Prospective Mathematics Teachers' Algebraic Proficiency From a Symbol Sense Perspective: The Case of Solving Inequalities

Al Jupri
Universitas Pendidikan Indonesia

Ririn Sispiyati
Universitas Pendidikan Indonesia

Procedural fluency and conceptual understanding are two aspects of mathematical proficiency discussed worldwide, including in Indonesia. In the algebra domain, algebraic proficiency concerns an ability to deal with symbolic representations that can be viewed from a symbol sense perspective. This algebraic proficiency is considered indispensable for prospective mathematics teachers for their future careers. This research aims to analyze prospective mathematics teachers' algebraic proficiency from the perspective of symbol sense. To achieve this aim, we set up a qualitative case study, involving 19 Indonesian mathematics education students (21-23 years old) as prospective mathematics teachers, in the form of a two-week online teaching and learning process (4 x 50 minutes) and its corresponding formative assessment for solving quadratic, cubic, and rational inequalities. The results revealed that the majority of the participants lack algebraic proficiency as they use procedural strategies more than symbol sense strategies to solve inequalities.

Keywords: algebraic proficiency, inequalities, prospective mathematics teachers, symbol sense

INTRODUCTION

Algebra is one of the core domains of mathematics taught at secondary school. This domain has been widely recognized as essential for the advanced study of mathematics, for other subjects, and for professional work (Carraher et al., 2006; De Lange, 2006; Doorman and Gravemeijer, 2009; Katz, 2007; Kop et al., 2020). Therefore, one of the preconditions for school students to pursue future careers is to be proficient in algebra. Algebraic proficiency refers largely to proficiency in symbolic representations and includes the aspects of conceptual understanding and procedural fluency (MacGregor and Price, 1999; McCallum, 2007; Stiphout et al., 2013). In previous studies, these two aspects of algebraic proficiency are often investigated by looking at structure and/or symbol sense (Bokhove and Drijvers, 2010; Hoch and Dreyfus, 2006; 2010; Kop et al., 2020; Novotna and Hoch, 2008; Stiphout et al., 2013).

Worldwide, algebra is considered a subject that is difficult for most students to learn and for teachers to teach (Bokhove and Drijvers, 2012; Chapman, 2006; Carraher et al., 2006; Van Amerom, 2003), and this is also true in Indonesia (Jupri et al., 2014a; Jupri et al., 2015; 2016). Previous studies investigating student difficulties in understanding early algebra, revealed that Indonesian students lack both algebraic procedural skills and conceptual understanding in dealing with equations, algebraic expressions, and word problems (Jupri et al., 2014a; Jupri et al., 2014b; Jupri and Drijvers, 2016; Jupri et al., 2016; Jupri et al., 2021). There are several factors that contribute to this, one of which is a lack of either the students or teachers' ability to understand algebra. The lack of ability in algebra from the teachers' side can be predicted from a lack of ability in prospective mathematics teachers. A previous study has shown that prospective mathematics teachers encountered difficulties in understanding quadratic and related equations and tend to have a better procedural fluency than conceptual understanding (Jupri and Sispiyati, 2020).

To investigate prospective mathematics teachers' ability in algebra, we carried out a qualitative case study involving mathematics education students who were taught using online media platforms—due to Covid-19 Pandemic—how to solve quadratic, cubic, and rational inequalities. Here, we present the findings and analyze them using the notion of symbol sense.

Algebraic Proficiency

Mathematical proficiency consists of five aspects, namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick, 2001). In the domain of algebra, algebraic proficiency refers to proficiency with symbolic representations (Brown and Quinn, 2007). The first two strands of mathematical proficiency, including conceptual understanding and procedural fluency, are central in discussing algebraic proficiency, particularly for addressing algebraic expressions, equations, and inequalities (Stiphout et al., 2013).

