Characteristics of Prospective Mathematics Teachers' Problem Solving in Metacognitive Awareness: Absolute Value Problems of Calculus Courses

Anita Adinda
UIN Syahada Padangsidimpuan

Heri Purnomo
Universitas Negeri Malang

Almira Amir
UIN Syahada Padangsidimpuan

Mariam Nasution
UIN Syahada Padangsidimpuan

Suparni
UIN Syahada Padangsidimpuan

Nur Choiro Siregar
The National University of Malaysia

Metacognitive awareness influences problem-solving thinking. Understanding metacognitive awareness is essential in the teaching and learning process. The purpose of this research is to determine the amount of metacognitive awareness and the characteristics of each level in problem solving. The extent of metacognitive awareness of 125 prospective mathematics teacher students from four different public and private universities was revealed in this exploratory qualitative study. To acquire research data, absolute value assignments, think-aloud transcripts, and semi-structured interviews were employed. Sources and methods are triangulated to assure data validity. The data was analyzed using fixed comparison analysis. The results of this study indicate that there are six levels of metacognitive awareness of prospective teacher students: tacit use, identity use, semi-aware use, aware use, strategic use, and reflective use. The six developed levels can identify all the characteristics of metacognitive awareness that arise in solving absolute value problems. Metacognitive awareness that has been developed can supporting the development of learning models, cognitive style characterization and tracing students' process of mathematical metacognitive awareness.

Keywords: absolute value, mathematical thinking, metacognitive awareness, metacognitive strategy, problem-solving

INTRODUCTION

Self-awareness is a component of metacognitive (Boekaerts, 1999; Karaali, 2015). Efklides (2006) also argues that metacognitive refers to students' knowledge and understanding of their cognitive processes and the regulation of these processes to achieve certain goals. Hacker suggests that metacognitive involves students' knowledge of how they learn, evaluating their learning needs, generating strategies to meet these needs, and then implementing them (Jaleel & Premachandran, 2016). Metacognitive is one of the factors that influence students' success in solving math problems (Aurah et al., 2011; Elia et al., 2016; Magiera & Zawojewski, 2011). Schraw and Dennison stated that metacognitive awareness allows individuals to plan, sequence, and monitor their learning in ways that directly improve performance (Bay et al., 2012). Schraw also stated that metacognitive is important for successful learning because it allows individuals to manage their cognitive skills and determine their weaknesses which can be improved by building new cognitive skills (Young & Fry, 2008). This shows that awareness is part of metacognitive. The awareness component will also improve students' metacognitive skills.

According to Magiera and Zawojewski (2011) and Wilson and Clarke (2004), metacognitive awareness refers to one's knowledge and understanding abilities that occur during the process of learning and problem-solving toward their own understanding of situations and assumptions of problems and considerations of students in connecting between their knowledge and what is needed in the problem situation. Yildirim & Ersözlü (2013) state that there is a significant positive correlation between metacognitive awareness and students' level of problem-solving on routine and non-routine questions. This shows that metacognitive awareness is important in the problem-solving process. Hess & Bacigalupo (2014) also stated that student awareness is very supportive of the process of solving problems. In addition to getting information, students also do not realize that the completion process they are working on is wrong. Abdellah (2015) emphasized that metacognitive awareness in students is important for increasing student learning achievement. The same thing was said by Hassan & Rahman (2017), who stated that metacognitive awareness had a positive relationship with learning achievement and mathematical problem-solving abilities. Meanwhile, Oz (2016) states that metacognitive awareness increases students' learning motivation. According to Rahman et al. (2010), students with high metacognitive awareness perform better in tests than students with low metacognitive awareness. Learners who are aware of their thinking are more strategic and have better performance than those who are not aware of it (Pahayahay & Cisneros-Pahayahay, 2017; Purnomo et al., 2021). This shows that metacognitive awareness is needed by students in learning, including solving mathematics.

In solving problems, students need to determine correctly what will be solved. The problem-solving process will be constrained if students need help understanding and getting the information in the problem. Many students usually immediately carry out the calculation process without realizing that there is important information in the problem. If given a problem, students with good metacognitive awareness will rethink what is asked by reading the problem carefully and understanding and knowing what is being asked in the problem. Then students will rethink the information known from the problem by looking for any information in the problem, connecting the information obtained to solve the problem supported by good background knowledge and looking for strategies from the information obtained.

