

Students as Prospective Teachers' Understanding of Integral Based on the APOS Theory in Terms of Gender Difference

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The present qualitative study aims to describe the understanding of integral by college students as prospective teachers based on the APOS theory with regard to gender differences. The research subjects were one female student (FS) and one male student (MS). The data were collected through assignments on integral and interviews. The results of data analysis showed that at the actions stage, FS and MS solved the integral problem procedurally. In the processes stage, FS explained the step of determining the area using a definite integral by installing the boundaries taken from the given equation. On the other hand, MS used a definite integral while still paying attention to the influence of the area on the x-axis. In the objects stage, both subjects showed some integral relationships as the totality of the process based on integral characteristics. At the schemes stage, the coherence of the two subjects' schemes was different.

Keywords: integral, actions, processes, objects, schemes

INTRODUCTION

Prospective mathematics teachers will become mathematics teachers who will assist students in learning, and teach them how to learn mathematics effectively and efficiently. The research on prospective mathematics teachers would have a significant impact on education (Juniati & Budayasa, 2022). Several studies have been conducted on prospective mathematics teachers, such as looking at the factors that affect the performance of prospective mathematics teachers, research on communication and creative reasoning of prospective mathematics teachers in solving mathematics problems (Budayasa & Juniati, 2019; Juniati & Budayasa, 2020; Misu et al., 2019; Palengka et al., 2019).

One of the objectives of learning mathematics is to build an understanding of a concept being studied. In constructing a concept, individuals carry out activities of searching, finding, and connecting knowledge that has been previously known with information obtained to build new knowledge about the concept they are learning.

The concept of “integral” is part of any calculus course in college and is mandatory to be mastered by a prospective mathematics teacher. In Indonesia, integral is introduced to students for the first time in high school. Therefore, integral is not new for students who are prospective mathematics teachers and studying in the Mathematics Education program; however, it is still not easy for prospective teachers to understand. Yudianto (2017) found that in solving integral problems, students and prospective teachers used predictions that were not detailed. Abdul-Rahman (2005) revealed that students tend to focus on integration techniques only and suppress concept awareness regarding geometric shapes when facing integration problems. Maharaj (2014) found that students had difficulty applying rules for integration. In addition, several studies reveal that students also have difficulty using integral concepts in the course of calculus (Bajracharya & Thompson, 2014; Bezuidenhout & Olivier, 2000; Cui & Bennett, 2006). This shows that students’ understanding of integral needs serious attention, especially for prospective teachers in order to ensure no mistakes when teaching the concept of integral later.

There are various theories used by researchers to examine the process of thinking when constructing a mathematical concept. Hidayatullah (2019) compared APOS and SOLO theories in examining the construction process of students’ mathematical understanding. Octac (2019) analyzed the mental construction of Masters students of mathematics education in linear algebra using the APOS theory. Dahl (2006) summarized and discussed the conceptual construction cycle model proposed by Pegg and Tall.

To examine the process of constructing an integral understanding of students as prospective teachers in this study, the APOS theory (Actions, Processes, Objects, Schemes) was used. Dubinsky (1991) stated that the APOS theory describes the stages of a person building their knowledge of a mathematical concept as the actions stage, processes stage, objects stage, and schemes stage. Several researchers have used APOS theory to analyze students’ mental activity in constructing their understanding of a mathematical concept (Afgani et al., 2017; Altieri & Schirmer, 2019; Listiawati & Juniati, 2021; Martínez & Parraguez, 2017; Prihandhika et al., 2019; Trigueros, 2019; Wulandari et al., 2019). In this study, the construction of integral understanding based on the APOS theory is the process of re-describing integral concepts through stages of actions, processes, objects, and schemes. In APOS theory, the actions stage is an object transformation carried out by individuals and is external. Parragues and Oktac (2010) mentioned that at the actions stage, individuals perform mathematical object transformations and perform calculations as a result of external stimuli so that each completion step that appears is triggered by the previous step. At the actions stage, individuals cannot anticipate or eliminate measures without explicitly doing so (Langi et al., 2021; Planell & Delgado, 2016). The thinking process at the actions stage is controlled by outside stimuli so that each step of solving the problem is carried out expeditiously and in an orderly manner so that nothing is missed. The actions stage in this study explored the students’ thinking process in building an understanding of the integral concepts that can be seen from solving the given integral problems.

The processes stage is an internal construction made by individuals through the same actions but does not have to be controlled by external stimuli (Baker et al., 2000; Planell & Delgado, 2016). Concurrently, Arnon et al. (2014) also stated that individuals experience movement from relying on external cues to having internal control when repeating actions and reflecting on them. The transformation from external control to internal control involves a mental mechanism called “interiorization.” This can be seen from the ability of individuals to not only perform each explicitly but to skip steps and reverse them (Dubinsky, 2005; Parragues & Octac, 2010). Individuals can make predictions and, simultaneously, imagine the response to a stimulus without making detailed steps because the transformation has been in their control. The processes stage in this study was studied through the explanation of the steps of calculating an area using integral.

The objects stage is a form of the totality of the process where action can be taken on the process (Dubinsky & McDonald, 2001). In constructing a mathematical concept at the objects stage, one realizes that the process is a totality of actions and forms it into mathematical objects (Baker et al., 2000). The formation of objects involves a mental mechanism called “encapsulation,” which is the transformation of processes into mathematical objects (Mamolo, 2014). In addition to encapsulation, the transformation includes de-encapsulating the object to the forming process. De-encapsulation plays a role in obtaining the properties of the process so that it can be used to perform actions on objects (Asiala et al., 1997). De-

encapsulation is not only carried out on one object; two objects can also be re-encapsulated into the forming process. These processes can then be coordinated and re-encapsulated to form new objects. In this study, students' understanding of the concept of integral at the objects stage was studied by explaining the relationship between two integrals with the same integrand.