Conceptual understanding can be interpreted as comprehension of mathematical concepts, operations, and relations; procedural fluency refers to skill in conducting procedures flexibly, accurately, efficiently, and appropriately (Kilpatrick, 2001). These two aspects are required simultaneously to develop a student's algebraic proficiency as well as eventual algebraic expertise. Algebraic expertise is a spectrum ranging from basic skills such as procedural work with a local focus and algebraic manipulation to strategic work which requires a global focus, algebraic reasoning and conceptual understanding (Drijvers et al., 2010). This strategic work, including a global focus which emphasizes algebraic reasoning, is part of the behavior of symbol sense (Bokhove and Drijvers, 2010; 2012).

Symbol Sense

Symbol sense involves an intuitive feel for when and when not to use symbols in the process of problem solving (Arcavi, 1994). Symbol sense, which can be seen as an analogy to number sense, refers to an ability to give meaning to symbols, including algebraic expressions, equations, inequalities, and formulas (Arcavi, 2005). Therefore, this ability indicates algebraic proficiency (Stiphout et al., 2013) and suggests a relational rather than instrumental understanding (Skemp, 1976). Symbol sense behavior is considered important for the successful learning of algebra in school as well as at higher education levels (Bokhove and Drijvers, 2010). In general, symbol sense has the following characteristics: an ability to manipulate and examine symbolic expressions in problem solving; confidence with symbols and symbolic expressions; the ability to create symbolic relationships for verbal or graphical information; the ability to select possible symbolic representations for a problem; the capability to check for symbol meanings during problem solving; and the understanding that symbols play a role as variables or parameters (Arcavi, 2005).

In previous studies, perspective of symbol sense has been used for understanding student difficulties in the concept of parameters (Drijvers, 2000; 2003); for investigating student algebraic expertise in a digital environment (Bokhove and Drijvers, 2010); for assessing students' algebraic proficiency (Stiphout et al., 2013), and for exploring students' graphing abilities in solving non-routine algebra tasks (Kop et al., 2020). In Indonesia, the notion of symbol sense has recently been used to understand the students' thought process in solving substitution problems (Jupri et al., 2016), and for understanding prospective mathematics teachers' algebraic proficiency in solving equations (Jupri and Sispiyati, 2020). In this study, symbol sense is used to investigate the algebraic proficiency in solving inequalities of prospective mathematics teachers.

METHOD

This qualitative case study was conducted through the following four steps. First, for a formative assessment on the topic of quadratic, cubic, and rational inequalities, we designed an appropriate teaching and learning module for 19 prospective mathematics teachers. These prospective teachers were all mathematics students (21-23-year-old), currently attending the Essential Concepts of School Mathematics course. The module (Table 1) consist of four different tasks, each dealing with a different type of mathematical problem: quadratic inequalities with linear factors; cubic inequalities with linear and quadratic factors; rational inequalities with linear forms for the numerator and denominator; and rational inequalities with quadratic forms for the numerator and denominator. We predicted that task two was more difficult than task one, and that task four is more difficult than task three (Table 1).

Second, we implemented the module through a two-week teaching and learning course, which covered the topic of inequalities, using a direct teaching approach via online media platforms, including Cloud Zoom Meeting, WhatsApp Web, and Google Classroom. The module was split into four lessons which lasted 50 minutes each. In the direct teaching approach, the lecturer explained the topic of inequalities, provided examples and explanations, asked questions, gave exercises, and provided feedback to students' written work after the course. The students were encouraged to pay attention to and answer questions from the lecturer, take notes, do the exercises, and present their written work via WhatsApp Web. During the learning and teaching process, the lecturer taught both procedural and more efficient symbol sense strategies for solving inequalities.

Third, after the two-week teaching and learning process, we gave each student a test to assess their algebraic proficiency in solving the inequalities in each of the four tasks. The students were given 30 minutes to complete the test. The students were observed on video via the Cloud Zoom Meeting platform during the test. We collected the students' written work via a Google Classroom platform.

