The (M, B, Q)Financial Models

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The financial disasters that have occurred in the past will continue to occur if managers do not develop methodological tools to address them. This shows that there is no remedy to control the complexity of the volatile behavior of the economic financial systems nor with the rapid development of technology.
Here, we propose the $(\mathrm{M}, \mathrm{B}, \mathrm{Q})$ *(financial, mathematical, quantum-physic (FMQ)) models to alert, prevent, pacify these random phenomenons. This financial model gives birth a family of triadic models of order* (p,q,r) *when* $p,q,r \in \square_{+} < \infty$. *In particular, we apply the (6,6,6) model in the capital markets and we give financial interpretations to its results.*

INTRODUCTION

The financial crisis that happened in the past[1] brought as consequence the achievement of many studies and investigations during and after the occurrence of every financial crisis concluding that the high volatile nature of the markets is directly related to the strong interrelation between them (Sandoval, 2010). Nevertheless, in spite of these numerous researches one still has not found the perfect tool to remedy these financial collapses.

Why does this happen?. We believe that, these catastrophes happened due to the lack of knowledgement of methodologies of help for the financial local, national and international to mitigate these contagious[2] emergencies of financial bankrupticy provoked by epidemic[3] stochastic complex phenomenon explained by a set of families of random variables. To face these catastrophes that left well-known bleaknesses, panic, economic instability, unemployment, between other effects in the humanity, we present the development and application of this novel FMQ methodology of financial utility which explains the conduct of the financial markets or the behavior of any volatile phenomenon on an ambience different from space and time to the already existing models.

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[1] Concrete facts of these world wide problems of financial unforeseeable and uncontrollable collapses already happened in different countries and continents of the planet from 1929 until 2008. Since there reminds to itself, on the Black Monday of 1987, the Russian crisis of 1998, the explosion of the Bubble Dot Com of 2001 and the more recent mortgage crisis of 2008 in the USA and the current ones of the European Union.

[2] King M. and Wadhwanni S. (1989) *Transmission of volatility between stock markets*, National Bureau of Economic Research, Paper Series 2910.

[3] Mathieu Mosolonka-Lefebvre, “Epidemics in markets with trade friction and imperfect transactions”, (Oct, 2013), arXiv:1310.6320V1[q-bio.PE].

The procedure to construct the $(\mathcal{M},\mathcal{B},\mathcal{Q})$ financial markets models consists, roughly speaking, of enclosing three conceptual ingredients of three scientific disciplines: financial markets $(\mathcal{M})$ from finance, fibre bundles $(\mathcal{B})$ from mathematics, and quantization $(\mathcal{Q})$ from quantum physics. This stage of integration of these disciplines called *finanphysics* might be understood, to grosso way, like a set of financial markets (the **actors** of de model) or a set of assets in every financial markets; the physical underlying ambience structured by the topological fibre bundle space (the **theater** of the model), like Wall Street building or NASDAQ, where the financial operations are executed subordinated by laws of exchange; and a set of abstract family of algebras (the **soul** of the model) that describe the evolutionary process of the quantization that they provoke and they generate dynamic activities of the financial markets due to the rate of change of state of their assets in the time.
Concretely, the FMQ ambience of integration of these three fundamental concepts are defined by three sixtuples

$$\mathcal{M} = \langle \mathrm{E}, \mathrm{F}, \tau, \Theta, \mathrm{X}, \mathrm{S}(0) \rangle \tag{1}$$

$$\mathcal{B} = \langle \mathrm{E}, \pi, B, \mathcal{F}, \mathrm{G}, \{\phi_j, V_j\} \rangle \tag{2}$$

$$\mathcal{Q} = \langle (\mathcal{M}, w), (C^\infty(\mathcal{M}), \{.,.\}), (\mathcal{M}', w'), (\mathrm{A}_\hbar, \mathring{a}_\hbar), \mathcal{A}, \Phi \rangle \tag{3}$$

Where the components of $\mathcal{M}$ are the probability space $\mathrm{E} = (\Omega, \mathcal{F}, P)$ that represents the underlying foundation of the stochastic process $\mathrm{X}(u,t)$, where $u \in \Omega$, and it gives us the information of the space of states of the financial markets in the time t at the occurrence of the event u and F is a filtration. On the other hand, $\mathcal{A}$ is an associative noncommutative algebra that has its origin in the algebra of continuos functions $C^\infty(\mathcal{M})$ over a Poisson manifold $\mathcal{M}$ and it is identified as a quantization of a classic mechanics $(\mathcal{M}, w)$ and[4] that belongs to the family $(\mathrm{A}_\hbar, \mathring{a}_\hbar)$ of deformed algebras on the quantum space $(\mathcal{M}', w')$, where $\hbar$ is the parameter of deformation and $\mathring{a}_\hbar$ is a noncommutative product on $C^\infty(\mathcal{M})[[\hbar]]$. The algebras $\mathcal{A}$ and $C^\infty(\mathcal{M})$ both are related routes the homomorphism Φ and amaizingly this relates to the composite rate $\tau=(\Gamma,\Xi,\Delta)$ of the financial markets where Γ, Ξ and Δ are the rate of the money market free of risk, the average rate of return, and the rate of dividends of the market process, respectively. Also Θ denotes the volatile nature of the financial markets, fluctuaring in the time. The components of $\mathcal{B}$ are known in the mathematical literature (Steenrod, 1999), the rest of the notations are given on the way of development of the model.

One of the main purposes of this study should be theoretically to formalize, by means of axioms and postulates, the interactions between these eighteen components and give them economic, financial meaning and from accounting point of view. Here we focus on two goals: The first, to develop the $(\mathcal{M},\mathcal{B},\mathcal{Q})$ financial models for capital markets and confront to its volatile behavior Θ over both configuration spaces, classic mechanics $(\mathcal{M}, w)$ and the quantum mechanics $(\mathcal{M}', w')$; the second one, to generalize these models to which we call triadic models of order (p,q,r), where $p,q,r \in \square_+ < \infty$ and to establish future research.

[4] Where w is a skew-symmetric tensor field on this Poisson manifold $\mathcal{M}$. However, this Poisson manifold can be some differentiable manifold (Berezin, 1975) and $C^\infty(\mathcal{M})$ is a set of differentiable functions on $\mathcal{M}$.