At the schemes stage, individuals have constructed coordination that links actions, processes, objects, and other schemes to solve problems. Action on the schemes stage can be an operation of a scheme or a comparison between one scheme and another (Clark, 2007). A scheme is an organizing unit of knowledge about objects or events based on past experiences and accessed to guide current understanding or action (Dubinsky, 1991). Schemes are coherent, which means one can connect the different components of the scheme when given a problem to decide whether the component is included in the scope of the scheme (Planell & Delgado, 2016). One of the characteristics of themes in APOS theory is understanding the relationships between actions, processes, objects, and other properties previously understood by individuals. In this study, the understanding of the integral concept of students who are prospective teachers at the schematic stage is studied through the explanation of integral characteristics by connecting actions, processes, objects, and other schemes that can be used in dealing with integral problems.

Learning a mathematical concept is an internal mental activity in each individual. The process of building an understanding of the mathematical concepts of each individual is different from other individuals because it is influenced by various factors. One of the factors that can be reviewed is gender differences considering that male and female students display characteristics that tend to be different in learning mathematics. Several studies have highlighted gender differences in learning mathematics. Contini et al. (2016) revealed that young males' math learning achievement was better than that of young females. Lestari et al. (2018) found differences in mathematical literacy between prospective female and prospective male teachers. There are also some studies stating that women's abilities are better than men's in terms of methods of verbal and visual solutions to solving geometry problems (Mainali, 2014; Puloo et al., 2018). In addition, Sari et al. (2017) described that boys' probabilistic thinking level is higher than that of girls. This can be an overview to conduct a study on how the process of integral reconstruction of students who are prospective teachers of mathematics is based on gender differences.

Based on the above description, this study concentrated on the process of building an integral understanding of students who are prospective teachers by using the theory of gender differences. The theory is used because it is believed to reveal the thinking processes of the students in mathematics education program when constructing integral through stages of actions, processes, objects, and schemes. Here, integral is the researchers' concern based on the need for students to master this concept as a compulsory subject in calculus courses, especially for prospective teachers of mathematics. Although several studies have examined integral problems (Jones, 2013; Jones et al., 2016; Misu, 2019), there is still little focus on the cognitive process in constructing an understanding of integral, especially by considering gender differences even though these differences also have an influence on the individual learning process. Previous researchers have focused more on the effect of gender differences on student learning outcomes but have paid less attention to how cognitive processes occur. Therefore, this study can be one of the references for the next generation of researchers on the importance of paying attention to the role of gender differences in students' cognitive processes when constructing mathematical understanding.

METHODS

Research Design

The purpose of the present qualitative study is to describe the process of constructing an understanding of integral by students who are prospective teachers based on the APOS theory as viewed from gender differences. The subjects were chosen based on who received the highest score on a Mathematical Ability Test results from each gender group. The data were collected through Integration Task (IT) sheets and accompanied by unstructured interviews. To test the credibility of the data obtained, time triangulation was performed.

Sample and Data Collection

The selection of research subjects was carried out through a written test using the Mathematical Ability Test (MAT) in the form of six validated mathematical questions selected from a bank of college entrance exam questions. The MAT was given to 22 Mathematics Education students. Based on the test results, one female student with a score of 78 and one male student with a score of 75 were obtained. The selected subjects had equivalent abilities as seen from the similarity in their test scores. The selected subjects were assigned an Integration Task developed by the researchers and accompanied by an unstructured interview. The IT questions can be described as follows:

1. Determine $\int (3x^2 + 2)^5 6x dx$!
2. What is your step in calculating the area delimited by the curve $y = g(x)$, the x -axis, $x = a$, and $x = b$?
3. Given two functions, namely, $f(x)$ and $g(x)$, if $g(x) = kf(x)$, then what is the relationship between $\int f(x)dx$ and $\int g(x)dx$?
4. Given functions $f(x)$, $g(x)$, and $h(x)$, if $f(x) = g(x) + h(x)$, then what is the relationship between $\int f(x)dx$ with $\int g(x)dx$ and $\int h(x)dx$?
5. If given a value of $a > b$ with a function $f(x)$, then what is the relationship between $\int_a^b f(x)dx$ with $\int_b^a f(x)dx$?
6. An area is delimited by the $f(x)$ curve, having symmetry on the y -axis with boundaries $x = -a$ and $x = a$. What is the relationship between $\int_0^a f(x)dx$ and $\int_{-a}^a f(x)dx$?

Descriptive Analysis

The data are analyzed based on the indicators of actions, processes, objects, and schemes, which are at the core of APOS theory. A description of the indicators for each stage is detailed in Table 1.

TABLE 1
DESCRIPTION OF ACTIONS, PROCESSES, OBJECTS, AND SCHEMES

Operational	Description
Actions	<ul style="list-style-type: none"> - Determine the appropriate rules to resolve the given integral problem. - Solve integral problems using appropriate integration rules.
Processes	<ul style="list-style-type: none"> - Explain the steps to determine the area by using integral.
Objects	<ul style="list-style-type: none"> - Explain the relationship between the two different integrals using the property of linearity of integral multiplication with a constant. - Explain the relationship between several integrals using the property of linearity on the integral summation. - Explain the relationship of two integrals with the same integral using the characteristics of the boundary change. - Explain the relationship of two integrals with the same integer using the nature of symmetry.
Schemes	<ul style="list-style-type: none"> - Explain the nature of integral by connecting actions, processes, objects, and other possible schemes to deal with integral problems.

RESULTS

After the subjects worked on the Integration Task that accompanied the interview, an overview was assembled to explore their integral understanding as prospective teachers according to the APOS theory.

