Information Communication Through Distorted Earnings Reporting

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In this paper we study how discretionary managerial reports can be used for information communication about the firm’s value. Equilibrium reporting strategy and investment level are derived for strategic investor (i.e., VC), and competitive investor (i.e., IPO) market settings. It is shown that when direct truthful information communication is unreliable because of incentive misalignments, costly discretionary biased reporting still allows perfect information communication. Comparative static shows that the level of earnings management and amount of capital invested differs in these two cases. Our model justifies the existence of lock up period during IPO and provides some arguments supporting a pattern of VC-IPO sequence since getting initial financing from strategic investor reduces the cost of signaling for the firm during subsequent IPO procedure. Presented analysis generates several empirical implications regarding strategic choices of earnings management depending on the investment structure.

INTRODUCTION

The issue of managerial discretion in the earnings reporting is a controversial one. On the one hand, there is a widely spread point of view that the discretion leads to manipulation. On the other hand, reporting discretion allows managers to provide some additional information to investors and other interested parties. This paper shows that there is no contradiction between the two premises. Specifically, we show that even biased reports can be used as a perfect means of information communication. Our results highlight the importance of allowing reporting discretion for improved information communication through earnings reports. In equilibrium, the manager uses reporting discretion to communicate private information, thus increasing the information content of reported earnings.

We examine information communication using a model in which a manager privately observes firm’s earning capacity, which is non-verifiable. After receiving the information, the manager releases an earnings report. An investor observes the report and determines the amount of money to be invested. Both the manager’s and the investor’s payoffs depend on the investor’s action, i.e., the amount invested. The direct true reporting leads to the manager’s incentives misalignment since the manager prefers to report in a manner that upwardly biases the investor’s beliefs about the firm’s profitability. If the discretionary reporting, though costly, is allowed, the investor rationally anticipates the direction and the degree of bias in reporting. In equilibrium, the manager selects the optimal bias to maximize her payoff, based on her rational conjecture about the investor’s best response to her strategy.

Section 2 describes the model. We consider communication game involving a privately informed manager and different types of rational investors: strategic investor (VC) in Section 3, and competitive investors (IPO) in Section 4. In section 5, we provide comparative statics for the strategic investor and the competitive investor model settings; the socially optimal level of investment and the value of deadweight
losses are derived and compared for both cases. We consider the calibration of the models in Section 6. In section 7, we determine the optimal retention level for manager, and show that the presence of firm’s own capital reduces the cost of signaling for the firm during an IPO. We summarize the paper and discuss its implications in Section 8.

THE MODEL

In this paper we model a communication game involving a privately informed manager and a rational investor. The manager is issuing an earnings report and the investor is choosing the optimal investment level based on the manager’s report. Before entering the game, the manager privately observes the true firm’s earnings capacity. There are two motives which influence the manager’s behavior. The manager is motivated by the desire to receive investment inflow into his company, therefore he is interested in the investor’s perceiving his company as a profitable one. However, higher reported earnings capacity, which is associated with higher future profitability, implies higher signaling costs levied on the company. Investor makes his decision about investment amount based on the manager’s report. The investor cannot rule out the possibility that the report is biased, therefore he has to estimate possible bias endogenously. After the investment is made, the company generates output according to the Cobb-Douglas production function.

\[ F = \alpha C_S^\gamma \cdot I^{1-\gamma} \]  

where \( I \geq 0 \) is an investment; \( C_S \geq 0 \) is the true firm’s earnings capacity, \( \alpha > 0 \) and \( \gamma \in (0; 1) \) are exogeneous parameters which are common knowledge. Parameter \( \alpha \) defines the scale of the company. Parameter \( \gamma \) defines relative importance of the production factors in the production function. The firm’s output is divided between the manager and the investor.

In the context of our model, \( C_S \) stands for quantity of a particular production factor, which is earned by the firm previously, but is not observable by outsiders.

Cobb-Douglas productivity function is a standard assumption in economic literature. The firm’s production function depends nonlinearly on two production factors, nonmonetary capacity (patents, know-how, contracts with customers etc.) and money invested. Both of these parameters should be balanced in order to reach optimal level of production. Too much of any of these factors increases firm’s production only marginally. This fact ensures that as far as the signal is costly, the manager will never ask for infinite investment and efficient level of investment could be reached.