LITERATURE REVIEW AND DEVELOPMENT OF THE MODEL

Considering the equations (1), (2) and (3) we focus the eighteen components and list in combinatorial representation in such a way taking one component at the time of every six fold we have $\left(\binom{6}{1}\right)^3$ ways of relating these components of the three conceptual ingredientes $(\mathcal{M}, \mathrm{B}, \mathrm{Q})$. For instance, we list some of them

$$(\mathrm{X}, \mathrm{G}, \mathcal{A}), (\tau, \mathrm{G}, \Phi : \mathcal{A} \to C^{\infty}(\mathcal{M})), (\Theta, \mathrm{G}, ((\mathcal{M}, w), (\mathcal{M}', w'))), (\mathrm{E}, \mathrm{E}, (C^{\infty}(\mathcal{M}), \{.,.\})), \cdots \quad (4)$$

where some of the combinations perhaps do not have any interrelation or financial meaning. The others components stay for future studies. However, for now, we are interested in the three first ones. So to acomplish our first goal we choose them.

We start with a Poisson algebra[5] of continuos functions $(C^{\infty}(\mathcal{M}), \{.,.\})$ under the ordinary commutative product and addition operations on the Poisson manifold, where we use the Poisson bracket $\{.,.\}$ to "deform" this product of classical observables of $(\mathcal{M}, w)$, and following de main idea of Dirac (Dirac, 1964) we put a new suitable noncommutative product $\mathring{a}_{\hbar}$, depending on the parameter $\hbar$, on this algebra and get a family of deformed algebras $\mathcal{A}_{\hbar}$ on the $(\mathcal{M}', w')$ which is in fact a family of associative multiplications $\mathring{a}_{\hbar}$ (Weinstein, 1994) over a fixed vector space. All this is a mathematical and physical environment that will be the nest of the model that we want to develop.

To continue, we put together these first three triads and their relationships, given in (4), in the following proposition.

Proposition A. Let $\mathcal{M}$ be a Poisson manifold and let the first three triads in (4). Then, the $(\mathcal{M}, \mathrm{B}, \mathrm{Q})$ financial models contribute to analysis of the financial markets if and only if it holds at least one the following conditions:

1. The algebra $\mathcal{A}$ is related to the stochastic process $\mathrm{X}(u,t)$, the underlying of the financial markets $\mathcal{M}$.
2. The homomorphism $\Phi : \mathcal{A} \to C^{\infty}(\mathcal{M})$ explains the composite rate of return τ=(Γ,Ξ,Δ) of the process of change in the financial markets.
3. The volatility Θ can be described on both configuration spaces, $(\mathcal{M}, w)$ and $(\mathcal{M}', w')$.

Proof. Since that the conditions (1) to (3) are necessary for the contribution of the $(\mathcal{M}, \mathrm{B}, \mathrm{Q})$ financial model to analyze the financial markets, we only proof the necessary condition of the proposition. The sufficient condition is obvious and we leave it for the reader.

For the first condition. Let $\mathrm{X} = \{X(u,t), u \in \Omega^n, t \in [0,T]^n\}$ be a vector stochastic process on E where $X(u,t)$ is a random vector and here all are n-dimensional. It is known from the definition that for every fixed value $t \in [0,T]^n$ the elementos $X(u,t)$, continuos vector random variables[6], gives us the n-dimensional joint distribution which inform the state of the market for wich $X(u,.)$ represent the distribution of the price (or the volatility Θ, or the sales, etc.) of n assets or n markets and conversely. On the other hand, fixing $u \in \Omega^n$ we have a realization or trayectory $X(.,t)$ of the price (or the volatility τ, or the sales, etc.) that give us the state of the market as a function of the time t.

[5] In general, it should be any algebraic structure, e.g. a Lie algebra or an associative algebra.

[6] To follow our goal, we consider continuous vector of random variables, leaving the discrete case for another ocasion, advising that the procedure will be the same.

First: We show that the family of random vectors $\{X(u,t)\}$ of the stochastic process X is a Poisson algebra $C^{\infty}(\mathsf{M})$ on the Poisson manifold M. For that, WLG, we simply take the natural manifold $\mathsf{M} \approx \mathrm{X} \approx \mathbb{R}^{2n}$ (according to Darboux's theorem they are locally isomorphics) whose classical observables are n-dimensional random vectors with continuous arguments.

That is, $\langle \mathrm{X}, +, \circ, \otimes \rangle$ is an associative commutative algebra of continuous random vectors on X (from now, we will assume troughout this paper that the observables are differentiable functions).

Where +: is the random vector addition, ∘: scalar multiplication, ⊗: vector multiplication.

Let $X_1, X_2, \cdots,$ be a collection of random vectors in X and let $k_1, k_2, \cdots$ be scalars in the field K real or complex. For any $X_1, X_2, X_3 \in \mathrm{X}$ and $k_1, k_2 \in \mathrm{K}$.

Easily these random vectors verify that $\langle \mathrm{X}, + \rangle$ is an abelian group satisfying[7] the postulates $A_1 - A_5$ and $\langle \mathrm{X}, +, \circ \rangle$ is a vector space satisfying the known postulates $A_1 - A_5$ and $B_1 - B_4$ being the last, namely,

B4: $(k_1 + k_2) \circ X_1 = k_1 \circ X_1 + k_2 \circ X_1$ distributive law. To keep showing, we have

$$\left.\begin{aligned} k_1 \circ (X_1 + X_2) &= k_1 \circ X_1 + k_1 \circ X_2 \\ (k_1 + k_2) \circ X_1 &= k_1 \circ X_1 + k_2 \circ X_1 \end{aligned}\right\} \text{ B5: bilinearity} \qquad (5)$$

C1: $X_1 \otimes X_2 \in X$ closure3.

$$\left.\begin{aligned} (X_1 + X_2) \otimes X_3 &= X_1 \otimes X_3 + X_2 \otimes X_3 \\ X_1 \otimes (X_2 + X_3) &= X_1 \otimes X_2 + X_1 \otimes X_3 \end{aligned}\right\} \text{ C2: bilinearity} \qquad (6)$$

D1: $(X_1 \otimes X_2) \otimes X_3 = X_1 \otimes (X_2 \otimes X_3)$ associativity.