The actions stage was explored by solving the first problem, which was to determine the results of the given integral problem using the appropriate rules. The answers of FS and MS are presented in Figure 1.

FIGURE 1
SUBJECTS' RESPONSES TO THE FIRST QUESTION

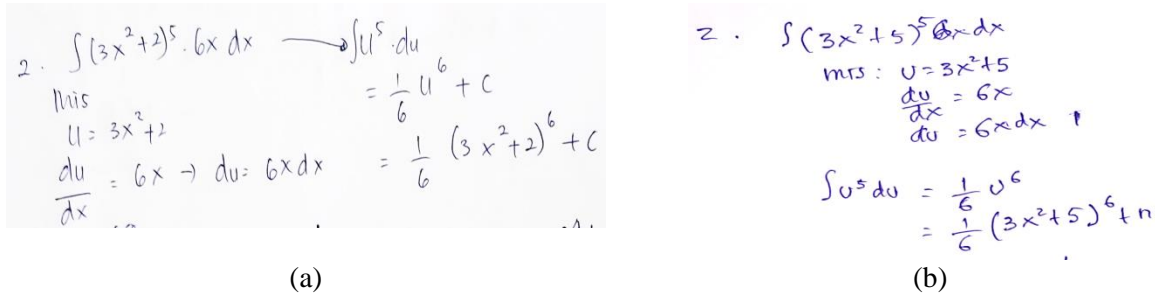


Figure 1. Responses to Question 1 by FS (a) and MS (b). Based on the subjects' written responses, an interview was conducted with each subject to confirm what they wrote. A transcript of these interviews is reported in Table 2.

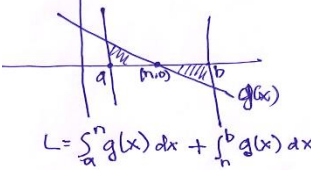
TABLE 2
INTERVIEW RESPONSES AT THE ACTIONS STAGE

Interview with FS		Interview with MS	
Researcher	What rules do you use to solve this integral?	Researcher	What rules do you use to solve this integral?
FS	Substitution rules, ma'am.	MS	Substitution, ma'am.
Researcher	Why do you use the substitution rule?	Researcher	Why do you use the substitution rule?
FS	Because $3x^2 + 2$ is derivated into $6x dx$. If assumption is done, the integral is simpler and easier to integrate.	MS	Because if $3x^2 + 2$ this is derived, the result is $6x dx$. Both of them are in the problem so the substitution rule can be applied.
Researcher	In your answer there is a plus c . What does this c mean?	Researcher	Here, you wrote that the integral result is $3x^2 + n$. What's n ?
FS	C is any constant.	MS	That signifies a constant, ma'am.
Researcher	So how many functions have derivatives $6x$?	Researcher	How many functions have derivatives $6x$?
FS	It is infinity, ma'am. Because this c is an infinite number of real numbers.	MS	It is infinity, ma'am. Because the point is there, ma'am, $3x^2$ plus any n constants, whereas the constants themselves are infinite.

FS and MS solved the given integral problem using the rules of substitution. This rule was chosen because its integrand consists of a function and its derivatives. FS applied procedurally the rule of substitution in the integral, where $3x^2 + 2$ is denominated as u with its derivatives $du = 6x dx$. Both subjects substituted u and du into the question so that it became $\int u^5 du$. Next, the problem was solved until it resulted in $\frac{1}{6}u^6 + c$ by FS while MS wrote the answer as $\frac{1}{6}u^6 + n$. FS and MS used different symbols to denote constants, namely, c and n . The last step was to replace the previously initialized value of u . FS answered $\frac{1}{6}(x^2 + 4)^6 + c$ and MS's answer was $\frac{1}{6}(3x^2 + 2)^6 + n$.

In the processes stage, both subjects explained the steps to determine the area. A transcript of the interview can be seen in Table 3.

TABLE 3
TRANSCRIPT OF INTERVIEW AT THE PROCESSES STAGE

Interview with FS		Interview with MS	
Researcher	What is your step in calculating the area delimited by the curve $y = g(x)$, the x -axis, $x = a$, and $x = b$?	Researcher	What is your step in calculating the area delimited by the curve $y = g(x)$, the x -axis, $x = a$, and $x = b$?
FS	I use definite integral, ma'am, so the area is $\int_a^b g(x)dx$.	MS	I could use definite integral so it becomes $\int_a^b g(x)dx$.
Researcher	Why use definite integral?	Researcher	Why use definite integral?
FS	Because one of the applications of integral is certainly to calculate the area under the curve. In this problem, there is also a limit, namely, $x = a$ and $x = b$ so it can be made in a definite integral form.	MS	Because definite integral is useful for calculating the area under the curve.
		Researcher	Can you imagine the shape of the area?
Researcher	Can you describe what you imagine about the area and explain how to calculate the area?	MS	If it's an integral, this means that the area is above the x -axis.
		Researcher	If a curve $g(x)$ intersects the x -axis, how do you calculate the formed area?
FS	<i>(Thinking for a moment)</i> If you calculate the area, you can immediately use the integral as I wrote earlier, ma'am.	MS	If it intersects the x -axis, it means that there is a cut-off point with the x -axis that causes the formed area to be above the x -axis and below the x -axis. Let's say it intersects at the point $(n, 0)$, meaning that later the area is the sum of the area above the x -axis with the area below the x -axis. Here's the picture.
Researcher	So the area can be calculated without having to draw the shape and the location of the area, right?		
FS	Yes, ma'am.		 $L = \int_a^n g(x) dx + \int_n^b g(x) dx$
		Researcher	The result of this equation will be positive or negative?
		MS	For the first integral, the result is positive because the area is above the x -axis, whereas for the second integral the result is negative because the area is below the x -axis.
		Researcher	Why is the second integral result negative even though it explains about the area?
		MS	So that the negative sign disappears, the result is multiplied by negative.
		Researcher	Is there any other possibility you can imagine for the shape of the area?
		MS	There's none, ma'am.