In such settings the investor’s profit is given by

\[ \pi_I = kF(C_S, I) - I \cdot (1 + r) \]  

where \( k > 0 \) is the investor’s share in firm’s output and \( r \geq 0 \) is the interest rate.

The manager’s profit is given by

\[ \pi_M = (1 - k)F(C_S, I) - \tau \cdot C_R \]  

where \( C_R \) is reported earnings, and \( \tau \) is the cost associated with reporting of these earnings. The parameter \( \tau \) could present the level of costs for finding new clients (if the number of clients is used as a signal about firm capacity), or spending resources for advertisement, or costs for new equipment, contracts or R & D.

STRATEGIC INVESTOR: GENERAL SETTING

In this section we consider information communication between privately informed manager and strategic investor. The manager has private information concerning the firm’s earnings capacity and is seeking for investment in the firm in order to maximize his profit (which is proportional to the firm's profit).
Strategic investor’s objective function is to maximize his profit from the investment in the company knowing that he is the only one investor and that for evaluating the firm’s future performance he could use biased signal from the manager.

There may be a variety of reasons why manager is looking for strategic vs. competitive investor even though, as will be shown in the next section, in the case of IPO she could earn larger profit than in the case of VC. For example, on this stage of the firm development, manager does not want to share any private information especially about current firm’s earnings capacity with a broad number of potential investors looking instead for attracting one investor as a main source of necessary investment. Venture capitalist or angel capitalist could be such a source. The case when a firm seeks for strategic investor’s capital and after that goes public, and implication of such strategy for the level of investments obtained as well as on the cost of signaling is considered in section (7.2). It is shown that presence of strategic investor’s capital in firm’s capital structure reduces the cost of signaling at the IPO stage.

Now let us consider general case of Cobb-Douglas production function described in (1). Let \( k \) be the share of the firm’s output, claimed by the investor, and let \( r \) be the interest rate, i.e., the opportunity cost of capital.

There are two choice variables available in our model: parameter \( k \) characterizing a level of manager’s retention of the company (manager gets \((1 - k)\) share of the firm’s output), and earnings. There is a lot of literature where earnings reported, or level of retention, or even both of these signals simultaneously, are used in order to convey the information regarding the firm’s type\(^1\).

In our model we use the following signaling pattern. Manager decides first which part of the company will be sold to outside investors. Next, having the number of shares for sale, e.g., parameter \( k \), already determined, manager makes a decision regarding the amount of earnings reported in order to maximize his profit. He uses this earnings reported as a signal to investor(s) about quality of the firm. We think this pattern well reflects a real-life situation since the level of retention is a more rigid parameter, and it is announced by the manager earlier. We suggest that making a decision regarding the retention level, manager knows the true earnings capacity of his firm. Therefore, he solves his maximization problem using backward induction, e.g., taking retention level as given (already preannounced) and choosing the earnings to report in a manner to maximize his profit, and knowing that investors can redo this backward induction reasoning as well in order to determine the optimal investment level.

Before proceeding with analysis of the manager’s problem, we determine the investor’s optimal investment level when there is no asymmetric information. If the investor knows the true earnings generating capacity \( C_S \), then the optimal investment level is a solution to the following maximization problem:

**The Strategic Investor’s Problem**

\[
\max_{I} \pi_I = \max_{I} \left[ k \cdot (\alpha \cdot C_S \cdot r^{1-\gamma} - r) \right] \tag{4}
\]

subject to \((PC): [k \cdot (\alpha \cdot C_S \cdot r^{1-\gamma}) - r] \geq 0 \text{ and } I \geq 0 \tag{5}\)

The participation constraint implies that investor expects from the project non-negative present value.

We determine optimal investment level in Lemma 1. This result serves as a benchmark for the subsequent analysis.

**Theorem 1** If the company’s true earnings capacity \( C_S \) is perfectly observed by investor, then optimal investment level is given by

\[
I^* = \left( \frac{(1-\gamma)\alpha k}{1+r} \right)^{1/\gamma} \cdot C_S \tag{6}
\]
Proof

All proofs are in the Appendix.