D2: $X_1 \otimes 1 = X_1$ where $1 = (1,1,\ldots,1)$ identity.

D3: $X_1 \otimes X_2 = \pm X_2 \otimes X_1$ symmetric/antisymmetric under interchange.

D4: $X_1 \otimes (X_2 \otimes X_3) = (X_1 \otimes X_2) \otimes X_3 + X_2 \otimes (X_1 \otimes X_3)$ derivation[8]

These postulates support the construction of the vector space and, in fact, it is an associative commutative algebra on X. Furthermore, to complete the definition of Poisson algebra we consider the postulates:

E1: $X_i \cdot X_j = X_j \cdot X_i$ for all X_i and X_j in X and an structure of Lie algebra such that it establishes:

E2: $(X_i, X_j) \rightarrow \{X_i, X_j\}$ which holds the postulates from D1 to D4 and it gives an associative commutative algebra. Hence, from all these $\langle \mathrm{X}, +, \circ, \otimes \rangle$ is a Poisson algebra over the vector space $\langle \mathrm{X}, +, \circ \rangle$ whose elements of this configuration space are random vectors with differentiable arguments and {.,.} is the Poisson bracket which satisfies the compatibility condition $\{X_1 X_2, X_3\} = X_1 \{X_2, X_3\} + \{X_1, X_3\} X_2$.

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[7] Clearly, these postulates are hold since the operations of addition, subtraction, multiplication, and scalar multiplication of random variables are hold.

[8] This derivative property is sometimes written in a more customary way: $\left[X_1, [X_2, X_3]\right] + \left[X_3, [X_1, X_2]\right] + \left[X_2, [X_3, X_1]\right] = 0$, and it is called the Jacobi identity.

Second: Now, we want to distinguish this associative commutative algebra in the classical mechanics by a statement that the observables in quantum mechanics, do not commute with one another. To achieve this, we deform the algebra X of random vectors to a family of algebras $\mathrm{X}_{\star_\hbar}$ that belongs to the formal power series $C^{\infty}(\mathrm{X})[[\hbar]]$ depending of the formal parameter ℏ, from the theory of formal deformation quantization, such that when ℏ⟶0 the resulted $\mathrm{X}_{\star_0}$ is the original no deformed algebra that we start with, that should be denoted by $C^{\infty}(\mathrm{X})$.

This is done by the $\star_\hbar$ which acts as a deformer of the initial Poisson`s algebra $\langle \mathrm{X},.,\{,\}\rangle = C^{\infty}(\mathrm{X})$ as a family of formal power series.

$$X_i \star_\hbar X_r = X_i \cdot X_r + \sum_{j\geq 1} B_j(X_i, X_r)\hbar^j \quad \text{for} \quad X_i, X_r \in C^{\infty}(\mathrm{X}) \qquad (7)$$

are members of the family $\mathrm{X}_{\star_\hbar}$ of algebras, where $B_j : C^{\infty}(\mathrm{X})\times C^{\infty}(\mathrm{X}) \to C^{\infty}(\mathrm{X})$ is a sequence of bilinear mappings for *j*=0,1,...

Hence, we have arrived to the *correspondence principle* and found that there is a family of (associative) algebras depending nicely in some sense upon a real parameter of deformation ℏ in the direction of the derivative such that $\mathrm{X}_{\star_0} = \mathrm{X}$ is the algebra of observables in classical mechanics, while $\mathrm{X}_{\star_\hbar}$ is the algebra of observables in quantum mechanics.

Therefore, the algebra $\mathsf{A} \in \mathrm{X}_{\star_\hbar}$, chosen in the construction of the $(\mathsf{M}, \mathrm{B}, \mathrm{Q})$ financial model holds its contribution to the analysis of financial markets. ⊲

For the second condition. We show that the homomorphism $\Phi : \mathsf{A} \to C^{\infty}(\mathsf{M})$, between the classical algebra and the quantum algebras explains the variation of the rate of return τ = (Γ, Ξ, Δ) , change of prices, volatility, etc. of the financial markets.

To show this, we consider the triad

$$(\tau, \mathrm{G}, \Phi : \mathsf{A} \to C^{\infty}(\mathsf{M}))$$

where: τ is the composite rate of return, $\mathcal{G}$ is a group, and Φ is the homomorphism.

A strategy consists of changing in the equation (2) the group structure $\mathcal{G}$ by a Poisson algebra $C^{\infty}(\mathsf{M})$ of differentiable random vectors[9] over the Poisson manifold M. Then using the algebras of the first condition of this proposition and the general definition of quantization (Berezin, 1975), we establish the homomorphism Φ :

We say that Φ over a field K is a homomorphism from the algebra A into the algebra $C^{\infty}(\mathsf{M})$ if for each couple of classical observables $f, g \in C^{\infty}(\mathsf{M})$ is given by

$$\Phi(f) = lim_{\hbar\to 0} f(\hbar, x)$$

and holds the following properties:

1. For any two points $x_1, x_2 \in \mathsf{M}$ there exists a function $f(x) \in \Phi(\mathsf{M})$ such that $f(x_1) \neq f(x_2)$,

[9] These random vectors capture the behavior of a set of assets or a set of financial markets or a set of any random phenomenon.

2. $\Phi(\frac{1}{\hbar}(f * g - g * f)) = i\{\Phi(f), \Phi(g)\}$, where $*_\hbar$ denote the multiplication on $\mathcal{A}$ and $\{.,.\}$ the Poisson bracket on $C^\infty(\mathcal{M})$.

On the other hand, from the definition of FMQ, equation (1), the third component τ = (Γ, Ξ, Δ) is the composite rate of return of the financial markets integrated by the rate of the process of the money market free of risk (Γ), the average rate of return of the market (Ξ), and the rate of dividends of the market (Δ), respectively. Assuming that these rates of the financial markets are described and explained by linear (or nonlinear) differentiable functions on a Poisson manifold in a period of time of maturity $[0,T]$ we conclude that this Poisson algebra $C^\infty(\mathcal{M})$ explains the behavior of the composite rate of return and, in fact again, $C^\infty(\mathcal{M})$ can be deformed as a family of formal power series

$$f *_\hbar g = f \cdot g + \sum_{j\geq 1} B_j(f,g)\hbar^j \quad \text{for} \quad f,g \in C^\infty(\mathcal{M}) \qquad (8)$$

Satisfying several conditios, see (Bates, 1995) for a complete detail.