Both subjects revealed that the area with the given limits could be calculated by applying definite integral. FS did not consider the shape or location of the area referred to in the question because she thought the instructions in the question to have been fulfilled to be brought into the form of the definite integral. Therefore, FS explained the steps to determine the area according to her understanding of the problem leading to the results in the form $\int_a^b g(x)dx$. It was different for MS, who described the shape and location of the area in accordance with the integral that he wrote, which is located in the first quadrant. MS also described another possible shape, namely, curve $g(x)$, intersecting the x axis at the point $(n, 0)$ so that two parts of the area are formed: one part above the x -axis and one part below the x -axis. To calculate the total area, MS summed up two integral pieces that represent the two areas formed, namely, $\int_a^n g(x)dx + \int_n^b g(x)dx$. MS revealed that the result of $\int_a^n g(x)dx$ is positive because it represents the area of the region above the x -axis while the result of $\int_n^b g(x)dx$ is negative because it represents the area of the region below the x -axis.

At the objects stage, both subjects described some integral relationships as a form of the totality of processes that can be seen from the resolution of IT problems no. 3–6. The answers of FS and MS were explained as follows.

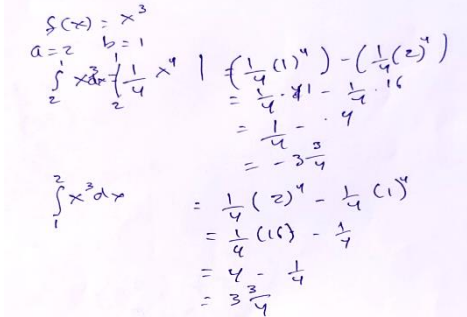
When given two different integral values (IT no. 3), namely, $\int f(x)dx$ and $\int g(x)dx$ with $g(x) = kf(x)$, FS and MS formed $\int f(x)$ as one object as well as $\int g(x)$ as a unitary object consisting of objects $f(x)$ as many as k as the forming elements so that they are written as $\int kf(x)dx$. When integrated, both subjects connected the two integrals as two interconnected objects, which is $\int g(x)dx$, the same as $k \int f(x)dx$. Both subjects revealed that when a function is multiplied by a constant, its integral result will also be multiplied by the same constant.

The second problem (IT no. 4) required clarifying relationships between $f(x)$, $g(x)$, and $h(x)$ with $f(x) = g(x) + h(x)$. Both subjects regarded $f(x)$, $g(x)$, and $h(x)$ as integrable objects. Specifically, for integrating $f(x)$, which is the sum of $g(x)$ and $h(x)$, both subjects revealed that $\int f(x)dx$ is also the sum of both integrals, i.e., $\int g(x)dx + \int h(x)dx$. When $f(x)$ is integrated, the forming objects $g(x)$ and $h(x)$ are also integrated, and then added. Furthermore, it can be seen that both subjects were aware of the linearity in the integral sum; that is, if $f(x)$ is integrated, then $g(x)$ and $h(x)$ are also integrated.

The third problem (IT no. 5) addresses the objects stage when funding the relationship between $\int_a^b f(x)dx$ and $\int_b^a f(x)dx$ if $a > b$. Both subjects gave the same answer but used different processes. The answers of FS and MS are listed in Table 4.

TABLE 4
TRANSCRIPT OF THE INTERVIEW AT THE OBJECTS STAGE

Interview with FS		Interview with MS	
Researcher	If given a value $a > b$ with a function $f(x)$, then what is the relationship between $\int_a^b f(x)dx$ and $\int_b^a f(x)dx$?	Researcher	If given a value $a > b$ with a function $f(x)$, then what is the relationship between $\int_a^b f(x)dx$ and $\int_b^a f(x)dx$?
FS	The result is like this, ma'am. <i>(Shows the answer she wrote)</i>	MS	If $\int_a^b f(x)dx$, this is most likely to be a negative integral result because the upper boundary is smaller than the lower boundary; then, $\int_b^a f(x)dx$ is likely to be positive because the lower boundary is larger than the lower boundary.

	<p>$a > b$ fungsi $f(x)$, hubungan antara $\int_a^b f(x) dx$ dengan</p> <p>$\int_b^a f(x) dx$ $a=2, b=1, f(x)=2x$ $\int_2^1 2x dx = x^2 \Big _2^1$ $= 1^2 - 2^2 = -4$ $\int_1^2 2x dx = x^2 \Big _1^2$ $= 2^2 - 1^2 = 4$ $a=3, b=1, f(x)=x^2$ $\int_3^1 x^2 = \frac{1}{3} x^3 \Big _3^1$ $= \frac{1}{3} \cdot 1^3 - \frac{1}{3} \cdot 3^3$ $= \frac{1}{3} - \frac{27}{3} = -\frac{26}{3}$ $\int_1^3 x^2 = \frac{1}{3} x^3 \Big _1^3$ $= \frac{1}{3} \cdot 3^3 - \frac{1}{3} \cdot 1^3 = \frac{26}{3}$</p> <p>$\int_a^b f(x) dx = -\int_b^a f(x) dx$</p>	Researcher	How can you conclude like that?
Researcher	Here, you make it into some examples, right? What can you deduce from your work?	MS	<p>I think the integer is the same, ma'am. Or let me show you some examples of the problem, ma'am, like this (showing some of the problems that are made and solved by himself).</p>  <p>From here it seems that if the upper boundary and the lower boundary are changed, it will affect the integral result.</p>
FS	Yes, ma'am, I tried some examples that I made myself. So, if $a > b$, then $\int_b^a f(x) dx = -\int_a^b f(x) dx$.	Researcher	So, what can you explain about the relationship of the upper and lower limit values to the integral result of this problem?
Researcher	Can you tell me something about the values of a and b on the integral results?	MS	So, if the value $a > b$, then the relationship will be $\int_b^a f(x) dx = -\int_a^b f(x) dx$.
FS	If I look here, ma'am, the integral whose lower boundary is smaller than its upper boundary will be negative if the lower boundary is exchanged. It means that if the limits are changed, then the sign will also change.		