The Manager’s Problem (SM)

Given investor’s strategy \( \tilde{I}(C_R) \) as a function of reported value \( C_R \), the manager’s maximization problem becomes:

\[
\max_{C_R} \pi_M = \max_{C_R} [(1 - k) \cdot \alpha \cdot C_S^Y \cdot \tilde{I}(C_R)^{1-\gamma} - \tau \cdot C_R] 
\]  

subject to (PC): \[(1 - k) \cdot \alpha \cdot C_S^Y \cdot \tilde{I}(C_R)^{1-\gamma} - \tau \cdot C_R \geq 0 \]  

In the equilibrium with information communication, we must have: \( \tilde{I}(C_R) = I^*(C_S) \), i.e. the equilibrium investment level based on the biased report \( C_R \) is equal to the optimal investment corresponding to \( C_S \).

Theorem 2 In the discretionary reporting game between privately informed manager and strategic investor there exists a unique equilibrium allowing information communication. This equilibrium is given by the following strategies profile:

\[
\tilde{I}^S(C_R) = \frac{k}{1-k} \cdot \frac{\tau}{1+r} \cdot C_R^S; \quad \tilde{C}_R^S(C_S) = (1 - \gamma)^{\frac{1}{\gamma}}(1 - k) \left( \frac{k}{1+r} \right)^{\frac{1-\gamma}{\gamma}} \cdot C_S 
\]  

Lemma 1 In the equilibrium strategic investor’s profit is:

\[
\pi^S_I = C_S \cdot \left[ \alpha k \left( \frac{(1-\gamma)ak}{1+r} \right)^{\frac{1-\gamma}{\gamma}} - (1 + r) \left( \frac{(1-\gamma)ak}{1+r} \right)^{\frac{1}{\gamma}} \right] 
\]  

and manager’s (SM) profit is

\[
\pi^{SM}_M = C_S \cdot \left[ \alpha (1 - k) \left( \frac{(1-\gamma)ak}{1+r} \right)^{\frac{1-\gamma}{\gamma}} - (1 + r) \frac{1-k}{k} \left( \frac{(1-\gamma)ak}{1+r} \right)^{\frac{1}{\gamma}} \right] 
\]  

The parameter \( k \) is determined endogenously in the model, but it is not used as a signal. This is because optimally chosen \( k \) is the same for all values of \( C_S \), as is shown below. Manager decides first which part of the company he will sell, and after that he chooses the level of earnings reported as a signal to investor. Let us consider for which value of parameter \( k \) manager’s profit is maximal. The manager maximizing his profit with respect to parameter \( k \) would face the following maximization problem:

\[
\max_k \pi_M = \max_k [(1 - k) \cdot \alpha C_S^Y \cdot \tilde{I}(C_R)^{1-\gamma} - \tau C_R] 
\]  

The solution to the manager’s maximization problem is presented in the following theorem:

Theorem 3 The optimal number of shares to be sold to the strategic investor is a decreasing function of the parameter \( \gamma \) of the production function and does not depend on any other parameters of the model:

\[
k = 1 - \gamma
\]
The intuition behind the result from Theorem 3 is the following: the larger weight $\gamma$ of the firm’s output is generated by the firm’s earnings capacity, the less desirable investments are; and, consequently, the smaller part of the firm will be sold to the outside investor.

**Lemma 2** Truthful reporting $C_R = C_S$ is not the manager’s best response to the (derived in Theorem 1) investor’s strategy $I^* = \left(\frac{(1-\gamma)ak}{1+r}\right)^\frac{1}{\gamma} \cdot C_S$, unless $\tau = \frac{1-k}{k} \cdot (1+r) \cdot \left(\frac{(1-\gamma)ak}{1+r}\right)^\frac{1}{\gamma}$.