TABLE 1
TASKS USED TO ASSESS MATHEMATICS EDUCATION STUDENTS

Tasks		Mathematical inequality
1.	A quadratic inequality with linear factors	$(x+1)(3x+2) \geq (2x-3)(x+1)$
2.	A cubic inequality with linear and quadratic factors	$(x^2+9x+20)(7x-11) \leq (4x-7)(x^2+9x+20)$
3.	A rational inequality with linear forms for the numerator and denominator	$\frac{x}{x-1} < \frac{5}{x+5}$
4.	A rational inequality with quadratic forms for the numerator and denominator	$\frac{x^2-2x}{x^2-2x+4} < 1$

Finally, we analyzed the students' written work using the perspective of symbol sense. The analysis comprised of classification of strategies used by students to solve the inequalities, and the identification of student difficulties encountered in solving the inequalities. We distinguished the solution strategies into procedural and symbol sense strategies. By symbol sense strategy we mean an inequality solving strategy that applies symbol sense characteristics, whereas the procedural strategy is an inequality solving strategy that does not apply symbol sense characteristics. In the procedural strategy, a standard procedure is used in the solution process. This process of analysis allowed us to evaluate whether the students had acquired both procedural fluency and conceptual understanding in a balanced manner from the course.

RESULTS

General Findings

Table 2 summarizes the findings from the students' written work in solving the inequalities. These findings are considered as a qualitative effect of the two-week teaching and learning process using the direct teaching approach via online media platforms, including Cloud Zoom Meeting, WhatsApp Web, and Google Classroom. As predicted, the students struggled more with task two than they did with task one, and with task four more than with task three. Regarding solution strategies, both procedural and symbol sense strategies emerged, where the procedural strategy appeared more frequent than the latter.

TABLE 2
ANALYSIS OF THE WRITTEN TEST (N = 19)

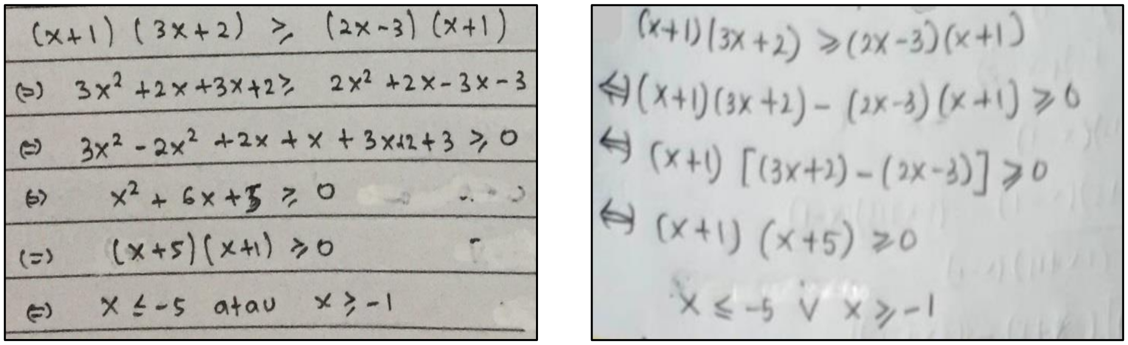
Task	Mathematical inequality	#Correct solution (%)	Solution strategies	
			#Procedural (%)	#Symbol sense (%)
One	$(x+1)(3x+2) \geq (2x-3)(x+1)$	12 (64.1)	14 (73.7)	5 (26.3)
Two	$(x^2+9x+20)(7x-11) \leq (4x-7)(x^2+9x+20)$	4 (28.6)	6 (31.6)	13 (68.4)
Three	$\frac{x}{x-1} < \frac{5}{x+5}$	13 (68.4)	13 (68.4)	6 (31.6)
Four	$\frac{x^2-2x}{x^2-2x+4} < 1$	4 (28.6)	19 (100.0)	0(0.0)

Findings for Task One

For task one, which involved solving the inequality $(x+1)(3x+2) \geq (2x-3)(x+1)$, we found that 64% of the students answered it correctly (Table 2). Although both the procedural and symbol sense strategies emerged, the former strategy is more commonly used by the students (74%). A typical procedural strategy used by the students to solve this inequality is as follows: they expand each side of the inequality to get $3x^2+5x+2 \geq 2x^2-x-3$. Next, they rewrite it into $x^2+6x+5 \geq 0$, and then into $(x+5)(x+1) \geq 0$. Finally, they conclude that $x \leq -5$ or $x \geq -1$ is the solution to the inequality. Incorrect solutions occurred when students made careless errors, for instance, in expanding or factorizing the process of algebraic expressions, and in concluding that the inequality $(x+5)(x+1) \geq 0$ is equivalent to $x \leq -5$ or $x \geq -1$.