Both subjects viewed $\int_a^b f(x) dx$ and $\int_b^a f(x) dx$ as objects on which an integration action can be carried out. FS first made an example of the problem according to the instructions given. She implemented the action of integrating according to the example of the problem she made on the objects $\int_a^b f(x) dx$ and $\int_b^a f(x) dx$ to then be concluded as when two integral pieces with the same integrand have an upper boundary and a lower boundary that exchange positions, then the two integral pieces will be opposite signs. On the other hand, MS first explained his prediction of those integrals, saying that it is likely that the result of integration $\int_a^b f(x) dx$ will be negative because its upper boundary is smaller than its lower boundary, whereas for $\int_b^a f(x) dx$ it is possible to be positive as the upper boundary is greater than its lower boundary. To prove the prediction results, MS made two examples of problems by assigning a and b values and function $f(x)$ according to the question instructions. After solving the problem he created, MS finally concluded that from the case there will be a relationship $\int_b^a f(x) dx = -\int_a^b f(x) dx$. MS explained that the integral result would be positive if the lower limit is smaller than the upper limit, otherwise it would be negative if the lower limit is bigger than upper limit.

The last problem (IT no. 6) was to explain the relationship between $\int_0^a f(x) dx$ and $\int_{-a}^0 f(x) dx$ if given an area delimited by the curve $f(x)$ and symmetry on the y -axis with the boundaries $x = -a$ and $x = a$. Both subjects first sketched according to the instructions from the question that can be seen in Figure 2.

FIGURE 2
SYMMETRICAL AREA SKETCH BY BOTH SUBJECTS

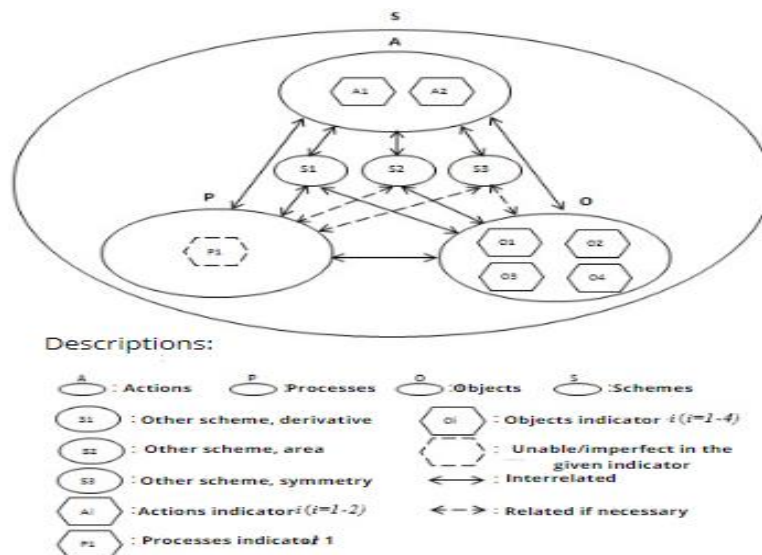


Figure 2 shows the similarity of the shape of the sketch by FS (a) and the sketch by MS (b) in the form of a straight curve that is symmetrical on *y-axis*. Furthermore, both subjects tried to understand the integral given based on the drawings they had made. Both subjects explained that $\int_0^a f(x)dx$ stated the area to the right of the *y-axis*, while $\int_{-a}^a f(x)dx$ stated the area as a whole. Based on the analysis, both subjects concluded that the relationship between the two integrals is $\int_0^a f(x)dx = \frac{1}{2} \int_{-a}^a f(x)dx$. This relationship occurs considering that the area formed from the curve is in the $f(x)$ form of a region that is symmetrical on the *y-axis*. MS further explained that the integral, which expresses the overall symmetrical area in the *y-axis*, has the same upper and lower boundaries but has a different sign. In addition, MS also imagined other symmetry curves, such as parabolic curves that allow the occurrence of these two integral relationships when calculating the area under the curve.

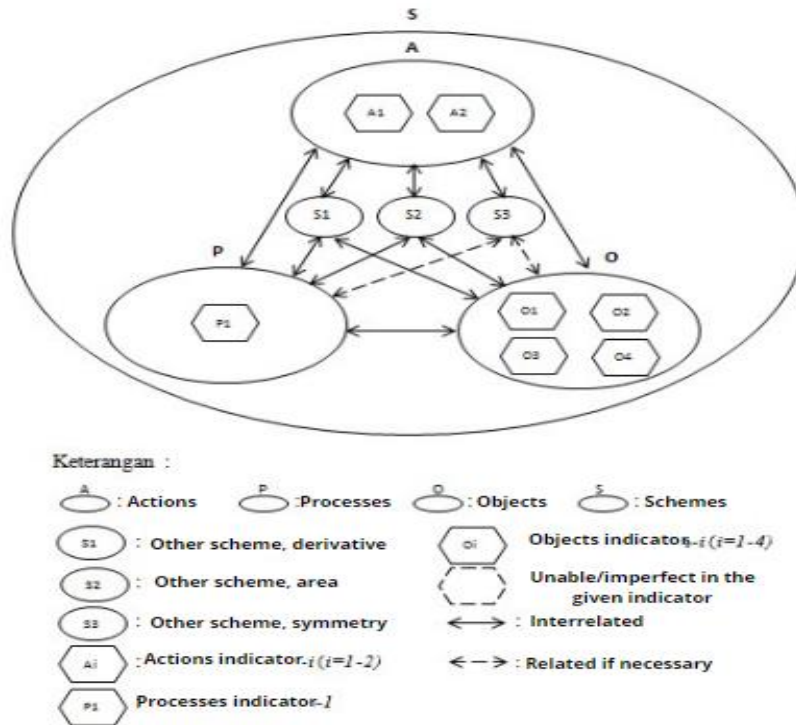
After solving the series of integral problems in this study, both subjects' schemes are reproduced as Figure 3 (FS) and Figure 4 (MS).