**COMPETITIVE INVESTORS: GENERAL SETTING**

In the previous section we examined the behavior of a strategic investor based on a manager’s discretionary earnings report. Now let us consider behavior of a pool of competitive rational outside investors during an IPO, where investment decisions are based on a manager’s report. Each competitive investor observes the manager’s report and the price to be paid for one share. Demand of the pull of competitive investors is perfectly elastic. The shares are normalized so that the whole company consists of 1 infinitely divisible share, from which $(1-k)$ is retained by management and $k$ is for sale to outside investors. Each investor, making investment $I$, gets $\left(\frac{I}{I_T}\right)k$ share of the company, where $I_T$ is the total amount invested. This implies that the price is $I / \left(\frac{I}{I_T}\right) = \frac{I_T}{k}$ per share. For the purpose of this paper one can think of a competitive investor as an individual who estimates the fair price and buys shares if actual price is equal or below his estimation. Each competitive investor solves the following problem:

*The Competitive Investor’s Problem*

$$\max_{I_T} \pi_I = \max I \left[k \left(\frac{I}{I_T}\right) \alpha \cdot C_S^\gamma \cdot I_T^{(1-\gamma)} - I \cdot (1+r)\right]$$  \hspace{1cm} (14)

subject to $(PC): \left[k \left(\frac{I}{I_T}\right) \alpha \cdot C_S^\gamma \cdot I_T^{(1-\gamma)} - I \cdot (1+r)\right] \geq 0$ and $I \geq 0$  \hspace{1cm} (15)

where, as before, parameter $k \in (0,1)$ defines the part of the company to be sold, $\alpha > 0$ and $\gamma \in (0; 1)$ are arbitrary industry-specific and publicly known parameters of production function (1), and $r$ is the interest rate. The competitive investor’s profit is perceived as a linear function of his investment amount $I$. Such function has interior optimum only if its slope is identically equal to 0. This yields to the competitive investor’s profit equal to 0.

**Theorem 4** If the company’s true earnings capacity $C_S$ is perfectly observed by competitive investor then optimal investment level is given by

$$I_T^* = \left(\frac{ak}{1+r}\right)^\frac{1}{\gamma} \cdot C_S$$  \hspace{1cm} (16)

Therefore, if $I_T < \left(\frac{ak}{1+r}\right)^\frac{1}{\gamma} \cdot C_S$, the investors would like to invest without any restrictions. If $I_T > \left(\frac{ak}{1+r}\right)^\frac{1}{\gamma} \cdot C_S$, the investors would decide not to invest at all. If $I_T = \left(\frac{ak}{1+r}\right)^\frac{1}{\gamma} \cdot C_S$, the investors agree to buy some finite amount of shares. For manager it is optimal to attract as much investment as possible. However, if signaling is costly, competitive investors discover the true $C_S$.

We would also like to mention here that the amount of total investment $I_T$ invested by competitive investors is always greater than the amount of total investment $I$ invested by strategic investor. This result arises from the comparison of formulas (6) and (16) and taking into consideration the fact that parameter
\(\gamma \in (0; 1)\). We provide more results and more intuition for relationship between levels of investments, manager’s profit and earnings reported for strategic investor and competitive investor model settings in section 5.

**The Manager’s Problem (CM)**

Analogically to the model of strategic investor, in the competitive investor case the manager is allowed some discretion in the reporting. Given competitive investor’s strategy \(\hat{I}(C_R)\) as a function of reported value \(C_R\), the manager’s optimization problem becomes:

\[
\max_{C_R} \pi_M = \max_{C_R} \left[ (1 - k) \cdot \alpha C_S^\gamma \cdot \hat{I}(C_R)^{1-\gamma} - \tau C_R \right]
\]

(17)

Since in the informative equilibrium we have \(\hat{I}(C_R) = I_T^*\), where \(I_T^*\) is defined in (16), we can state that

**Theorem 5** In the discretionary reporting game between privately informed manager and competitive investor there exists a unique equilibrium with information communication. This equilibrium is given by the following strategies profile:

\[
I(C_R)^{CI} = \frac{1}{1-\gamma} \frac{k}{1-k} \cdot \frac{r}{1+r} \cdot C_R^{CI}; \quad C_R^{CI} = (1 - \gamma)(1 - k) \frac{1}{\tau} \left( \frac{k}{1+r} \right)^{1-\gamma} \cdot C_S
\]

(18)

**Lemma 3** In the equilibrium competitive investor’s profit is:

\[
\pi_I^{CI} = 0
\]