A typical symbol sense strategy observed from the students' written work is as follows: a student observes that the inequality $(x+1)(3x+2) \geq (2x-3)(x+1)$ has the same factors $(x+1)$ on both sides. Next, after adding both sides of the inequality with the additive inverse for $(2x-3)(x+1)$, the student uses a distributive property of multiplication over addition to obtain $(x+1)[(3x+2)-(2x-3)] \geq 0$. Finally, after rewriting the inequality into $(x+1)(x+5) \geq 0$, the student concludes $x \leq -5$ or $x \geq -1$. Incorrect solutions occurred, for instance, when students cancel out the same factors $(x+1)$ from both sides of the inequality to obtain $(3x+2) \geq (2x-3)$, and conclude $x \geq -5$. This result is an incorrect solution. Figure 1 illustrates examples of students' written work for the inequality of task one.

FIGURE 1
REPRESENTATIVE EXAMPLES OF STUDENTS' WRITTEN WORK, USING A PROCEDURAL STRATEGY (A) AND A SYMBOL SENSE STRATEGY (B) TO SOLVE THE INEQUALITY



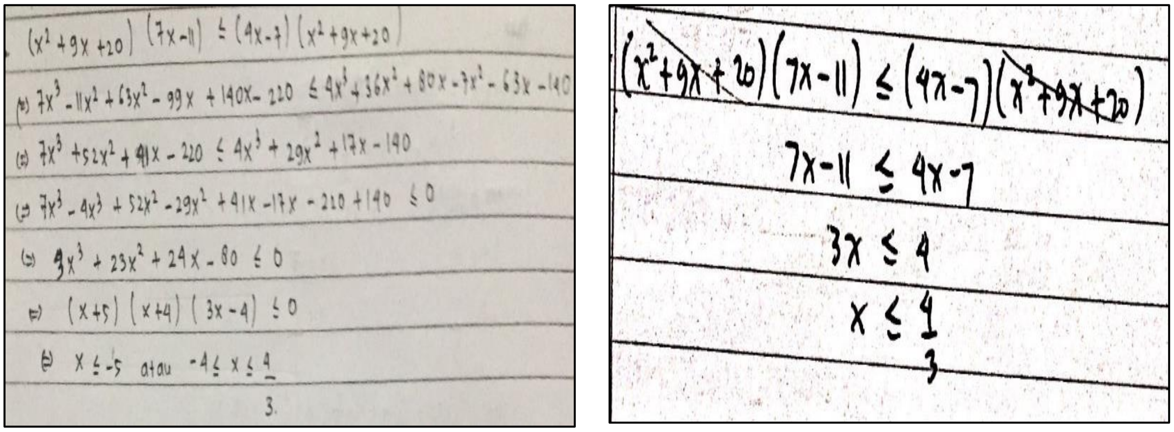
(A) (B)

Findings for Task Two

We found that only four out of 19 (28.6%) students could solve the inequality of task two (Table 2), which involved solving the inequality $(x^2+9x+20)(7x-11) \leq (4x-7)(x^2+9x+20)$ correctly. Surprisingly, the use of symbol sense strategies appeared more often (68.4%) than procedural strategies. A typical procedural strategy observed for solving this type of inequality from the students' written work is as follows: a student expands the linear and quadratic factors on each side of the inequality to get $7x^3 + 52x^2 + 41x - 220 \leq 4x^3 + 29x^2 + 17x - 140$. Next, they rewrite it into $3x^3 + 23x^2 + 24x - 80 \leq 0$, and then into $(x+5)(x+4)(3x-4) \leq 0$. Finally, they conclude $x \leq -5$ or $-4 \leq x \leq 3/4$ to be the solution of the inequality. Incorrect solutions occurred when students made careless errors in expanding the linear and the quadratic factors, or in factorizing the cubic expression $3x^3 + 23x^2 + 24x - 80$ into linear factors of $(x+5)(x+4)(3x-4)$.