Based on Figure 3, it can be said that in building her understanding of integral, FS connected actions, processes, and objects with other schemes, namely, derived schemes, area, and symmetry. The scheme shows that the process indicators are not perfectly done by the female subject (FS) when explaining the steps of determining the area using integral. In addition, the process indicators would relate, if necessary, to the regional broad scheme and symmetry. The same thing is also shown in the object indicator, namely, describing that the relationship of two integrals with the same integrand using the characteristics of symmetry would be related if necessary to the symmetry scheme.

FIGURE 3
FS SCHEME



**FIGURE 4
MS SCHEME**



Based on Figure 4, it can be said that in building his understanding of integral, MS connected actions, processes, and objects with other schemes, namely, derivative, area, and symmetry. The MS scheme shows the processes indicator that would be related, if necessary, to the regional area scheme and symmetry. The same thing is also shown in the object's indicator, namely, describing that the relationship of two integrals with the same integrand using symmetry characteristics would be related if necessary to the symmetry scheme.

DISCUSSION

Based on the results of the research, it is revealed that there are several similarities and differences in the thinking processes exhibited by FS and MS in building their integral understanding based on the APOS theory. Starting at the actions stage, both solved the integral problem procedurally using the substitution rule. The substitution rule was chosen because the integral is an integrand that contains a function and its derivatives. In its application, the two subjects wrote the integral solution in a comprehensive and step-by-step manner according to the instructions and requests in the question. Kazunga and Bansilal (2020) stated that in APOS theory, action is a reaction to external stimuli felt by individuals. Parragues and Octac (2010) explained that at the actions stage, individuals can perform calculations and mathematical object transformations due to external stimuli where the previous step triggers each next step. In this case, the outside stimuli received by FS and MS are the given integral questions while the response is in the form of complete problem solving with a process that is structured according to the rules of substitution. There is a difference in writing constant symbols in the final result of both subjects' answers. Nevertheless, both subjects had the same understanding of the meaning of the symbols they wrote, namely, the constant symbol (c or n) of the general form of integral. In contrast, Maharaj's research (2014) found that students did not know the meaning of the constants they wrote on the integration results and were not even aware of integrals as antiderivatives. Both subjects of this study interpreted that there are infinitely many functions that have

derivatives as given in the problem due to the influence of the addition with constant, which is also unknown. The thinking process of both subjects at the actions stage shows the same procedural ability in understanding the concept of integral.

At the processes stage, both subjects explained the steps of determining the area bounded by several equations using integral. FS directly applied the definite integral with the boundaries taken from the equations given in the problem so as to produce $\int_a^b g(x)dx$. Baker (2000) and Dubinsky (2001) mentioned that the process in APOS theory is an internal construction that is no longer controlled by external stimuli and is processed after repeating the action. Through the mental mechanism of *interiorization*, FS imagined the integral as a representation of the shape and the area in question because it contained all the equations given in the problem. Abdul-Rahman (2005) revealed that students tend to focus on integration techniques only and suppress concept awareness regarding geometric shapes when facing integration problems. FS did not tell the shape of the area in question but focused on calculating the area. On the other hand, MS's explanation was accompanied by a description of the possible forms of the area formed from the equations given. The integral boundaries used were adjusted to the location of the formed area so that two integral possibilities can be obtained that can represent the calculation of the area, namely, $\int_a^b g(x)dx$ and $\int_a^n g(x)dx + \int_n^b g(x)dx$. Furthermore, MS imagined that the results of these integrations would be either positive or negative depending on the position of the region relative to the x -axis. MS internalized not only in terms of solving the integral in question but also in imagining the form of regions that can occur from the given equations. Mainali (2014) and Puloo et al. (2018) revealed that women's abilities are better than men's in terms of methods of verbal and visual solutions in solving geometry problems. At the processes stage, both subjects provided verbal solutions about the steps of determining the area of the bounded area using several equations using integral, but MS showed better performance when visualizing geometric shapes of the proposed problem.

In APOS theory, when someone constructs a mathematical concept and realizes that the process is a totality formed into mathematical objects and on which actions can be applied, it means that the individual has been at the objects stage (Baker, 2000; Dubinsky & McDonald, 2001). When given two different integrals, namely, $\int f(x)dx$ and $\int g(x)dx$ with $g(x) = kf(x)$, both subjects formed $\int f(x)$ as one object as well as $\int g(x)$ as a unity of objects consisting of objects $f(x)$ and using as many as k as their forming elements so that they are written as $\int kf(x)dx$. When integrated, both subjects connected the two integrals as two interconnected objects, which is $\int g(x)dx$ and the same as $k \int f(x)dx$. Both subjects revealed that when a function is multiplied by a constant, its integral result will also be multiplied by the same constant. From this thinking process, it can be said that both subjects performed the mechanism of *encapsulation*. In APOS theory, encapsulation is a mental mechanism that transforms a process so that individuals can summarize $g(x)$ into an object (Arnon et al., 2014). $f(x)$ is an element wrapped in a unitary object as a forming element of $g(x)$, which consists of $f(x)$ by k times.

