRIBUTION DENSITY GARCH MODEL

	ZAMBIA			MAURITIUS		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
$Q^2(20)$	14.454 (0.807)	2.5565 (1.000)	15.126 (0.769)	6.2379 (0.999)	8.2732 (0.990)	8.1947 (0.990)
ARCH(2)	4.7590 (0.0926)	0.4768 (0.7879)	2.9820 (0.2251)	0.0526 (0.9740)	0.3062 (0.8581)	0.3010 (0.8603)
AIC	3.4355	2.5853	1.2474	2.8017	-0.4722	-3.4229
SBIC	3.4410	2.5922	1.2543	2.8072	-0.4653	-3.4160
Log-Like	-8054	-6059	-2921	-6567	1113	8033

Q^2 (20) are the Ljung-Box statistic at lag 20 of the squared standardised residuals. ARCH (2) refers to the Engle (1982) Lagrange Multiplier (LM) test for the presence of ARCH effect at lag 2. P-values are given in parentheses. AIC, SBIC and Log-Like are Akaike information criterion, Schwartz Bayesian information criterion and Log-Likelihood value respectively.

TABLE 7
DIAGNOSTICS STATISTICS-COMPARATIVE DISTRIBUTION DENSITY EGARCH MODEL

	ZAMBIA			MAURITIUS		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
$Q^2(20)$	22.902 (0.294)	2.5024 (1.000)	541.55 (0.000)	5.4065 (0.999)	10.275 (0.963)	30.711 (0.059)
ARCH(2)	5.6020 (0.0607)	0.7475 (0.6882)	26.1514 (0.000)	0.0468 (0.9769)	0.1225 (0.9406)	0.2505 (0.8823)
AIC	3.4515	2.5689	0.4975	2.7770	-1.0974	-7.0612
SBIC	3.4584	2.5771	0.5058	2.7838	-1.0891	-7.0529
Log-Like	-8091	-6019	-1161	-6508	2580	16568

TABLE 8
DIAGNOSTICS STATISTICS-COMPARATIVE DISTRIBUTION DENSITY GJR-GARCH MODEL

MAURITIUS **ZAMBIA**

GARCH

Model	Gaussian	Student-t	GED	Gaussian	Student-t	GED
RMSE	0.958287	0.958946	0.958958	2.268713	2.319338	2.319551
MAE	0.166886	0.142725	0.142666	1.051456	1.049991	1.048422
MAPE	5.100966	4.847559	4.855454	179.0995	65.73144	63.16215
TIC	0.973056	0.999808	0.999888	0.967945	0.997914	1.000000

EGARCH

Model	Gaussian	Student-t	GED	Gaussian	Student-t	GED
RMSE	0.967187	0.958957	0.958953	2.268427	2.319362	2.319551
MAE	0.302038	0.142643	0.142559	1.074202	1.049803	1.048422
MAPE	9.119916	4.854806	4.852406	273.4574	65.20528	63.16215
TIC	0.857438	0.999912	1.000000	0.945777	0.998156	1.000000

GJR

Model	Gaussian	Student-t	GED	Gaussian	Student-t	GED
RMSE	0.958837	0.958954	0.958956	2.317271	2.319370	2.319551
MAE	0.145465	0.142561	0.142622	1.075894	1.049748	1.048422
MAPE	4.767736	4.852479	4.854187	148.2264	65.04887	63.16216

TABLE 9
FORECAST PERFORMANCE-COMPARATIVE DISTRIBUTION DENSITY

	ZAMBIA			MAURITIUS		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
$Q^2(20)$	14.504 (0.804)	2.5163 (1.000)	15.905 (0.722)	5.7001 (0.999)	7.8694 (0.993)	8.1900 (0.991)
ARCH(2)	4.7986 (0.0908)	0.4810 (0.7862)	2.7148 (0.2573)	0.0791 (0.9612)	0.2204 (0.8956)	0.2975 (0.8618)
AIC	3.4359	2.5777	1.3973	2.8502	-5.3280	0.2233
SBIC	3.4428	2.5859	1.4055	2.8571	-5.3198	0.2316
Log-Like	-8054	-6040	-3271	-6680	12503	-518

Turning to distribution densities (Tables 6, 7 & 8); the non-normal (Student-t and GED) distributions clearly outperformed the Gaussian. For instance, the log-likelihood function strongly increased when fatter tailed distribution densities were used for both indices. Furthermore, using the non-normal densities of Student-t and GED produced lower AIC and SBIC than the normal distribution. From the preceding evidence, all the three GARCH models performed well with non-normal distribution densities. All models appeared effective by describing the dynamics of the series as shown by the Ljung-Box statistics for the squared standardised residuals with lag 20 which were non- significant at 1 per cent level for both indices. The LM test for the presence of ARCH at lag 2 indicated that conditional heteroscedasticity were removed for all three GARCH models regardless of the distribution density (with the exception of GED-EGARCH) which were all non-significant at standard level.