Furthermore, students will think about the steps of the strategy by knowing the stages or steps of the strategy taken and checking whether the steps taken are appropriate. Furthermore, students will think of different ways by trying other methods or strategies and then comparing the answers. Finally, students will think about the correctness of the answers obtained by checking whether the answers follow the problem and finally, students will conclude the answers.

The ability of students also varies in solving problems. Some students consciously pay attention to the problems by solving them hierarchically (Subanji et al., 2021). However, some students only carelessly answer when faced with questions (Sophianingtyas & Sugiarto, 2013). This also applies to students at tertiary institutions because many students need metacognitive awareness in solving problems. This is evident when students get problems such as absolute scores, and many students cannot identify the problem given. They need to realize that the strategies used by students in solving problems are inappropriate. When students cannot get results, they assume that their mistakes result from needing to be careful in the algebraic process. They also need to realize that there is still information about the problems they get. So, they need to be corrected in determining the strategy. Elia et al. (2016) stated that student errors in solving absolute value problems resulted from wrong strategies. Students' habits need to be corrected in taking strategies to work on problems procedurally. Purnomo et al. (2021) stated that when students solve problems, they are used to solving them procedurally without fully understanding the problem. This is the result of the different levels of awareness of students.

Jaleel & Premachandran (2016) examined differences in students' metacognitive awareness in terms of gender, school origin and school management. This study uses the Metacognitive Awareness Inventory (MAI) to determine the metacognitive awareness of high school students. The level of metacognitive awareness of students. This study divides the levels into low, moderate, high, and very high awareness. The results of this study indicate that there is no significant difference in students' metacognitive awareness in terms of gender, school origin and school management. Pahayahay & Cisneros-Pahayahay's research (2017) assesses the level of metacognitive awareness that plays an important role in teaching-learning. In this study, the authors measure metacognitive awareness with the Metacognitive awareness Inventory (MAI), which relates to students' awareness of their position in the algebra learning process in tertiary institutions. This study divides the level of metacognitive awareness into three levels, namely high, medium, and low metacognitive awareness. The results reveal that there is no significant difference between the level of students' metacognitive awareness by male and female respondents.

Misu & Masi (2017) examined differences in metacognitive awareness between male and female students based on mathematical ability in the Mathematics Education Department of Halu Oleo University. The subjects of this study were students of class 2016 in the Department of Mathematics Education from Halu Oleo University. This research is also based on the development level of Swartz & Perkins. The results showed that the metacognitive awareness of male and female students in the Mathematics Education Department at Halu Oleo University was generally at the aware use level. Misu & Masi's research (2017) identified a level of metacognitive awareness based on the level developed by Swartz & Perkins. There are four levels of metacognitive awareness (Swartz & Perkins, 2017). Level 1 tacit use: Students make decisions without thinking about them. Level 2 aware use, students become aware of the strategy or decision-making process. Level 3 strategic use: Students organize their thinking by choosing a strategy for decision-making. Finally, in level 4 reflective use, students reflect on thoughts before, during and after the process, reflect on progress and how to improve it. This level was adopted by several other researchers, namely Fisher (1998).

Previous researchers have discussed absolute value (Almog & Ilany, 2012; Amram et al., 2019; Ciltas & Tatar, 2011; Curtis, 2016; El-khateeb, 2016; Elia et al., 2016). Student's difficulties in solving absolute value problems allow for differences in students' metacognitive awareness in solving absolute value problems. Differences in students' metacognitive awareness in solving absolute value problems can be a level of metacognitive awareness. Many math teacher candidates still need help solving absolute value problems. The results of previous studies indicate that students experience difficulties in solving absolute value equations and inequalities. One of the causes of student difficulties in solving problems involves many technical steps (Amram et al., 2019). Students also experience conceptual errors and errors in the operation of absolute value problems. Therefore, this study focuses on the metacognitive awareness of prospective mathematics teacher students in the subject of absolute value.

Metacognitive awareness is part of metacognitive abilities. Metacognitive awareness activities are based on the description of activities developed by Magiera & Zawojewski (2011) and Wilson & Clarke (2004). There are several students' metacognitive activities in solving absolute value problems. First, students seek information from absolute value equations or inequalities. Second, students can look for strategies to find solutions. Third, students reconsider whether the information they have obtained is correct and whether there is still information that needs to be disclosed. Fourth, students can look for strategies from the information they get. Fifth, students reconsider whether their strategy is appropriate following the absolute value equation. Sixth, students reconsider whether the steps they have taken are correct. Finally, students reconsider whether the solution they get follows the absolute value equation.