When given functions $f(x)$, $g(x)$, and $h(x)$ with $f(x) = g(x) + h(x)$, and asked to describe the integral relationship of each of those functions, both subjects regarded each given integral function as an object-object relationship. This shows that both subjects went through the mental mechanism of *encapsulation*, which is to see these integrations as the totality of the processes by which the actions of integrating can be carried out. Both subjects realized that each object could be integrated. This is in line with the opinion of Borji and Planell (2020) who said that individuals with object conceptions can see dynamic structures, i.e., processes, as static structures where action can be applied. Specifically, for integrating $f(x)$, which is the sum of $g(x)$ and $h(x)$, both subjects revealed that $\int f(x)dx$ is also the sum of both integrals, i.e., $\int g(x)dx + \int h(x)dx$. When $f(x)$ is integrated, the forming objects $g(x)$ and $h(x)$ are also integrated, and then added. From this, it can be seen that both subjects made a conclusion by considering the characteristics of the integral they face, namely, if the integrand is in the form of summing several functions, then the integration process is carried out on each number and then summed.

At the objects stage, both subjects explained the relationship between the values $\int_a^b f(x)dx$ with $\int_b^a f(x)dx$ if $a > b$. FS formed the given integral into objects and then applied actions to the objects. This is shown through the steps taken by FS, namely, by first making an example of the problem according to the instructions given. Abdul Rahman (2005) revealed that the use of examples to describe mathematical concepts has become a part of integral instruction in mathematics. When using examples to clarify definitions and exemplify specific rules, students' thinking tends to be limited only to the appropriate examples and does not see the common ground behind the selected examples. FS applied the integrating action according to the example she had created against the objects $\int_a^b f(x)dx$ and $\int_b^a f(x)dx$. FS achieved the objects stage by concluding that when two integrals with the same integrand have an upper boundary and a lower boundary that exchange positions, the two integrals will be of opposite sign. There was a slight difference shown by MS at the objects stage. MS formed the given integral into objects and then applied actions to the objects. MS predicted the results of these integrations without doing it explicitly but by imagining integration and focusing more on the boundaries of the integrations to conclude that the integration results would be positive or negative. In their research on how to overcome integral problems through anticipation, Yudianto et al. (2018) mentioned that students solve integral problems and analyze the problems given through preliminary guesses (predictions), then solve problems not in detail (forecasting) but the results are correct. Although MS in this study had predicted the relationship of the two integrals given to him, he continued by making an example of the problem according to the instructions given to prove the prediction. MS applied actions, namely, integrating according to the example he made on the objects $\int_a^b f(x)dx$ and $\int_b^a f(x)dx$, and then making reflections according to the results obtained. The objects stage was achieved by both subjects by concluding that when two integrals with the same integrand have an upper boundary and a lower boundary that exchange positions, the two integrals will be of opposite sign.

The objects stage was also indicated by both subjects when describing the relationship between $\int_0^a f(x)dx$ and $\int_{-a}^a f(x)dx$ if given an area delimited by a curve $f(x)$ and symmetry on an axis y with boundaries $x = -a$ and $x = a$. Both subjects *encapsulated* both integrals given as cognitive objects, where the form $\int_0^a f(x)dx$ is used to calculate the area on the right of the y -axis while $\int_{-a}^a f(x)dx$ calculates the overall area. Both subjects were aware that by applying integrating actions to these objects, the value of the area would be obtained; even though they did not write it directly, they could also imagine integrating them. In the end, both subjects concluded that $\int_0^a f(x)dx = \frac{1}{2} \int_{-a}^a f(x)dx$. There are differences in how both subjects think to form objects on this problem. FS *encapsulated* based only on the description in the problem that the curve formed is symmetrical while MS *encapsulated* not only based on the description in the problem but also on recognizing the characteristic of its integral boundary, which is the same number but with different sign, i.e., symmetry in the y -axis. Furthermore, both subjects de-encapsulated through the statement that the area on the right is equal to the area on the left of the y -axis, so that the overall area is twice the area on the right of y -axis or twice the area on the left of the y -axis. This is in line with the opinion of Asiala et al. (1997) and Arnon (2014), which states that de-encapsulation plays a role in obtaining the properties of the process so that it can be used to perform actions on objects and also to return to the forming process.

After solving some of the integral problems in this study, both subjects' schemes can be seen in Figure 3 (FS) and Figure 4 (MS). Both subjects' schemes indicate the relationship of all indicators of actions, processes, objects, and other schemes built by each subject. The coherence of FS's scheme, in contrast to MS's scheme, depended on the ability to decide whether the derived scheme and/or the area scheme can be used in dealing with integral problems. The equations and differences of the schemes are described as follows.

In reconstructing its integral concept, both subjects connected actions, processes, and objects with other schemes, namely, derivatives. Both subjects involved derivative schemes when applying substitution rules in solving integral problems. Both subjects understood that derivatives play a role in the rules of integral

substitution, especially when identifying integrand parts through prediction. When making predictions, it means that both subjects exhibited an interiorization mechanism, where FS imagined doing the action of deriving the function while also imagining the form of the problem when applying the rules of substitution later. By the time both subjects had succeeded in identifying the integrand parts, both had regarded each part as cognitive objects through the mechanism of encapsulation. Because they had formed an object, both subjects implemented actions in the form of substitution rules on the integral problem by involving the concept of derivatives. Both subjects built their understanding that one of the rules in integration is the substitution rule, which applies when solving the integral that contains a function along with the derivative of the function. This is in line with the opinion of Arnon et al. (2014), who explained that a scheme is an individual's mental structure when facing a mathematical problem, comprising a collection of actions, processes, and objects as well as other schemes.