(19)

and manager’s (CI) profit is:

\[
\pi_M^{CI} = (1 - k)\gamma \alpha \frac{1}{\tau} \frac{k}{1+r} \cdot C_S
\]

(20)

Calculating the last step of backward induction process gives us the solution to the manager’s maximization problem with respect to parameter \(k\). The result is presented in the following theorem:

**Theorem 6** The optimal number of shares to be sold to the competitive investor is a decreasing function of the parameter \(\gamma\) of the production function and does not depend on any other parameters of the model:

\[
k = 1 - \gamma
\]

(21)

This implies that the optimal stake of the firm for sale to outside investors is always equal to \(1 - \gamma\). Therefore, the manager’s decision regarding the size of the firm’s stake for sale is not informative to the investor. Thus information communication takes place only through \(C_R\), but not through \(k\).

**Lemma 4** Truthful reporting \(C_R = C_S\) is not manager’s best response to the (derived from Theorem 4) investor’s strategy \(I^* = \left( \frac{ak}{1+r} \right)^{1/\gamma} \cdot C_S\), unless \(\tau = (1 - \gamma) \frac{1-k}{k} \cdot (1+r) \cdot \left( \frac{ak}{1+r} \right)^{1/\gamma} \cdot C_S\).
COMPARATIVE STATICS FOR THE STRATEGIC INVESTOR AND COMPETITIVE INVESTOR MODEL SETTINGS

We have considered the communication game between privately informed manager and investor for two different types of investors: strategic investor and competitive investor. It is shown that efficient information communication in equilibrium is reached in both cases, but the strategies played by manager and investor are different for these two model settings.

Let us denote $\pi_M^{VC}$, $C_R^{VC}$, and $I^{VC}$ manager’s profit, reported earnings and obtained investment for this strategic investor model setting, and $\pi_M^{IPO}$, $C_R^{IPO}$, and $I^{IPO}$ manager’s profit, reported earnings and obtained investment for the competitive investor’s model setting respectively.

Next let us consider the question for which setting (competitive vs. strategic model setting) manager’s profit/reported earnings/investment obtained are higher. The answer to this question is stated in the next theorem.

**Theorem 7 1.** Manager’s profit $\pi_M^{VC}$ in the case in the case when a part of the company is sold to the strategic investor is always less than manager’s profit $\pi_M^{IPO}$ in the case when a part of the company is sold to the competitive investors. The ratio of these two profits is fully determined by exogenous parameter $\gamma$ (Fig.1, $F(\gamma)$ graph):

$$\frac{\pi_M^{VC}}{\pi_M^{IPO}} = (1 - \gamma)^{\frac{1}{\gamma}}, \quad \text{or} \quad \pi_M^{VC} < \pi_M^{IPO} \quad \forall \gamma$$

2. The level of the reported earnings $C_R^{VC}$ in the case when a part of the company is sold to the strategic investor is always less than the level of the reported earnings $C_R^{IPO}$ in the case when a part of the company is sold to the competitive investors. The ratio of these two values is fully determined by exogenous parameter $\gamma$ (Fig.1, $F(\gamma)$ graph):

$$\frac{C_R^{VC}}{C_R^{IPO}} = (1 - \gamma)^{\frac{1}{\gamma}}, \quad \text{or} \quad C_R^{VC} < C_R^{IPO} \quad \forall \gamma$$

3. Investment obtained by the company in the case when a part of the company is sold to the strategic investor $I^{VC}$ is always less than investment obtained by the company in the case when a part of the company is sold to the competitive investors $I^{IPO}$. The ratio of these investments is fully determined by exogenous parameter $\gamma$ (Fig.1, $F(\gamma)$ graph):

$$\frac{I^{VC}}{I^{IPO}} = (1 - \gamma)^{\frac{1}{\gamma}}, \quad \text{or} \quad I^{VC} < I^{IPO} \quad \forall \gamma$$
Therefore, the firm always attracts more investments, spending more on signalling, from competitive investors than from strategic investor. The question is if these investments are socially optimal and what are, if any, deadweight losses associated with these two regimes?