Then, the homomorphism $\Phi : \mathcal{A} \to C^\infty(\mathcal{M})$ influences in the valuation of the performance of the composite rate of return τ=(Γ,Ξ,Δ) in the $(\mathcal{M},\mathcal{B},\mathcal{Q})$ financial model . ⊲

For the third condition. To show that the analysis of the volatility Θ can be done on both configuration spaces: on the classical mechanics space $(\mathcal{M}, w)$ and on the quantum mechanics space $(\mathcal{M}', w')$.

First: On the space $(\mathcal{M}, w)$. There are many studies of the volatility Θ on this space, in particular the volatility of the financial markets. Namely, if a capital market system has N assets (N degrees of freedom) in its configuration linear real sapce $\mathbb{R}^{2n}$ of dimension $2n$ and the observables are differentiable functions $f,g,h,\cdots$ where each observable describes the behavior of a particular characteristic of a financial market or it describes the continuous change of a characteristic of an asset, in the time, marked by $f(p,q) \in \mathbb{R}^{2n}$ and $(p,q) = (p_1, p_2, ..., p_n, q_1, q_2, ..., q_n)$ where p_i and q_i are the moments and the coordinates of position, respectively. For instance, the rate of return of an asset or the rate of return of a market(s) can be decomposed in its amplitude of variation (volatility) and its tendency (the sign that indicates the direction of its displacement), then we may define its volatility by a real differentiable function $f(t,\Delta t)$ depending of the time and the period of maturation of the negotation. Similarly, one might mention a variety of ways to focus the study of the volatility of the financial markets on the classical space of configurations.

Second: On quantum space $(\mathcal{M}', w')$. Here the space of states of configurations is the Hilbert space, but we avoid it, see (Dirac, 1964), and recall the space of formal power series $C^\infty(\mathcal{M})[[\hbar]]$ of noncommutatives algebras of differentiable functions $f(x_1, x_2, \cdots, x_n)$ of n variables, that here will be random vectors, with square summable.

Since the operators $\hat{p}_k, \hat{q}_k$ in $L^2(\mathbb{R}^n)$ can be compared with the classical momentum and the coordinates $\mathrm{p}_k, \mathrm{q}_k$ by the formulas

$$(\hat{q}_k f)(x) = x_k f(x) \quad \text{and} \quad (\hat{p}_k f)(x) = \frac{\hbar}{i}\frac{\partial f}{\partial x_n}$$

From this, we can see clearly that the observables of the classical world and the quantum world are related. Then, since that the volatility of any chaotic phenomenon can be described by differentiable functions we are able to analize this volatility Θ on these two configuration spaces that are immersed in the fibre bundle space $\mathcal{B}$. ◁

DEFINITION OF THE $(\mathcal{M},\mathcal{B},\mathcal{Q})$ FINANCIAL MODEL

Here we keep on using the already established notations. Let $\mathcal{M} = B$ be a Poisson manifold, the base of the fibre space $\mathcal{B}$, defined in the equation (2), provided with a Poisson algebra of differentiable functions $f,g,h,\cdots$ under the usual multiplication on $C^{\infty}(\mathcal{M})$ in the classical ambience $(\mathcal{M},\omega)$ and let $E = \mathcal{M}'$ the total space of $\mathcal{B}$ providad with a family of associative noncommutative algebras $C^{\infty}(\mathcal{M})[[\hbar]]$ of formal power series $f \star_{\hbar} g \in C^{\infty}(\mathcal{M})[[\hbar]]$ with underlying space $(\mathcal{M}',\omega')$ in the quantum ambience. We define the $(\mathcal{M},\mathcal{B},\mathcal{Q})$ financial model as a cross section[10]

$$\Psi : (C^{\infty}(\mathcal{M}),\{.,.\}) \to (C^{\infty}(\mathcal{M})[[\hbar]],\star_{\hbar})$$

such that for any f,g,h in $C^{\infty}(\mathcal{M})$ the following expression

$$\Psi(f,g) = \left|f \star_{\hbar} g\right| = \left| f\cdot g + \sum_{j=1}^{\infty} B_j(f,g)\hbar^j \right| \le R \tag{9}$$

exists. Where for analogy to the consequences of the definition of fibre bundle (Steenrod, 1999), we have

1. $C^{\infty}(\mathcal{M}) \cong C^{\infty}(\mathcal{M})[[\hbar]]/\hbar$ as algebras and the star product $\star_{\hbar}$ in $C^{\infty}(\mathcal{M})[[\hbar]]$ given by $\star_{\hbar} : C^{\infty}(\mathcal{M})[[\hbar]] \times C^{\infty}(\mathcal{M})[[\hbar]] \to C^{\infty}(\mathcal{M})[[\hbar]]$ whose values are determined on the subspace $C^{\infty}(\mathcal{M}) \subset C^{\infty}(\mathcal{M})[[\hbar]]$.
2. $\Pi\Psi\big((f,g), f \star_{\hbar} g\big) = I_{C^{\infty}(\mathcal{M})}$, where Π is the projection of the $\mathcal{B}$.
3. $|f\cdot g| \le R$, where $\cdot$ is the usual multiplication of the algebra in the classical world and R is a nonnegative real value selected by the user, e.g. the investor. It should be selected under the warning: "Smaller is R stricter is the model ".
4. The coefficients $B_j : C^{\infty}(\mathcal{M}) \otimes C^{\infty}(\mathcal{M}) \to C^{\infty}(\mathcal{M})$ are polydifferentiables [11]
5. The Poisson bracket on $C^{\infty}(\mathcal{M})$ is defined by

   $$\{f,g\} = \frac{f \star_{\hbar} g - g \star_{\hbar} f}{2\hbar}\Big|_{\hbar=0} = \frac{1}{2}(B_1(f,g) - B_1(g,f)), \forall f,g \in C^{\infty}(\mathcal{M}))$$

   this bracket acts as a derivation on both parameters and it satisfies the Jacobi's identity.
6. In this model there exists a parameter of deformation $\hbar$ that indicates the scale of deformation (now formal variable), that governs the commutativity of this new algebra. Once again, here $\hbar$ takes positive real values and for each fixed value we obtain a member of a family of financial models. Then, there are infinite many models. In particular, we are concerned for $0<\hbar\le 1$, as we show in the following section

[10] See (Steenroad, 1999, pag. 3), details of fibre bundles.
[11] In particular, for the B_1 first term for the associativity condition of expansion (8) we have $B_1(fg,h) + B_1(f,g)h = f\,B_1(g,h) + B_1(f,gh)$, the 2-cocycle of Hochschild (Weinstein, 1994).