A typical symbol sense strategy observed from the students' written work is as follows: similar findings to task one, a student observes that the inequality $(x^2+9x+20)(7x-11) \leq (4x-7)(x^2+9x+20)$ has the same factors $(x^2+9x+20)$ on both sides. Next, after adding both sides of the inequality with the additive inverse for $(4x-7)(x^2+9x+20)$, the student uses a distributive property of multiplication over addition to obtain $(x^2+9x+20)[(7x-11)-(4x-7)] \leq 0$. Finally, after rewriting the inequality into $(x^2+9x+20)(3x-4) \leq 0$ or $(x+4)(x+5)(3x-4) \leq 0$, the student concludes $x \leq -5$ or $-4 \leq x \leq 4/3$ is the solution to the inequality. Incorrect solutions with the symbol sense strategies occurred, for instance, when students cancelled out the same factors $(x^2+9x+20)$ on both sides of the inequality to obtain $(7x-11) \leq (4x-7)$ and conclude $x \leq 4/3$. This solution is incorrect. Figure 2 shows representative examples of students' written work for the inequality of task two.

FIGURE 2
REPRESENTATIVE EXAMPLES OF STUDENTS' WRITTEN WORK FOR TASK TWO USING A PROCEDURAL STRATEGY (A) AND A SYMBOL SENSE STRATEGY (B) TO SOLVE THE INEQUALITY



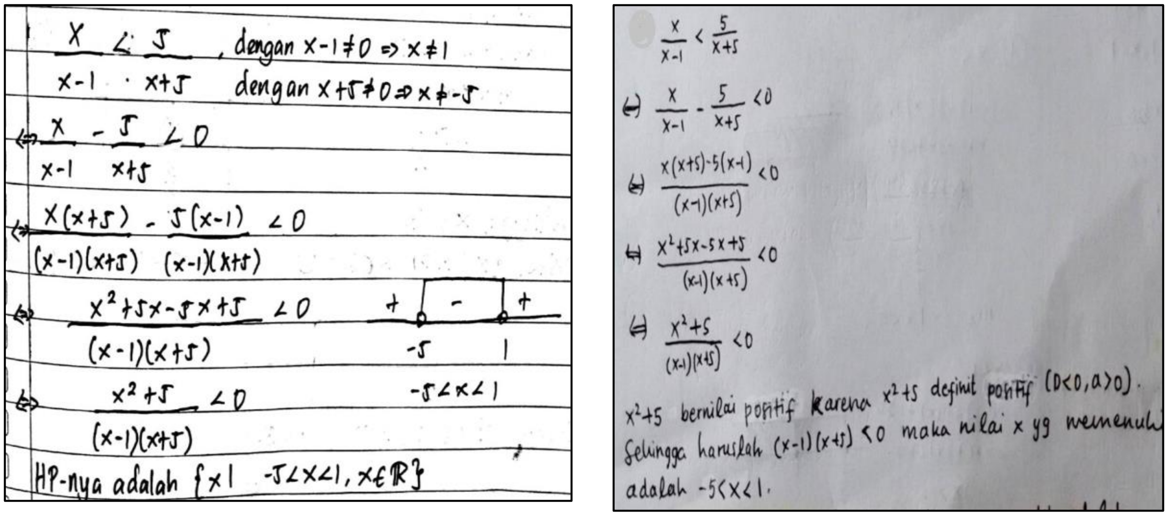
(A) (B)