Several studies examine the level of metacognitive awareness. From this research, there are two frameworks of reference in classifying the characteristics of the level of metacognitive awareness. The two research frameworks use the Metacognitive Awareness Inventory (MAI) developed by Schraw and Dennison (1994) and the level of metacognitive awareness by Swartz & Perkins (2017). Metacognitive Awareness Inventory (MAI) is used to see students' level of metacognitive awareness in learning. Metacognitive awareness is based on MAI references, not on problem-solving. At the same time, the last problem that is often experienced by students so far is in the context of problem-solving. Therefore, it is important to explore the level of metacognitive awareness and its characteristics. Swartz & Perkins' metacognitive awareness also shows that there is still a jump, and the description of the indicators is also limited. The aspects that characterize each level still cannot represent all types of metacognitive awareness that arise in students. So, there is a need for a thorough search to determine the characteristics of metacognitive awareness in solving absolute value problems.

METHODS

Participants

The descriptive study with a qualitative approach (Mockford, 2008; Creswell, 2014) was carried out with 125 students who were given the assignment were students from 4 different public and private universities in Indonesia who had studied absolute value problems in calculus and real analysis courses. Research subjects were taken at each level with a fixed comparison method. Subjects selected for interviews met the criteria as shown in Table 1 below.

TABLE 1
CRITERIA FOR METACOGNITIVE AWARENESS

Main Category	Metacognitive Strategy
Tacit Use	The individual makes decisions without thinking about them and applies strategies without special awareness of solving problems.
Aware Use	An individual was aware of what and why he was implementing a strategy and made decisions with clear reasons.
Strategic Use	Individuals organize their thinking to select strategies and have specific strategies.
Reflective Use	Individuals already reflected on their thoughts before, during and after the resolution process.

Swartz & Perkins (2017)

Data Collection

The data in this study are answer sheets of think-aloud transcripts and transcripts of student awareness interviews in solving absolute value problems. Research data were collected from absolute value assignments, think-aloud transcripts, and semi-structured interviews. Students think aloud transcript answer sheets come from student problem-solving responses to absolute value problem assignments. The task was previously validated with three validators both in terms of content and construct so that the task can reveal the level of students' metacognitive awareness.

Researchers conducted interviews with students after students finished working on problem assignments in a think-aloud manner. Interviews were conducted to reconfirm the characteristics of students' metacognitive awareness while working on absolute value assignments. The interviews were conducted after students had finished working on absolute value problems so that the interview process did not lead students to do metacognitive.

Data Analysis

Analysis of student written assignments in solving absolute value problems based on the correctness of the completion carried out by the subject guided by the complete instructions and the key. The subject's answers were analyzed based on predetermined indicators. Subjects were determined at a certain level of metacognitive awareness based on their answers and records of students' problem-solving activities and then clarified by interviews. Based on these answers, 125 students were grouped into six levels of metacognitive awareness of the absolute value problem, as shown in Table 1, along with the distribution of the number of student responses shown in Table 2.

TABLE 2
NUMBER OF STUDENTS AT EACH LEVEL OF METACOGNITIVE AWARENESS

The Level of Metacognitive Awareness	The Number of Students
Tacit Use (TU)	19 students
Identify Use (IU)	20 students
Semi-aware Use (SaU)	43 students
Aware Use (AU)	31 students
Strategic Use (SU)	7 students
Reflective Use (RU)	5 students

The data analysis process in this study includes: (1) Examining all existing data from various sources, namely student assignment sheets, recordings of think-aloud results while solving problems, and interview results. (2) Data reduction was carried out by selecting and selecting each incoming data from students' written answers, recordings of students' think-aloud while solving problems, and interview results. (3) After the written answer data, recording and interview results are obtained. The data for each student is classified and identified according to the characteristics of metacognitive awareness. (4) Conclude the level of metacognitive awareness of each student. (5) validating data obtained by triangulating sources and methods (Moleong, 2012). Source triangulation was carried out by comparing the results of the researcher's analysis with the results of other sources regarding the characteristics of each student's level of metacognitive awareness in solving absolute value problems. While method triangulation was carried out by comparing written answers, the results of think-aloud recordings, and the results of interviews with each student in solving problems. Student answers for each question were also compared. If the data generated is the same, then the level meets the valid criteria. (6) Data reliability is measured by fixed comparison analysis, namely comparing certain data categories with other data categories so that categories with the same and consistent characteristics are obtained (Barney & Anselm, 1995). If the two subjects occupy the same level, have the same characteristics, and apply to each level, then the level meets the reliable criteria. (7) Integrating the level with its characteristics according to the theoretically developed characteristics is integrated with the same characteristics (empirical results) so that it becomes a characteristic of a level of metacognitive awareness.