7. **A particular case:** for each pair of physsical observables *f*,*g* in $C^{\infty}(\mathrm{M})$ we define the Poisson bracket $\{f,g\}:C^{\infty}(\mathrm{M})\times C^{\infty}(\mathrm{M})\rightarrow C^{\infty}(\mathrm{M})$ that gives us a third observable expressed like

$$\{f,g\}=\sum_{j=1}^{n}(\frac{\partial f}{\partial q_j}\frac{\partial g}{\partial p_j}-\frac{\partial g}{\partial q_j}\frac{\partial f}{\partial p_j})$$

Now relating the last one with the formal power series $f\star_{\hbar}g$ given in (8) such that the commutator $[f,g]=f\,\mathring{a}_{\hbar}\,g-g\,\mathring{a}_{\hbar}\,f$, by Dirac‘s suggestion (Dirac, 1964), can be expressed like $i\hbar\{f,g\}$ plus terms of order $\hbar^2$. That is,

$$[f,g]=f\,\mathring{a}_{\hbar}\,g-g\,\mathring{a}_{\hbar}\,f=i\hbar\{f,g\}+\mathrm{O}(\hbar^2) \qquad (10)$$

With this equation (10) and taking the conditions of (8) we arrived to the implied condition

$$\{f,g\}=\frac{1}{2}B_1(f,g) \quad \text{(in the real field)} \qquad (11)$$

Then, the definition of the proclaimed financial model, equation (9), can be written in a simpler way by means of

$$\Psi(f,g)=\left|f\,\mathring{a}_{\hbar}\,g\right|\leq\left|f\cdot g+B_1(f,g)\hbar+O(\hbar^2)\right|\leq R$$

or equivalently

$$\Psi(f,g)=\left|f\,\mathring{a}_{\hbar}\,g\right|=\left|f\cdot g+2\{.,.\}\hbar+O(\hbar^2)\right|\leq R$$

or equivalently

$$\Psi(f,g)=\left|f\,\mathring{a}_{\hbar}\,g\right|=\left|f\cdot g+2\hbar\sum_{j=1}^{n}(\frac{\partial f}{\partial q_j}\frac{\partial g}{\partial p_j}-\frac{\partial g}{\partial q_j}\frac{\partial f}{\partial p_j})+O(\hbar^2)\right|\leq R$$

This particular statement of the model is simpler and applicable to concrete cases in the financial field, where $q=(q_1,q_2,\cdots,q_n)$ and $p=(p_1,p_2,\cdots,p_n)$ are the coordinates of momentum and position and *f*(*p*,*q*) is in $\square^{2n}$.

The most understandable case happens when the Poisson manifold M is $\square^{2n}$.

In particular, if the manifold $\mathrm{M}=\square^{2}=\{(p,q)\}$ is equipped with a symplectic form $w=2dp\wedge dq$ and the corresponding Poisson bivector is $\alpha=2\partial_p\wedge\partial_q$.

In this case, the Poisson bracket on $C^{\infty}(\square^{2})$ is given by

$$\{f,g\}=\frac{\partial f}{\partial x_p}\frac{\partial g}{\partial x_q}-\frac{\partial g}{\partial x_q}\frac{\partial f}{\partial x_p}$$

then, replacing this in the last expression it results

$$\Psi(f,g)=\left|f\,\mathring{a}_{\hbar}\,g\right|=\left|f\cdot g+2\hbar(\frac{\partial f}{\partial x_p}\frac{\partial g}{\partial x_q}-\frac{\partial g}{\partial x_q}\frac{\partial f}{\partial x_p})+O(\hbar^2)\right|\leq R$$

and it is the model in its even more simpler form.

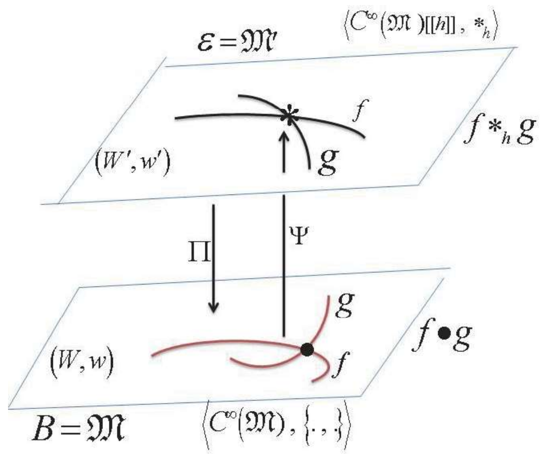

Figure 1: Panoramic spatial visualization of the definition of the (M, B, Q) financial model on both classical and quantum configuration spaces and its topological and algebraic interactions.

Again, the values of the upper bound R of the model, e.g. $R = 0, \ldots, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1, 2, 3 \ldots$, will depend on the rigorousness of the investor allowing an appropriate largeness ("Financial band") in the model. That is to say, the investor will always wish that the radius R of the financial band be the smallest (the most tightest) possible, which indicates low variability (small variance). For instance, R=0 means that the functions describing the change of price, (valuation of the rate of return, volatility, etc.) of the assets are constants (variability zero). In other words, it coincides with what, for analogy, other known models sustain (e.g. the CAPM) the variability of its portafolio of assets have minimal variance (or minimal risk). The opposite counterpart of this situation occurrs when R>1. That is to say, there exists high variability of the behavior of the financial assets, noted volatile nature (noted risk). The most advisable signal for a good performance of the model is to mantein $0<R\leq 1$, as we show in the following section.

The Figure 1 shows a picture of the definition of the (M, B, Q) financial model in the ambience of the fibre bundle B , in which the financial markets operate for the actions of algebras of differentiable functions, as we explain in the following section of results and applications.