Findings for Task Three

We observed that 13 (68.4%) students correctly solved task three, which involved finding the solution to the inequality $\frac{x}{x-1} < \frac{5}{x+5}$ (Table 2). We observed both procedural and symbol sense strategies in the students' written work. In this case the procedural strategy was used more frequently (68.4%) than the symbol sense strategy. A typical procedural strategy observed is as follows: from the inequality $\frac{x}{x-1} < \frac{5}{x+5}$, a student rewrites it into $\frac{x}{x-1} - \frac{5}{x+5} < 0$, and into $\frac{x^2+5}{(x-1)(x+5)} < 0$, after which they check whether the value of the inequality is positive or negative, for several numbers less than -5, between -5 and 1, and greater than 1. Finally, they conclude that $-5 < x < 1$ is the solution to the inequality. Incorrect solutions occurred when a student does cross multiplication to the inequality $\frac{x}{x-1} < \frac{5}{x+5}$ to obtain $x(x+5) < 5(x-1)$, which is equivalent to $x^2 + 5 < 0$, and conclude that the inequality has no solution.

A typical symbol sense strategy observed in the students' work is as follows: after rewriting the inequality $\frac{x}{x-1} - \frac{5}{x+5} < 0$ into $\frac{x^2+5}{(x-1)(x+5)} < 0$, a student sees that the numerator has a positive value for every real number, that is $(x^2 + 5) > 0$. As the entire inequality is less than zero, the student concludes that the inequality is equivalent to $(x-1)(x+5) < 0$. Finally, it can be concluded that $-5 < x < 1$ is the solution to the inequality. We did not find any incorrect solutions with this symbol sense strategy in the students' written work. Figure 3 shows representative examples of students' written work for the inequality of task three.

FIGURE 3
REPRESENTATIVE EXAMPLES OF STUDENTS' WRITTEN WORK FOR TASK THREE USING A PROCEDURAL STRATEGY (A) AND A SYMBOL SENSE STRATEGY (B) TO SOLVE THE INEQUALITY



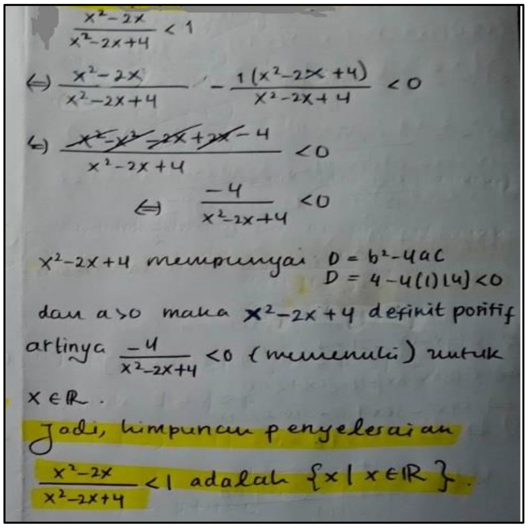
(A) (B)

Findings for Task Four

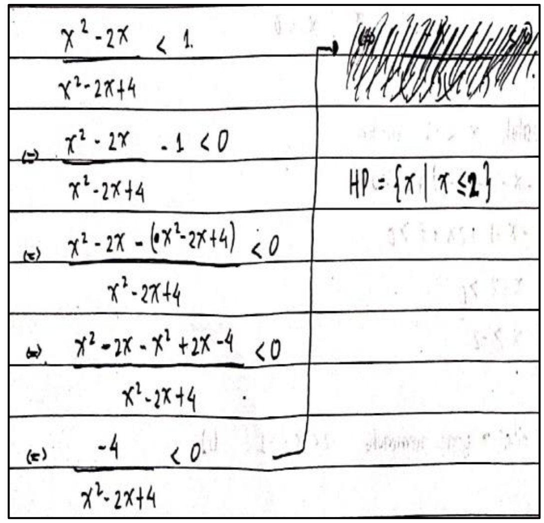
For task four, which involved solving the inequality $\frac{x^2-2x}{x^2-2x+4} < 1$, we found only four (28.6%) students solved this task correctly. It is surprising that all 19 students used the procedural strategy, and no one used the symbol sense strategy in dealing with this type of inequality. A typical observed procedural strategy from students' written work is as follows: a student rewrites the inequality $\frac{x^2-2x}{x^2-2x+4} < 1$ into $\frac{x^2-2x-(x^2-2x+4)}{x^2-2x+4} < 0$, which is equivalent to $\frac{-4}{x^2-2x+4} < 0$. As the numerator $-4 < 0$ and denominator $x^2 - 2x + 4 > 0$ for every real number x, then it can be concluded that every real number satisfies the inequality. Incorrect solutions often occurred because students could not conclude that the inequality $\frac{-4}{x^2-2x+4} < 0$ is satisfied by every real number. They incorrectly concluded that the inequality has no real solution, or it is satisfied by an interval of real numbers. Figure 4 shows representative examples of students' written work for the inequality of the task four using procedural strategies.