RESULTS

Based on the six levels of metacognitive awareness shown in Table 2, one student from each level was represented as level of metacogtitive awareness of prospective mathematics teachers. Subject 1 (S11) represents TU, Subject 2 (S22) represents IU, Subject 3 (S23) represents SaU, Subject 4 (S14) represents AU, Subject 5 (S25) represents SU, and Subject 6 (S16) represents RU as shown in the following table 3.

TABLE 3
METACOGNITIVE AWARENESS OF PROSPECTIVE MATHEMATICS TEACHERS

Code	Metacognitive Awareness Level
S11	Tacit Use
S22	Identify Use
S23	Semi-aware Use
S14	Aware Use
S25	Strategic Use
S16	Reflective Use

The following is a description and explanation of the characteristics of each level of student metacognitive awareness in problem solving of absolute value problem.

Tacit Use

Students (S11) at this level can be seen when students are only trying to find the value of x without thinking about the information in the problem. It can be seen from the answers in the student think-aloud transcript below.

FIGURE 1
TRANSCRIPT OF THINK-ALOUD S11 ON TACIT USE

Translated version:
Problem 1. $|x-1|+x^2-3x \geq -\frac{13}{4}$. Means looking for x. how is this.. answer.. What should I do. Then I did not find the answer. Continue to play. I don't know. Confused. Confused. x^2-2x-1 also cannot be factored. -13/4 movable. Equalize the denominator. If you look for the factor, what do you use, it doesn't work... there isn't a factor. Then I don't know anymore..
Number 2. The person asked means the number of x. There is an absolute value. But how about that? Transferred. What are you doing? Continue to look for x this time. Here you go. This is all you can
Problem 3. Also look for x yes. But this is an absolute fraction. -2 moved segments. Continue to play the denominator. Why so confused.

Based on the think-aloud transcript in Figure 1, the underlined sentence shows that S11 re-read what was asked and always restated what to look for in the problem. S11 tries to find the value of x in question, but S11 is always confused in solving the problem. S11 does not know what to do, so it is suspected that S11 solves the problem with an unclear strategy. S11 does not try to find out what information is in the problem before solving the problem. S11 repeatedly said the word "confused", indicating that S11 did not understand what he was doing. The sentences "What to do with it" and "What to do with it" indicate that S11 does not know what steps to take next. The sentence "Then look for this x times yes" indicates that S11 is unsure about what it will do. S11 thought of no other way when inequality cannot be factored in. In question number 2, S11 knows there is an absolute sign, but S11 cannot draw any information from the absolute sign. S11 did the same thing in question number 3; S11 experienced confusion about what he would do and what he had done.

Next, the researcher (P) interviewed S11 to get confirmation of the answers he had made. The following is an excerpt from the interview with S11 based on completing questions 1, 2 and 3.

P : Do you understand what is being asked?
S11 : Understood.
P : Try to state what is asked in the questions above!
S11 : from Problem 1, what is asked is x, which satisfies the equation from Problem

2, which asks for the sum of all x that satisfies the equation from Problem 3, what is asked is x, which satisfies the equation

P : What do you think about the absolute value sign?

S11 : The absolute value is always positive.

P : Why did you omit the absolute value sign in solving the problem?

S11 : Because the absolute value is always positive. So just remove the sign.

Based on interviews with S11 regarding completing questions 1, 2, and 3, S11 knows and understands what is being asked in the problem: finding the value of x that satisfies the equations and inequalities. S11 said that the information contained in the problem is that there is an absolute value mark and a fraction sign. It shows that S11 cites information only from what is stated in the problem. S11 cannot obtain new information from existing absolute signs and does not obtain other information on the problem. S11 also needed to understand absolute value material better. The absolute value of a real number is always positive, but S11 assumes that the absolute sign can be omitted in simplifying an absolute from x. The results of solving S11 show that the absolute sign has no effect whatsoever. The results of this interview also show that S11 only focuses on obtaining a solution x without looking at whether the steps made are appropriate.