In order to answer these questions let us consider socially efficient investments $I_{OPT}$ as a function of exogenous parameters $\alpha$, $\gamma$ and interest rate $r$. This means that these investments will maximize the whole production profit of the firm taking into consideration all possible expenses:

$$I_{OPT} = \max_i [\alpha C_S \gamma \frac{I^{1-\gamma}}{1+r} - (1+r)I]$$

(25)

There is no parameter $k$ here, since there is no difference between manager’s and investor’s shares in the firm production from the point of view of social efficiency.

**Theorem 8** 1. Socially optimal level of investments is given by

$$I_{OPT} = \left(\frac{(1-\gamma)\alpha}{1+r}\right)^{\frac{1}{\gamma}} \cdot C_S$$

(26)

**Theorem 9** Strategic investor in equilibrium always makes less investments in the firm than is optimal. These investment levels are related as follows (Fig.1, $G(\gamma)$ graph):

$$I_{SI}^* = (1-\gamma)^{\frac{1}{\gamma}} \cdot I_{OPT}, \quad i.e. \quad I_{SI}^* < I_{OPT} \quad \forall \gamma$$

(27)

Investment level chosen by competitive investors is always socially optimal, i.e.

$$I_{CI}^* = I_{OPT} \quad \forall \gamma$$

(28)
Deadweight losses for strategic investor setting are determined as the difference between profit of the firm with socially optimal investments and profit of the firm with investments provided by strategic investor, plus signaling expenses:

\[ DWL_{SI} = \pi(I_{OPT}) - \pi(I_{SI}) + \tau C_{R}^{SI} \] (29)

Similarly, deadweight losses for competitive investors setting are determined as the difference between profit of the firm with socially optimal investments and profit of the firm with investments provided by competitive investors, plus signaling expenses:

\[ DWL_{CI} = \pi(I_{OPT}) - \pi(I_{CI}) + \tau C_{R}^{CI} \] (30)

**Theorem 10** (Fig. 2) 1. If \( \gamma = \frac{1}{2} \), deadweight losses for strategic investor model setting equals deadweight losses for competitive investors model setting.

2. If \( \gamma < \frac{1}{2} \), deadweight losses for strategic investor model setting are less than deadweight losses for competitive investors model setting.

3. If \( \gamma > \frac{1}{2} \), deadweight losses for strategic investor model setting are bigger than deadweight losses for competitive investors model setting.

**FIGURE 2**

![Graph showing the comparison of deadweight losses for strategic and competitive investors model settings.]

**CALIBRATION OF THE MODELS**

Douglas production function obtained in various empirical research in economic literature (list of literature here). Let interest rate parameter be equal 5%, or \( r = 0.05 \). There is a quite homogeneous approximation for the value of parameter \( \gamma \), and \( \gamma = 0.7 \) (insert literature here). Consequently, since \( \gamma = 0.7 \), results obtained for general cases and stated in Theorem 2 and Theorem 6 imply that \( k = 1 - \gamma = 0.3 \),
so the output of the company is split between the manager and the investor in proportion 7 : 3. The evaluation of parameter \( a \) (the measure for technical change) is approximated by the 3.6.

First of all, let us stress out here that the value of parameter \( k = 1 - \gamma = 0.3 \), theoretically obtained in Theorem (6) for the IPO model setting, is almost identical to the empirical results for the public float during an IPO. Thus, in J. Ritter [2003] database a mean for the parameter \( k \) for years 1992-2002 equals 30.9%. Habib and Ljungqvist [2001] provide the following estimation for the mean of parameter \( k \) for years 1991-1995: \( k = 0.32\% \).

Our next observation relates to time-series change of parameter \( \gamma \). As it follows from empirical economic literature, see for example Fraser [2002] with data for USA firms, Konishi and Nishiyama [2001] with data for Japanese firms, parameter \( \gamma \) fluctuates dramatically over years, with a tendency to grow over time. Thus, for example, \( \gamma \) was less than \( \frac{1}{2} \) before 1986, and it grows from \( \frac{1}{2} \) to \( \frac{3}{4} \) from 1986 to 2001. Additionally, as follows from Ritter [2003] data, the number of IPOs grows dramatically through these years as well. Total number of initial public offerings was 112 for 1975-1979 years, 2392 for 1980-89 years, and 4145 for 1990-99 years. From the other hand, as was shown in the Theorem (10), deadweight losses for strategic investor model setting (VC) are bigger than deadweight losses for competitive investors (IPO) model setting if parameter \( \gamma > \frac{1}{2} \). Thus, linking the analytical results from the Theorem 10 with empirical data we can conclude that the increasing in the number of IPOs can be attributed to changes in economic environment captured by parameter \( \gamma \). I.e. For “old” economy venture capitalist, as a source for new investment in the company, is more efficient. For “new”, intangible-intensive economy IPO is economically efficient form of attracting new investment.