The intention of the definition with bounded makeup has meaning of limiting the explosive change of the values of the formal power series, likewise explosive change of the model, that only must range inside of the radius R and in this way we might control the chaotic variability of the functions involved in the expansion of the series that describe the conduct of prices, valuations of exchange, future valuations of interest, indexes of "stocks", which financially are interpreted and perceived as risky changes and are indicators of the volatile nature of the financial markets.

APPLICATION OF THE (M, B, Q) MODEL

We appeal the model constructed in the previous section and its definition in (9) and all notations already established for a concrete application and the model contribution in the analysis of the capital markets.

We initiate choosing a financial arbitrary market, for instance WLG, let's suppose that we choose the financial market NASDAQ that has affiliates N dynamic financial assets $a_1, a_2, \cdots, a_N$ in its portfolio of markets, where at least two assets are interchangeable.

We are interested in measuring the change of some charaterístics of these assets (for example, change in the prices, change in financial return[12], change in the volatility, etc.), in the time. As before, let's suppose that the behavior of these assets are described by differentiable functions on a Poisson manifold $\mathbb{R}^D$ of dimension *D*. So, let's $f_{1D}, f_{2D}, \cdots, f_{ND}$ be functions on the space $C^\infty(\mathbb{R}^D)$, as we konw they form a Poisson algebra $\left\langle C^\infty(\mathbb{R}^D), \{.,.\} \right\rangle$ and these functions describe the trajectories of changes of every asset a_i, respectively, for *i*=1,2,3,⋯,*N*. Also, we choose $p = (p_1, p_2, \cdots, p_D)$ and $q = (q_1, q_2, \cdots, q_D)$, such that $f(p,q) \in \mathbb{R}^{2D}$ are the coordenates.

To clarify and undertand better, we restrict this example to the simplest case of two arbitrary financial assets *A* and *B* selected from the set $\{a_1, a_2, \cdots, a_N\}$, with their corresponding functions $f_{12} = f(p,q)$ and $f_{22} = g(p,q)$, of the NASDAQ stock exchange, in dimension *D*=2. We want to measure, for example, its return according to the volatility and its tendency in a horizon of the time *t*∈[0,*T*] of one year (12 months). Concretely, let $f(p,q) = p\,sin(q)\sqrt{T}$ and $g(p,q) = q\,sin(p)\sqrt{T}$ in $C^\infty(\mathbb{R}^4)$ be functions of return of each asset, respectively, where *p* and *q* denote their generalized volatility depending of the time *t*∈[0,*T*] such that *sin*(*p*) and *sin*(*q*) describe their tendencies of the assets *A* and *B*, respectively. Since the volatility $\Theta(p,q)$ measures, for example, the variation (or the risk) of the returns of an investment of the assets *A* and *B* in the horizon [0,*T*]×[0,*T*], by definition of volatility and its generalized form given by $\Theta_T = \theta\sqrt{T}$, where θ is the anualized volatility[13], we have that (*f*·*g*)(*p*,*q*)=*psin*(*q*).*qsin*(*p*) represents the joint return[14] of the assets *A* and *B*, on the classical space, whose fluctuation of the return on the squared [0,*T*]×[0,*T*] is shown in the following surface of return, see Figure 2.

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[12] Remember that the return of an assets is the profit or full loss that there experiences the owner of an investment in a period of specific time.

[13] For a daily yield of the stock with standard deviation Θ_{SD} and period *P* of return $\theta = \dfrac{\theta_{SD}}{\sqrt{P}}$. A common assumption is

$$P = \frac{1}{252}.$$

[14] The purpose of defining the joint return as a product rather than a weighted sum, is to protect the originality of the algebra of Poisson of differentiable functions over the ordinary product of functions and because (*f*·*g*)(*p*,*q*)=*psin*(*q*).*qsin*(*p*) will be the first term of the expansion of the series of deformation in the quantum space.

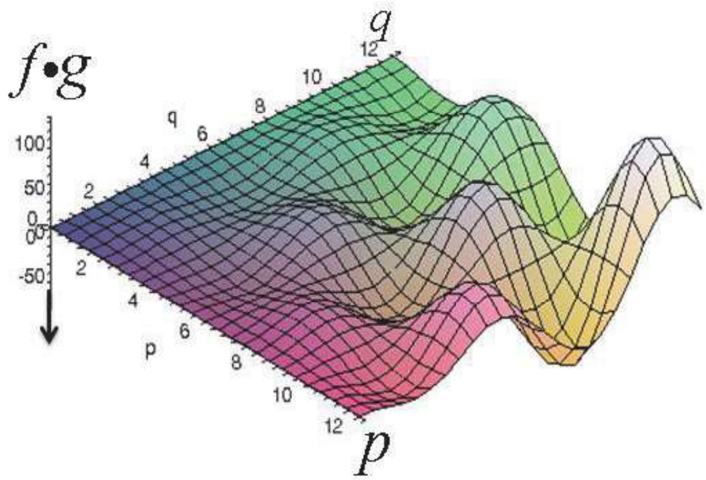

Figure 2: Surface of the joint return $(f{\cdot}g)(p,q)=psin(q).qsin(p)$ of two assets in the classical space.

We notice that the joint return changes in the horizon (the square) $[0,T]\times[0,T]$ such that on having this evaluated in these apexes (0.0),(0,1),(1,0),(1,1) result the minimum return (minimal volatility) on (0,0), means no investment is still executed, $(f{\cdot}g)(0,0)=0$ (there is no risk), the horizon of investment or negotiation still does no initiate. Similarly, the evaluation on the vertex (1,1) result $(f{\cdot}g)(1,1)=0.7080734183$ which represents the maximum return (there is volatility) on this horizon. As in finance we know that to major horizon of investment (or credit) major awaited return (major interest to be paid). One notice that in this case the tendency (the sign) of the return is described by a differentiable trigonometric function which approximates a time series tendency in the continuos time.