A symbol sense strategy should have been carried out as follows: the numerator and the denominator of the inequality $\frac{x^2-2x}{x^2-2x+4} < 1$ have the same algebraic term $(x^2 - 2x)$. If this term is replaced with P, the inequality becomes $\frac{P}{P+4} < 1$, which is correct for every real number $P \neq -4$. Therefore, $x^2 - 2x \neq -4$ is satisfied for every real number x. Unfortunately, no student used this symbol sense strategy to solve the inequality.

FIGURE 4
STUDENTS' WRITTEN WORK FOR TASK FOUR USING A PROCEDURAL STRATEGY WITH A CORRECT SOLUTION (A) AND WITH AN INCORRECT SOLUTION (B)



(A)



(B)

DISCUSSION

From the findings described in the previous section, we need to highlight the solution strategies and corresponding difficulties encountered by mathematics students in dealing with inequalities. Task one and two aimed to determine the student's solution strategies and difficulties in solving quadratic and cubic inequalities. For task one, which involved solving the inequality $(x+1)(3x+2) \geq (2x-3)(x+1)$, if a student identifies the same linear factors $(x+1)$ on each side of the inequality and applies the distributive property of multiplication over addition to obtain, for instance, $(x+1)[(3x+2)-(2x-3) \geq 0$, this demonstrates that the student has read through and has checked the meaning of the entire inequality. This solution strategy involves the use of symbol sense characteristics (Arcavi, 1994; 2005; Jupri and Sispiyati, 2020). Therefore, we assigned this a symbol sense strategy in solving the inequality. If the student directly expands both sides of the inequality to obtain $3x^2+5x+2 \geq 2x^2-x-3$, this means that they, to some extent, used a procedural strategy to solve the inequality. This description on the use of procedural and symbol sense strategies is also applied to task two, which involved solving the inequality $(x^2+9x+20)(7x-11) \leq (4x-7)(x^2+9x+20)$. In our view, the use of symbol sense strategy indicates a more conceptual and relational understanding of the inequality (Skemp, 1976), whereas the use of the procedural strategy indicates a better procedural fluency (Kilpatrick, 2001; Stiphout et al., 2013). Mastering these two aspects of proficiency, demonstrates algebraic proficiency for mathematics education students as prospective mathematics teachers.

Regarding student difficulties in solving inequalities of task one and two, which related to the use of the procedural strategy, errors in expanding and factorizing algebraic expressions were observed. From the perspective of symbol sense, these difficulties involve an inability in manipulating symbolic expressions in the process of problem solving (Arcavi, 2005; Bokhove and Drijvers, 2010; Jupri and Sispiyati, 2020; Jupri et al., 2021). The difficulties encountered by the students, related to the use of the symbol sense strategy, include errors of canceling out the same factors on both sides of an inequality and therefore, concluding an incorrect solution to the inequality. From the perspective of symbol sense, these difficulties demonstrate an inability to read and check the meanings of symbolic expressions (Arcavi, 2005; Bokhove and Drijvers, 2010; Jupri and Sispiyati, 2020; Stiphout et al., 2013).