Based on think-aloud answers and interviews with S11, S11 only knows what is being asked and tries to find the value of x by simplifying the inequality and ignoring the absolute value sign. The result of the S11 solution process only changes the form of inequality without obtaining the desired result. S11 cites the information in the problem and does not try to find other information. S11 solves the problem with an irrelevant strategy and does not pay attention to whether the strategic steps are appropriate. S11 also needs to know what steps it will take to solve the problem. Based on the above, the characteristic S11 metacognitive awareness that is fulfilled only considers what is asked. S11 ignores and discards the absolute value flag in absolute value operations.

Identify Use

Students (S22) at this level reread the questions, repeat what they are looking for in each question and try to find what is being asked. It is shown based on the following student's think-aloud transcript.

FIGURE 2
TRANSCRIPT OF THINK-ALOUD S22 ON IDENTIFY USE

Translated version:

Problem 1. Find the value of x that satisfies the equation $|x-1|+x^2-3x \geq -\frac{13}{4}$. Means looking for x. There is $|x-1|$. That means there are two possibilities. First $|x-1|+x^2-3x \geq -\frac{13}{4}$. Changed to $x-1+x^2-3x \geq -\frac{13}{4}$. ... becomes $x^2-2x \geq -\frac{9}{4}$.. . The second one $-(x-1)+x^2-3x \leq -\frac{13}{4}$ ... the sign is that you don't know if it was changed or what... After simplifying it to $x^2-4x+1+\frac{13}{4} \leq 0$.

Problem 3. Find all values of x that satisfy $\left|\frac{x^2-\frac{1}{2}x+\frac{1}{16}}{2x-\frac{1}{2}}\right| > 2$. . Means looking for x. From the problem, there are two possibilities. First $\frac{x^2-\frac{1}{2}x+\frac{1}{16}}{2x-\frac{1}{2}} > 2$ .. $x^2-4\frac{1}{2}x+\frac{17}{16} > 0$ ... Second $-\left(\frac{x^2-\frac{1}{2}x+\frac{1}{16}}{2x-\frac{1}{2}}\right) > 2$, the negative sign can be changed to less than min 2. ... $x^2+3\frac{1}{2}x-\frac{15}{16} < 0$

Based on think aloud, S22 always rereads the questions, repeats what is looked for in each question and tries to find what is being asked. It shows that S22 has thought about what was asked. S22 has used absolute sign information using two possibilities as stated in the answer sheet. The sentence "I do not know if the sign is changed or what" in question 1 shows that S22 seems confused when using the inequality sign. S22 stops in his tracks when he obtains a new inequality. It means the S22 needs to be rethinking what steps it will take. He also did not think of other ways when he encountered problems in finding solutions to inequalities. In question number 3, S22 also uses the same information, namely by describing the absolute value sign using two possibilities. But S22 realized that it had an algebraic error. S22 was unable to complete its work. This shows that S22 needs to consider whether the steps are correct.

Next, the researcher (P) interviewed S22 to get confirmation of the answers and think aloud he made. The following is an excerpt of an interview with S22 based on solving questions 1 and 3.

P : Do you understand what is being asked?
S22 : Understood.
P : Try to state what is asked in the problem!
S22 : Find the set of solutions ma'am.
P : How do you know what to ask?
S22 : From the question ma'am
P : What information do you know from the questions?
S22 : There is an absolute sign.
P : From the absolute value sign, what do you get?
S22 : From that absolute value, there will be two possibilities
P : How do you find out this information?
S22 : From the form of the question ma'am
P : Is the information you get used to solving the problem?
S22 : Useful, ma'am
P : Why?
S22 : Because from the absolute value information, we have two possibilities in which the value of x will be obtained
P : In solving questions 1 and 3 that you made, did the process really end there?
S22 : I do not know what to do anymore, ma'am

Based on interviews with S22 regarding the completion of questions number 1 and 3, S22 knows and understands what is being asked in the problem, namely finding a set of solutions to inequalities. S22 said that the information there is an absolute sign. S22 knows that if there is an absolute value sign, two possibilities can be made. This shows that S22 already has information that can be obtained from the questions. However, S22 needs to know what steps to take next to find a solution to the inequality. This means that S22 still needs to rethink the steps that have been completed yes, whether they are appropriate or not. The results of this interview also show that S22 has thought about what is being asked and has some information, but S22 still needs to rethink how to use the information he has obtained to find a solution.

Based on think aloud answers and interviews with S22, it can be concluded that S22 has rethought the information contained in the questions. S22 can retrieve information from the problem, but S22 cannot solve the problem. Based on the above, the characteristics of S22 metacognitive awareness are rethinking what is being asked and rethinking the information contained in the problem.