TABLE 1

SIGNIFICANT VALUES FROM VOLATILITY ON THE (M, w) CLASSICAL SPACE

(p,q)	$(f{\bullet}g)(p,q)=psin(q).qsin(p)$
(0,0)	0
(0,1)	0
(1,0)	0
(0.1,0.1)	0.00009966711080
(0.3,0.3)	0.007859897330
(0.5,0.5)	0.05746221174
(0.6,0.6)	0.1147756042
(0.7,0.7)	0.2033580499
(0.8,0.8)	0.3293438472
(0.9,0.9)	0.4970168483
(1,1)	$sen(1)^2=0.7080734183$

We also notice, from the Table 1, that some other values evaluated on the boundary and in the interior of the square $[0,T]\times[0,T]$ are all betmeen 0 and 0.7080734183. Moreover, no matter what other values of p and q in the square are chosen, all are between the minimum 0 and the maximum 0.7080734183.

These results are ratified with the shape of the surface of the return in the Figure 2. In the origin (0,0) there is neither return nor volatility, there si no activity of financing, but we agree that when we move away from the origin the expected return increases (or we can say larger is the horizon of negotiation larger expected volatility) up to the maximum $(f\cdot g)(1,1)=0.7080734183$. We point out that these computations are independent from the type of currency and units of measurement due to existence of symmetries of measurement.

On the other hand, if we attach the coefficients α_1 and α_2 to each function f and g of the assets A and B, respectively, where they represent the fraction of the entire investment in a portfolio of the market, then the entire total expected return will be the weighted sum of the returns of each asset and easily one might use the $Var(\alpha_1 f+\alpha_2 g)$ or the coefficient β, etc. to measure the volatility or risk of this portfolio, as the model CAPM proclaims. Here we propose and explain another method of analysis of markets.

Now, we continue with this analysis and it is what we are really interested in: the return of the assets A and B in the quantum space of configurations. We use the particular case (part 7) of the definition of the $(\mathrm{M}, \mathrm{B}, \mathrm{Q})$ model.

We initiate re-calling the functions $f(p,q)=psin(q)$ and $g(p,q)=qsin(p)$ in $C^\infty(\mathbb{R}^4)$ that describe the return of saying assets and that support the same characteristics. We count one dimension for the time t of the period $[0,T]$ of the financial activity located in the commutative Poisson algebra, and we proceed to deform this algebra by means of the noncommutative product $\star_\hbar$ to obtain a family of noncommutative algebras depending on the parameter $\hbar$ such that $f\star_\hbar g$ is in the space of the formal power series $C^\infty(\mathbb{R}^4)[[\hbar]]$.

Speaking concretely, we use:

$$\Psi(f,g)=\left|f \star_\hbar g\right|=\left|f\cdot g+2\hbar\left(\frac{\partial f}{\partial p}\frac{\partial g}{\partial q}-\frac{\partial g}{\partial p}\frac{\partial f}{\partial q}\right)+O(\hbar^2)\right|\leq R$$

and obtain

$$\left|(f \star_\hbar g)(p,q)\right|=\left|p\,sin(q)q\,sin(p)+2\hbar\left(sin(q)\,sin(p)-pq\,cos(p)\,cos(q)\right)+O(\hbar^2)\right|\leq R \qquad (12)$$

The fascinating situation at this moment is that we can make changes and control the model by means of the parameter $\hbar$ and the radius R. This indicates that for every partner of interchangeable assets whose shapes are described by their differentiable functions f and g we can obtain a family of models changing $\hbar$ and R. That is to say, we have on our face a multitude of families of models that form classes of equivalence and this result is precisely because the quantization deformation of the original algebra of Poisson with which we initiate. In particular, since $\hbar$ it is a real number and if we do that $\hbar\longrightarrow 0$ in the definition of the model we obtain the case that we have just analyzed above. That is to say, the analysis of assets that it does to itself in the classical space, or what happens in the classical space, is one particular case of what happens in the quantum space.

On the other hand, the role of the parameter R is that it offers the investor the opportunity of being strict or sparing in the control of the volatile nature of his assets. It is always preferred that the volatile nature of an assets is small. This will be achieved mantein $R<1$ and that the changes of prices or another volatile characteristic of the financial assets change in small radius (the financial band is narrow). If the fluctuation of the function of the asset goes eventually out of the radius it will mean "financial alert" for the managers of the assets. For example, leting $\hbar = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$ and R=1 when $O(\hbar^2)\longrightarrow 0$ we evaluate in (12) and place de results in the Table 2. For ħ=0, we have the classical case, given in the Table 1.

$$\left| f\,\text{å}_{\frac{1}{2}}\, g \right| = \left| pq\, sin(p)\, sin(q) + sin(q)\, sin(p) - pq\, cos(p)\, cos(q) \right| \le 1$$

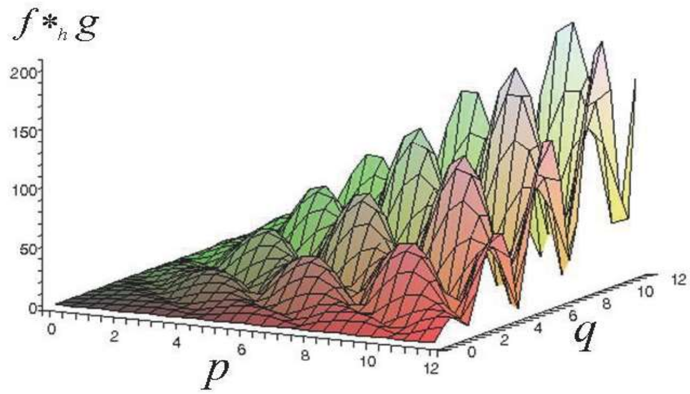

Figure 3: Surface of the joint return $(f\,\text{å}_{\hbar}\, g)(p,q)$ of two assets in the quantum space for ħ=1/2.

We proceed to evaluate like earlier $(f\,\text{å}_{\frac{1}{2}}\, g)(p,q)$ in the horizon of the square [0,T]×[0,T]. First, we evaluate in the apexes (0.0),(0,1),(1,0),(1,1) and find again the minimal yield in (0,0). That is to say, $(f\,\text{å}_{\frac{1}{2}}\, g)(0,0) = 0$ means the result coincides with that of the classical mechanics, the investment is not even executed, there is no risk, there is no volatility. Then, evaluating in the vertex (1,1), we see that $(f\,\text{å}_{\frac{1}{2}}\, g)(1,1) = 1.124220255$ is the maximum yield that contains the maximum risk or maximum volatility. Notice that $(f\,\text{å}_{\frac{1}{2}}\, g)(1,1) > R = 1$ It means that the fluctuation of the model at (1,1) gets out of the band with R=1, "financial alert". Nevertheless, if the investor is less strict and bets up to R=2 his investment would accept this risk. All other evaluations in the square are in between these minimum and maximum values, as we see in the Table 2.