The comparison between models with each distribution density indicated that, giving the different measures used for modelling volatility, GED-EGARCH provided the best in-sample estimation for both Mauritius and Zambia, clearly outperformed EGARCH with Gaussian and Student-t as well as GARCH and GJR models.

TABLE 10
RANKING PERFORMANCE FORECAST

Gaussian

	MAURITIUS			ZAMBIA		
Model	GARCH	EGARCH	GJR	GARCH	EGARCH	GJR
RMSE	1	3	2	2	1	3
RAE	2	3	1	1	2	3
MAPE	2	3	1	2	3	1
TIC	2	1	3	2	1	3
Total	7	10	7	7	7	10

Student-T

Model	GARCH	EGARCH	GJR	GARCH	EGARCH	GJR
RMSE	1	2	3	1	2	3
RAE	1	3	2	3	2	1
MAPE	1	3	2	3	2	1
TIC	1	2	3	1	2	3
Total	4	10	10	8	8	8

GED

Model	GARCH	EGARCH	GJR	GARCH	EGARCH	GJR
RMSE	3	1	2	1	1	1
RAE	3	1	2	1	1	1
MAPE	3	1	2	1	1	2
TIC	1	3	2	1	1	1
Total	10	6	8	4	4	5

Summary of Best Performing Model

	Mauritius	Zambia
Gaussian	GARCH/GJR	GARCH/EGARCH
Student-t	GARCH	GARCH/EGARCH/GJR
GED	EGARCH	GARCH/EGARCH

Table 10 ranked the GARCH models when evaluated against each other with the introduction of the three different distribution densities for the disturbance term. The evidence in tables 9 and 10 indicated that clearly, no single model completely dominates the other for both indices. However, as Table 11 indicates, the symmetric GARCH model slightly outperformed the asymmetric GARCH models in Mauritius by providing the best out-of-sample forecast, while the GARCH and EGARCH provided the best out-of-sample forecast for the Zambian market. This contradicted the evidence found in Malaysia and Singapore where asymmetric GARCH models clearly outperformed the symmetric GARCH (Nor and Shamiri, 2007).This finding also showed that forecasting with heavy-tailed distribution densities yield no significant reduction of the forecast error than when normal distribution was assumed.

CONCLUSION

Over the last three decades, many academics and analysts have paid particular attention to stock market volatility since it can be used to measure and forecast in a wide range of financial applications, including portfolio selection, value at risk, asset pricing, hedging strategies and option pricing. This paper was aimed at developing a model to forecast the performance of the symmetric GARCH and asymmetric GARCH (i.e. GJR and EGARCH) models with the introduction of different distribution densities.

The statistical results from the symmetric GARCH point towards the fact that in Mauritius, the current shocks to the conditional variance will have less or no impact on future volatility, while in Zambia, the impact of current shocks to the conditional variance on future volatility is either persistent or explosive. The use of EGARCH and GJR with normal and non-normal distribution appears justified to model the asymmetric characteristics of both indices. Generally, the asymmetric coefficients are statistically significant at standard levels for both indices. The evidence also showed that news impact is asymmetric in both stock markets as the asymmetric coefficients for all densities are unequal to zero.

Besides, for both markets, the empirical evidence indicated that the coefficient estimates of the EGARCH were generally positive and statistically significant at standard levels, suggesting that positive instead of negative shocks imply a higher next period conditional variance of the same sign. This meant that negative shocks would have no greater effects on volatility than positive shocks as expected. The evidence from GJR showed that in both markets, negative shocks will generally have a greater impact on future volatility than positive shocks of the same magnitude, confirming the existence of leverage effect.

The comparison between models with each distribution density indicated that, given the different measures used for modelling volatility, GED-EGARCH provided the best in-sample estimation for both Mauritius and Zambia, clearly outperformed EGARCH with Gaussian and Student-t as well as GARCH and GJR models. With respect to forecasting evaluation, the results indicate that clearly, no single model completely dominated the other for both indices.

Finally, there are areas where further studies might be useful. For example, future research should focus on modelling and forecasting GARCH models with high frequency trading (i.e. intra-day) data. Further research should also consider exploring variety of models including other conditional variance models such as APARCH and long memory models such as FIEGARCH, FIAPARCH and CGARCH in order to allow a greater insight into the dynamics of these two markets. Lastly, similar study should be conducted in other African stock markets in order to provide a wider insight into the relevance of GARCH models in financial application in Africa frontiers.

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