Semi-Aware Use

Students (S23) at this level think about what is known. Students think of information from absolute value signs. Students take the completion step by dividing the two parts of the inequality or inequality by applying the rules to absolute values. Students think about how to connect the information they get to solve problems. This is shown based on the Figure 3 student's think-aloud transcript.

Based on S23's think-aloud, the sentence "After I pay attention" shows that S23 has paid close attention to each question. It means that S23 has thought carefully about what is asked and thought about the information contained in the problem. In questions 1, 2 and 3, S23 thinks about the information he gets to solve the problem. It can be seen that S23 always thinks about the absolute value signs he sees and what can be taken from the absolute value signs. S23 always seeks a solution using the absolute value definition, then S23 simplifies each of the cases it has made. In problem 1, S23 had difficulty finding the roots of the inequality, so he used the formula and found the roots in the form of imaginary numbers. S23 does not compare it in terms of the absolute value it makes. In questions 2 and 3, S23 did not compare the value of x he obtained with the absolute value requirement. S23 also needs to check whether the solution it gets is correct or not. It indicates that S23 needs to know whether its steps are correct.

FIGURE 3
TRANSCRIPT OF THINK ALOUD S23 ON SEMI-AWARE USE

Translated version: For question number 1, after I noticed there was an absolute value sign. Because there is this absolute value, I divided it into two parts: when $x - 1 \geq 0$ and when $x - 1 < 0$. After I simplified it, I tried to find the factor but could not. Finally, I tried using the a-b-c formula. It turns out that all the values of x that satisfy the inequality contain imaginary numbers in the sense that the roots are not real or unreal roots. So, the equation in number 1 is the result of a complex number equation because there are imaginary numbers. For number 2, after I pay attention to the problem, there are two signs of absolute value. Since there are two signs of this absolute value, I made it into four parts. Because according to the definition, if there is an initial sign, it means that there are two possible parts. This means that if there are two absolute signs, then there are four parts. Out of the four possible parts, I get four values of x. After I add them up, the total value of x that satisfies the equation is 2. For question number 3, after I paid attention because there was an absolute value as well, I divided it into two parts. The inequality is difficult to simplify because it is a fraction. But I still simplify each part according to the steps that I know. After I simplify, I look for the roots. From these roots, I finally get all the values of x that satisfy the inequality.

Furthermore, S23 was interviewed by the researcher (P) to get clarification on his answers. The following is an excerpt from the interview with S23 based on completing questions 1, 2 and 3.

P *: What information do you know from the questions?*
S23 *: Equality/inequality absolute value.*
P *: How do you find out this information?*
S23 *: Based on the form of the questions presented*
P *: Is the information you get used to solving the problem?*
S23 *: Useful because with that absolute value, later, we can use it by dividing the two parts.*
P *: How do you connect the information you get to solve the problem?*
S23 *: By applying rules on absolute values.*
P *: How do you solve the problem?*
S23 *: Problem 1. To find all the values of x, I first divide the two parts, namely for x≥1 and x<1*
Problem 2. In my opinion, there are four parts to the value of x, namely when the absolute value is inside ≥0 and <0 and when the absolute value is outside is ≥0 and <0. After that, x is added up.
Problem 3. To find all values of x, I divide the two parts for the inequality.
P *: Are the steps to answer your problem correct?*
S23 *: Maybe there is something wrong, ma'am*

Based on the results of the interviews, it was shown that S23 thought about what he knew. S23 considers the information from the absolute value sign. S23 takes the solution step by dividing the halves of the inequality or inequality by applying the rule to absolute values. It means S23 thinks about how to relate the information it gets to solving problems. From the results of the interview, S23 said that "Maybe there is something wrong, ma'am". It shows that S23 needs to figure out the steps he has made because he did not carry out the inspection. It proves that S23 still needs to rethought whether the steps are correct.

Based on the answers of think-aloud transcripts and interview transcripts, the characteristic of S23's metacognitive awareness is that they have rethought what was asked. S23 has also thought about the information in the problem. After S23 got the information, S23 thought about what to do next. The S23 considers the information received and can relate it to solving problems. Nevertheless, S23's mistake was not rethinking whether his strategy was following the right steps. S23 needed to figure out what he was doing because he thought there was a calculation error. Nevertheless, the S23 does not rethink what went wrong. Based on the above, S23's metacognitive awareness characteristics are rethinking what is being asked, rethinking the information in the problem and rethinking the information it has obtained and being able to connect the information to solve problems.