TABLE 2

EXPERIMENTAL DATA AFTER $(f \star_{\hbar} g)(p,q)$ IS EVALUATED FOR $\hbar = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$.

(p,q)	$(f\star_0 g)$	$(f \star_{\frac{1}{4}} g)$	$(f \star_{\frac{1}{2}} g)$	$(f \star_{\frac{3}{4}} g)$	$(f\star_1 g)$
(0,0)	0	0	0	0	0
(0,1)	0	0	0	0	0
(1,0)	0	0	0	0	0
(0.1,0.1)	the	0.00013285621	0.0001660453	0.00019923439	0.00023242349
(0.5,0.5)	same	0.07611774115	0.0947732705	0.1134287999	0.1320843293
(0.6,0.6)	as	0.1515739677	0.1883723312	0.2251706947	0.2619690582
(0.7,0.7)	in	0.2675452891	0.3317325284	0.3959197677	0.4601070070
(0.8,0.8)	Table 1	0.4313156514	0.5332874555	0.6352592596	0.7372310632
(0.9,0.9)		0.6473257961	0.7976347443	0.9479436914	1.098252640
(1,1)	.70807342	0.9161468361	1.124220255	1.332293673	1.540367091

Since $\hbar \in \mathbb{R}^{+}$ is the parameter of deformation in the direction of the derivative evaluated in $\hbar$ as $\longrightarrow 0$ we land on the classical space and when $\hbar \longrightarrow \infty$ the expression (12) diverges and there is a formal power series por each $\hbar$, as we show above, then there are infinitely many members of this family as we expected.

From Table 2 we can conclude that the returns (same is for volatility) in the quantum space are larger than the returns in the classical space. What does it mean?. In physics it is proclaimed that the quantum description of a physical phenomenon is more detailed and real than the classical one, and so there are certain phenomenon that the difference between which is displayed in their quantum description, whereas their classical description does not show this difference. Therefore, the values of $(f \star_{\hbar} g)(p,q)$ are greather than $(f\bullet g)(p,q)$ (In general, it can be show) and the managers of the financial institutios do not know even more they do not susspect the real volatility of the financial phenomenons until the catastrophes of bankrupticy occurs, because they believe and think in clssical world and ignore the quantum reality.

CONCLUSIONS

In this paper, we have attempted to construct the $(\mathcal{M}, \mathcal{B}, \mathcal{Q})$ financial model of order (p,q,r) when $p,q,r \in \mathbb{R}_{+} < \infty$. In particular, focusing the triad $(\tau, \mathcal{G}, \Phi : \mathcal{A} \rightarrow C^{\infty}(\mathcal{M}))$ we get the model of order (1,1,1) which is a particular case of the model of order (6,6,6) defined in (1), (2), and (3), respectively. However, we could also consider in this study other models of order (2,2,2), (3,3,3),$\cdots$,(6,6,6) called triadic financial models. In general, models of order (p,q,r) when $p,q,r \in \mathbb{R}_{+} < \infty$ might be established in the ambience of topological fibre bundles and in the soul of quantum fields. We leave these studies to other future researchers.

We conclude from the definition and application of the financial model in (9) and from its consequences that the chaotic change in the quantum space is greather than the chaotic change that it happens in the classical space and for the commandment of distinction between these two mechanics the first one has more realistic behavior of random phenomenons and this is one of the fundamental reasons that the managers cannot deal with the financial crisis because this world is unknown for them. The example above shows just one case and there should be many others to figure out.

Finally, we believe that we are exploring a mysterious area of the financial sciences where one can find fruitful responses that help to analyze the chaotic behavior of the financial markets from this new perspective.

ENDNOTES

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1. Concrete facts of this world wide problems of financial unforeseeable and uncontrollable collapses already happened in different countries and continents of the planet from 1929 until 2008. Since there reminds to itself, on the Black Monday of 1987, the Russian crisis of 1998, the explosion of the Bubble Dot Com of 2001 and the more recent mortgage crisis of 2008 in the USA and the current ones of the European Union.

2. King M. and Wadhwanni S. (1989) *Transmission of volatility between stock markets*, National Bureau of Economic Research. Paper Series 2910.

3. Mathieu Mosolonka-Lefebvre, "Epidemics in markets with trade friction and imperfect transactions", (Oct, 2013), arXiv:1310.6320V1[q-bio.PE].

4. Where *w* is a skew-symmetric tensor field on this Poisson manifold M. However, this Poisson manifold can be some differentiable manifold [3] and $C^{\infty}(M)$ is a set of differentiable functions on M.

5. In general, it should be any algebraic structure, e.g. a Lie algebra or an associative algebra.

6. To follow our goal, we consider continuous vector of random variables, leaving the discrete case for another occasion, advising that the procedure will be the same.

7. Clearly, these postulates are hold since the operations of addition, subtraction, multiplication, and scalar multiplication of random variables are hold.

8 This derivative property is sometimes written in a more customary way: $[X_1, [X_2, X_3]] + [X_3, [X_1, X_2]] + [X_2, [X_3, X_1]] = 0$, and It is called the Jacobi identity.

9. These random vectors capture the behavior of a set of assets or a set of financial markets or a set of any random phenomenon.

10. See [7] Steenrod N., pag. 3.

11. In particular, for the B1 first term for the associativity condition of expansion (8) we have B1(fg; h) + B1(f; g) h = fB1(g; h) + B1(f; gh) which is the 2-cocycle of Hochschild [8].

12. Remember that the return of an assets is the profit or full loss that there experiences the owner of an investment in a period of specific time.

13. For a daily yield of the stock with standard deviation θ_{SD} and period *P* of return $\theta = \frac{\theta_{SD}}{\sqrt{P}}$ A common assumption is P = 1/252

14 The purpose of defining the joint return as a product rather than a weighted sum, is to protect the originality of the algebra of Poisson of differentiable functions over the ordinary product of functions and because (f _ g)(p; q) = psin(q):qsin(p) will be the first term of the expansion of the series of deformation in the quantum space.

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