Finally, let us present some numerical outcomes for the presented above calibration.

**Proposition 1** Given exogenous parameters of the model as follows: \( \gamma = 0.7 \), and \( r = 0.05 \), the ratios of manager’s profit, reported earnings and obtained investment for the strategic investor model setting to manager’s profit, reported earnings and obtained investment for competitive investor model setting equals:

\[
\frac{\pi_{SI}}{\pi_{CT}} = 0.6 \quad \frac{c_{RSI}}{c_{ReT}} = 0.6 \quad \frac{\iota_{SI}}{\iota_{CT}} = 0.18
\]

(31)

Proposition 1 provides more testable implications of the model which however are contingent on availability of proprietary data related to private agreement between strategic investor (VC) and financed firm.

**LEVEL OF RETENTION AND INFLUENCE OF INITIAL CAPITAL FOR IPO MODEL SETTINGS**

**Qualitative Components of Retention for IPO Model Settings**

What we have considered so far in our model as a parameter \( k \) was the investor’s share in firm’s output. All investments were used directly by the firm, none part of these investments were used directly by manager. Manager’s profit depended on the level of investments only since these investments produced an output in the firm’s production function.

Now let us consider the following question. What will happen if during an IPO manager will be permitted to sell some part of his own shares as well as issue new ones for sale to the investor? What part of his own shares if any will he sell to the outside investors in order to maximize his profit?

Let us denote \( N \) an initial number of stocks. \( N_1 \) is the number of stocks to be sold to the outside investors by manager, this means that the amount earned from selling these \( N_1 \) shares goes into manager’s pocket, not to the firm. Finally, \( N_2 \) represents the number of newly issued stocks, and the amount earned from selling these \( N_2 \) additional shares goes directly to the firm. \( \iota \) denotes investment made by particular investor.

\[ N = N_1 + N_2 \]
$I_T$ represents the total amount of investments. Some part of these investments, namely $N_1 \frac{I_T}{N_1+N_2}$ goes to manager, and the rest of these investments, $N_2 \frac{I_T}{N_1+N_2}$ goes to the company.

Under such designation, the manager’s profit can be written as:

$$\pi_M = N_1 \cdot \frac{I_T}{N_1+N_2} \cdot (1+r) + \alpha C_S \gamma \cdot \left( I_T - \frac{N_1}{N_1+N_2} I_T \right)^{1-\gamma} \left( \frac{N-N_1}{N+N_2} \right) - \tau C_R \quad (32)$$

The first term in equation (32) represents the amount obtained by manager from selling his own $N_1$ stocks to the investors. The second term measures the manager’s share in firm’s output after $N_1 + N_2$ shares are sold and investments are made. The last term reflects the signalling costs paid by company prior to selling stocks in order to communicate firm’s earnings capacity to the prospective investors.

**The Manager’s Problem** can be written as

$$\max_{C_R,N_1,N_2} \pi_M = \max_{C_R,N_1,N_2} \left[ \max_{N_1} \left[ \alpha C_S \gamma \left( I_T - \frac{N_1}{N_1+N_2} I_T \right)^{1-\gamma} \left( \frac{N-N_1}{N+N_2} \right) - \tau C_R \right] \right] \quad (33)$$

**Each Competitive Investor Solves:**

$$\max_{I} \pi_I = \max_{I} \left[ \alpha C_S \gamma \left( I_T - \frac{N_1}{N_1+N_2} I_T \right)^{1-\gamma} \cdot \frac{I}{I_T} \cdot \frac{N_1+N_2}{N+N_2} - (1+r) I \right] \quad (34)$$