Second, we need to discuss solution strategies and difficulties for task three and four on solving rational inequalities. For task three, which involved solving the inequality $\frac{x}{x-1} < \frac{5}{x+5}$, if a student rewrites the inequality into $\frac{x^2+5}{(x-1)(x+5)} < 0$, for instance, the student finds $x^2 + 5 > 0$ and concludes $(x-1)(x+5) < 0$. This process indicates that the student has read and has checked the meaning of the entire inequality. These actions are characteristic of using a symbol sense approach to solve the inequality (Arcavi, 1994; 2005; Jupri and Sispiyati, 2020; Stiphout et al., 2013); therefore, this strategy is assigned as a symbol sense strategy. If the student rewrites the inequality into $\frac{x^2+5}{(x-1)(x+5)} < 0$, but directly applies a standard algorithm for solving an inequality using a number line method directly afterwards, then this strategy is perceived as a procedural strategy. This description on the use of symbol sense and procedural strategies is also applied for solving task four, which involved solving the inequality $\frac{x^2-2x}{x^2-2x+4} < 1$.

Observed student difficulties for solving task three, included doing a cross multiplication from $\frac{x}{x-1} < \frac{5}{x+5}$ to $x(x+5) < 5(x-1)$. This indicates that the students are not aware that multiplying both sides of the inequality with $(x-1)(x+5)$ is not allowed because is not known whether x is zero or not. From the perspective of symbol sense, these difficulties indicate an inability to read through and manipulate symbolic expressions in the process of problem solving (Arcavi, 2005; Bokhove and Drijvers, 2010; 2012; Jupri and Sispiyati, 2020). Observed difficulties for task four include an inability to conclude that the inequality $\frac{-4}{x^2-2x+4} < 0$ is correct for every real number. From a symbol sense perspective, these difficulties indicate the students' inability to read and check the meaning of symbolic expressions (Arcavi, 2005; Bokhove and Drijvers, 2010; 2012; Jupri and Sispiyati, 2020; Stiphout et al., 2013).

CONCLUSION

Procedural and symbol sense strategies for solving inequalities emerge in mathematics education students' written work, in which the use of procedural strategies is more frequent than that of symbol sense strategies. In our view, the frequent use of procedural strategies demonstrates the students' procedural fluency, and the use of symbol sense strategies demonstrates the students' conceptual understanding. By conceptual understanding we mean the students' ability in solving inequalities by applying symbol sense characteristics, such as, the ability to read through and manipulate algebraic expressions, as well as to check the meanings of the expressions during the solution process. From these findings, we assume that the two aspects of algebraic proficiency, namely procedural fluency and conceptual understanding, seem to be acquired in an imbalanced manner by prospective mathematics teachers.

The difficulties encountered by mathematics education students in solving inequalities include careless errors in expanding and factorizing algebraic expressions, cancelling out the same factors from both sides of an inequality, and in concluding the meaning of an inequality. From the perspective of symbol sense, errors in expanding and factorizing demonstrate an inability to manipulate symbolic expressions; mistakes in cancelling out the same factors and concluding the meaning of an inequality demonstrate the inability to read through and check for meanings of algebraic expressions. These errors occur because the students' lack proficient knowledge of both procedural fluency and conceptual understanding, the two important aspects of algebraic proficiency.

We propose two recommendations. First, regarding the more frequent emergence of procedural strategies, and considering the limited number of participants in this study, we recommend repeating this study with larger cohort to further investigate whether mathematics education students in Indonesia have a better procedural fluency than conceptual understanding. Also, a deeper investigation on the application of symbol sense characteristics in solving inequalities can be further investigated by using tasks that are designed specifically to investigate additional characteristics of symbol sense strategies. Second, concerning difficulties encountered by mathematics education students either in applying procedural or symbol sense strategies, we recommend further investigation, for instance, by diagnosing students' difficulties in conducting basic procedural skills in solving inequalities, including expanding, factorizing, cancelling out, and checking the meanings for algebraic expressions. In this way, careless and unnecessary errors can be anticipated in the learning and teaching process for prospective mathematics teachers. This in turn will improve the quality of prospective mathematics teachers and mathematics education in the future.

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