Aware Use

Students (S14) at this level think about information and can relate the information they get to solve problems. S14 knows what steps it should take to solve the problem. This is shown based on the Figure 4 student's think-aloud transcript.

FIGURE 4
TRANSCRIPT OF THINK ALOUD S14 ON AWARE USE

Translated version: First find all values of x that satisfy $|x-1|+x^2-3x \geq -\frac{13}{4}$. The trick is, because there is an absolute sign, there are two possible forms of inequality, namely x $x-1+x^2-3x \geq -\frac{13}{4}$ where $x-1 \geq 0$, and the second $-(x-1)\ +x^2-3x \geq -\frac{13}{4}$where $x-1<0$. So, solve them one by one first. The first is $-1+x^2-3x \geq -\frac{13}{4}$ where x-1≥0, it is simplified to find the result $4x^2-8x+9 \geq 0$. Solve the quadratic equation x with $ax^2+bx+c=0$. Because the coefficient $a=4>0$, the inequality will always be greater than zero. so $4x^2-8x+9 \geq 0$ holds for all x∈R. Likewise, the form of inequality in which the two methods are the same, the coefficient $a=4$ so that the inequality will always be positive as well, so that the result is x∈R as well. Therefore, the combination is x∈R. So, all the x values that satisfy are x∈R. Second, find the sum of all x values that satisfy the equation $\left|3x-2+|x-2|\right|=5$.. Because there is an absolute sign, the two possible equations are $3x-2+|x-2|=5$ and $3x-2+|x-2|=-5$. The first thing to look for is the first equation first, simplified so that a possible equation is found, namely there are $3x+|x-2|=7$.. Because there is another absolute sign, there are two more possible equations, namely $3x+x-2=7$ where the value of x is $\frac{9}{4}$where $x \geq 2$ and the second equation is $3x-(x-2)=7$ where the value of x is $\frac{5}{2}$ where $x<2$. Then find the set of $x=\frac{9}{4}$ where $x \geq 2$, i.e. the result is $x=\frac{9}{4}$.. Then $x=\frac{5}{2}$ where $x<2$, i.e the result is x is an element of the empty set. So, the value of x at $3x-2+|x-2|=5$ is $x=\frac{9}{4}$. For the second equation, the method is the same, find the possible equations, find the set so that the value of x found in $3x-2+|x-2|=-5$ is $x=-\frac{5}{2}$. Then the sum of all x values is $-\frac{5}{2}+\frac{9}{4}=-\frac{1}{4}$

Based on think-aloud, S14 read the questions carefully. S14 is also trying to find a solution to the problem. This shows that S12 thought about what was asked. S14 also uses information from the absolute value sign to find possible equations or inequalities in each question. Based on the information obtained, S14 looks for the value of x by describing each possibility. It means that S14 has already thought about the information it has received to solve the problem. In problem 1, S14 said, "Because the coefficient a=4>0, the inequality will always be greater than zero". It suggests that S14 rethinks the information from the new inequality so that he can take appropriate steps to find a solution. Likewise, in question 2, S14 remembered to compare the x value he obtained with the absolute conditions he made. S14 also remembers to add up the value of x it gets. It means S14 knows what steps to take next. It shows that S14 has thought through its strategic steps to solve the problem.

Furthermore, S14 was interviewed by the researcher (P) to get confirmation of the answers he made. The following excerpt from an interview with S14 is based on solving questions 1 and 2.

P : What information do you know from the questions?
S12 : Contains absolute value equations and inequalities.
P : Is the information you get used to solving the problem?
S12 : Yes, it is useful. Because from that information, we will get new information.
P : How did you get the information based on the information you got?
S12 : By paying close attention to what is in the problem.
P : How do you connect the information you get to solve the problem?
S12 : From the knowledge that I have learned before.
P : How do you solve the problem?
S12 : Using systematics and the steps that I have learned, namely by determining possible equations or inequalities, then looking for solutions that lead to what the problem asks what for.
P : Does your problem-solving strategy follow similar problem-solving steps?
S12 : It seems appropriate.
P : What steps did you take? Were they correct?
S12 : That is right too. It is according to what I learned.