**Theorem 11** If the manager could choose both, which number of his own shares $N_1$ out of $N$ shares available to sell, and how many new shares $N_2$ to issue for sale for outside investors, he would never sell his own shares. The only nontrivial equilibrium is given by:

$$N_1 = 0 \quad (35)$$

and

$$N_2 = \frac{1-\gamma}{\gamma} N \quad (36)$$

or, stated in terms of parameter $k$, investor’s share in the firm’s output is given by:

$$k = 1 - \gamma \quad (37)$$

Summarizing, given the information asymmetry about the firm’s capacity, in the equilibrium, manager will not sell his own shares, but only sell new issued shares. This result theoretically supports lock-up period. Manager will voluntarily choose not to sell his own shares since, for example, if he decides to sell all of his shares this will brake down the equilibrium, what will result in zero investments.

**Influence of Initial Capital on Additional Investment for IPO Model Settings**

What we have considered so far were two one period models: one for strategic investor and another one for competitive investors settings. In both these settings we assumed that the firm was endowed with some earnings capacity, but did not have own capital. But how the firm’s and competitive investors’ strategies would change if the firm has some capital, what is typically the case during an IPO, for example? We show below that the presence of own capital in firm’s capital structure reduces the cost of signaling therefore making the attracted investments less expensive.

Let us denote the firm’s own capital as $I_0$. **The Manager’s Problem** can be written as
\[ \max_{C_R} \pi_M = \max_{C_R} [(1 - r) \alpha C_S \gamma (I_0 + I_T)^{1 - \gamma} - \tau C_R] \]  

and Each Competitive Investor solves

\[ \max_{I} \pi_I = \max_{I} [k \alpha \frac{1}{I_T} C_S \gamma (I_0 + I_T)^{1 - \gamma} - (1 + r) I] \]  

where \( \text{const} = \frac{\tau k}{(1-k)(1-\gamma)(1+r)} \) and \( I_T \) denotes the first derivative of the total investments provided to the firm by competitive investors with respect to earnings reported \( C_R \).

Therefore, if \( I_0 \) equals to zero, we have that \( I_T = \frac{\tau k}{(1-k)(1-\gamma)(1+r)} \cdot C_R \), the same result as was obtained in Theorem 5 for competitive investor model setting without initial capital. But if \( I_0 > 0 \), then derivative of total investments \( I_T \) with respect to reported earnings \( C_R \) is increasing function of initial capital \( I_0 \). (Fig.3)

This means that the firm with its own initial capital \( I_0 \) faces smaller costs of signalling, i.e. it is able to attend the same investment with smaller reporting costs.

**CONCLUSION**

We have analyzed the distortion in the discretionary reports caused by the incentive misalignment. We model an investment decision process where a rational investor makes his decision based on earnings report realized by a rational manager. We incorporate three key features of the reporting environment. First, the manager is endowed with relevant private information. Second, the manager is allowed to bias reported earnings. Third, both the investment and the cost of signaling depend on the reported earnings. We then examine the information communication process through earnings announced. We derive an equilibrium where the manager overstates or understates actual earnings capacity depending on the existing cost function rate. We have shown that discretionary reporting with costly signals may assure perfect information communication where the direct revelation mechanisms do not work due to the incentive incompatibility. Overall, our result reveals that, in our setting, there is an informational advantage to allowing reporting discretion when the manager is endowed with private information.

The model provides testable implication. It suggests that while accounting earnings should have high association with stock returns (because of information content), earnings themselves are not good proxies for asset returns because of bias in reporting.

There are several extensions to this model that may further our understanding of strategic information communication in accounting. One such extension would allow the parties to negotiate different contracting arrangements, leading to more socially efficient outcome. Examining the optimal contracting based on the communication of private information might provide a better understanding of the role of accounting in the optimal contracting and suggest accounting standard setting implications for making such contracting more efficient. Another extension would be to introduce uncertainty of the production outcome. Also, the investor could be perceived as a bank providing a risky loan to the firm. Such an analysis would shed light upon the limits of the credit lines imposed by banks basing on the accounting data provided by firms.

**ENDNOTES**

2. Public float is determined as a number of shares issued (global offering) to post-issue shares to be outstanding.