Both subjects linked integral schemes to area schemes. FS's scheme differed from MS's when facing the problem of determining the area using integral. The act of calculating the area using integral was done imperfectly in process indicators, where FS explained what she imagined about calculating the area using definite integral but did not consider the influence of the location of the area on the Cartesian field. This can be caused by not mastering drawing the function chart, which is one of the preconditions for the integral material. MS started by describing the shape of the area bounded by the curve with some equations. This is somewhat different from the research of Mainali (2014) and Puloo et al. (2018), which found that women's ability is better than men's regarding verbal and visual solutions to solving geometry problems. Based on the images he made, MS calculated the area formed using integral even though the process had to go through two attempts to calculate it. In the first calculation, MS had not realized the importance of considering the location of the area against the x -axis so the results of the area calculation resulted in a zero value. As the interview went on, MS finally realized his mistake and decided to recalculate the area he had described. There was a change in strategy made by MS, which meant he accommodated the information he obtained when interviewed. Based on research by Rutherford (2011), Confrey (2012), and Marchand (2012), when a person changes the scheme, he has to adjust to the conditions at hand, i.e., accommodation occurs. Through accommodation, MS changed the scheme he had so that it was in accordance with the reality he faced, namely, that the area formed was unlikely to have an area of zero value. In the second calculation, MS used the sum of definite integral by summing the integral that represented the area above the x -axis with the integral that represented the area below the x -axis. Thus, it could be said that both subjects understood integral as a representation of the area under the curve.

Both schemes by the subjects were also formed through the resolution of integral problems that give rise to an understanding of the properties of integration. The property of linearity of integral multiplication with constants was built by both subjects through the act of integrating two different functions that are regarded as cognitive objects, where one object is the result of multiplying the other object by a constant. The nature of the linearity of integral summation was also built when viewing each of the summed numbers as cognitive objects on which integrating actions can be performed. In this problem, both subjects carried out a coordination mechanism by performing a sum operation after integrating each number. There was also the nature of boundary changes built by both subjects through the relationship of two integrals with the same integrand. The two given integral parts were regarded as different objects. Both subjects applied integrating actions on these objects by making several examples of problems to make conclusions about the integral nature of the problem. The next integral characteristic built by FS was the presence of symmetry. The schemes shown by both subjects involved the shape of the area bounded by a symmetry curve and, at the same time, applied previously understood symmetry properties to explain the relationship between the two given integrals.

The thinking process of both subjects showed the scheme they had about the concept of integral. It can be said that the integral concept schemes built by both subjects were a collection of actions, processes, objects, and other schemes, namely, derivatives, area, and symmetry. This happened because both subjects have innate knowledge of the concept of derivation, area, and symmetry. According to Čadež and Kolar (2015), every individual has innate knowledge in the form of schemes that can be held in hand when doing something. When facing integral problems, there is a process of scheme adaptation in the mind of both

subjects, as revealed by Yilmaz (2011) and Cohen (2012), where one performs the adaptation process because the knowledge previously possessed is often not in accordance with the situation at hand. Ultimately, both subjects built their knowledge of the concept of integral by reflecting on the prevailing properties of integral and linking with other schemes.

CONCLUSION

The understanding of integral of both subjects in this study based on the APOS theory at the actions stage tends to be similar as demonstrated through the resolution of procedurally integral problems. External stimuli strongly influenced the thinking process of both subjects. The outside stimuli received by the male subject (MS) and female subject (FS) were integral questions while the response was in the form of complete problem solving with a process structured according to the rules of substitution. Each step was carried out explicitly and guided by the external instructions.

At the processes stage, both subjects showed differences in the thinking process when reconstructing their integral concepts. FS directly formed a definite integral by applying the boundaries taken from the equations given in the problem. The integral was a representation of the shape and the area in question because it contained all the equations given in the problem. FS did not consider the influence of the shape and location of the area in question but rather focused on imagining that the result of the integral was the value of the area. This was the case with MS, who also implemented integral but took into account the influence of the area location. MS revealed several integral possibilities that could represent the area following the position of the area against the x -axis that can be formed based on the given equation.

In the objects stage, both subjects showed some integral linkages/relationships as the totality of the processes based on integral characteristics. Both subjects also provided the same conclusion about integral, namely, that when a function is multiplied by a constant, the integral result will also be multiplied by the same constant. Integrating a function that is the sum of several functions involves integrating each number so that its integral is the sum of the integral of the forming functions. When two integrals with the same integrand have an upper and lower boundary that exchange positions, the two integrals will have opposite signs. There are differences in how each subject thought to form objects on two integral problems representing an area of symmetry on the y -axis. FS encapsulated based only on the description in the problem that the curve formed was symmetrical. At the same time, MS encapsulated not only based on the description of the problem but also recognized the characteristic of its integral limitation, which is the same number but with a different sign, i.e., symmetry on the y -axis.

Both subjects applied schemes on derivation and area in understanding integral. The coherence of FS's scheme, in contrast to MS's scheme, depended on the ability to decide whether the derived scheme and/or the area scheme can be used in dealing with integral problems. Both subjects' schemes were dynamic; hence, there are indicators of actions, processes, and objects related to derivatives schemes and area schemes, but there are also some cases related only if needed. Both subjects' schemes were developed by understanding the properties of integral, namely, the property of linearity of integral multiplication with constants, the property of linearity of integral summation, the property of boundary change, and the property of symmetry.

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