Based on the results of the interviews shows that S14 has fulfilled the characteristics of awareness of thinking about what is being asked. S14 said that he obtained information "by paying close attention to what was in the problem." It suggests that S14 considers the general information from the problem. S14 also says it connects information to solve problems from the knowledge I have previously learned. S14 demonstrates that it can relate known information to solve problems. S14 also said that the strategy he had made had followed the steps that were appropriate to the problem.

Based on the answers to think-aloud transcripts and interview transcripts, the characteristic of S14's metacognitive awareness is to have rethought what was asked. S14 has also thought about the information in the problem. After S14 got the information, S14 thought about what to do next. S14 considers the information it gets and can relate it to solving problems. S14 also rethought its strategic steps to solve the problem.

Strategic Use

Students (S25) at this level try to find solutions that satisfy inequalities or absolute value equations. S25 originally used the definition to describe the absolute value sign. The S25 further simplifies each case. S25 thinks about information, connects information in solving problems, thinks about the steps to be taken and thinks about other strategies or strategies that are easier to find solutions. This is shown based on the Figure 5 student's think-aloud transcript.

Based on think aloud the answer, in question 1, S25 stated that we would look for x that satisfies the inequality. S25 also tries to find the value of x that satisfies the inequality. It suggests that the S25 has rethought what was asked. Next, S25 retrieves information from absolute signs using 2 cases. In this case, S25 uses the term review. Then S25 looks up the x value of each review it makes. S25 looks for a different way to find a solution because the solution obtained previously does not satisfy the inequality. S25 analyzes the values of the terms in the inequality one by one. It shows that the S25 has thought about the information and can relate to it to find a solution. S25 has also used different ways to solve problems and compared answers from each method to conclude solutions from inequalities. Based on that, S25 has thought about its strategic moves and rethought different ways to find solutions. In question 2, S25 does the same thing: stating what is sought, trying to find what is being asked, taking absolute value information based on definitions and looking for relationships from the information obtained to solve the problem. S25 also looked for a different way because the first method experienced problems in problem 2. Based on the think-aloud transcript, S25 has rethought what was asked, the information, the relationship of information to solve the problem, the strategic steps, and the different ways or specific strategies to solve problems.

FIGURE 5
TRANSCRIPT OF THINK ALOUD S25 ON STRATEGIC USE

Translated version:

In problem number 1, what you are looking for is x which satisfies $|x-1| + x^2 - 3x \geq -\frac{13}{4}$. The form of inequality is absolute value inequality. According to what I know, if there is an absolute value sign, it means that 2 reviews can be used. The first review is $x \geq 1$. From this review I look for the roots of inequality. The dissimilarity cannot be factored. So, I use the abc formula. From the abc formula, I get the imaginary value of x. In the second review $x < 1$ I also cannot find the roots by factoring. So, I also use the abc formula to find the roots. The value of x that I get is also imaginary. Because the form is imaginary, it means that there is no real number x that satisfies the inequality. After I tried any x it turned out that the inequality satisfies. I look again at the inequalities in the problem, it turns out that all x will definitely satisfy the inequalities. If analyzed the left side of the inequality will always be greater than the right side. Because for any real number x, the absolute value will always be greater than zero, $x^2 - 3x$ will also always be greater than $-\frac{13}{4}$. Therefore, I conclude that the x satisfies are all x real numbers.

In problem 2, what you are looking for is the sum of all x values that satisfy $\left|3x - 2 + |x-2|\right| = 5$. Because there are absolute signs, the first thing I did was review from absolute signs. I reviewed from $3x - 2 + |x-2| \geq 0$. But from this review I'm still having trouble finding the value of x because there is another absolute sign. Then I reviewed $3x - 2 + |x-2| < 0$, but similar to the first review I had trouble finding x. so I try to review from $x \geq 2$. from this review I get absolute value again. So, I review again from $x \geq 1$ obtained $\frac{9}{4}$ and satisfy. Judging from $x < 1$ it is obtained $-\frac{1}{4}$ but does not fulfill $x \geq 2$. Then I review from $x < 2$, I get $|2x| = 5$. Then I review from $x \geq 0$, it is obtained that $x = \frac{5}{2}$ does not satisfy $x < 2$. Judging from $x < 0$ it is obtained $x = -\frac{5}{2}$ and fulfills. So, it can be concluded that the number of x that satisfies is $\frac{9}{4} + (-\frac{5}{2}) = -\frac{1}{4}$

Next, S25 was interviewed by the researcher (P) to get confirmation of the answers he made. The following excerpt from an interview with S25 is based on solving questions 1 and 2.

P	*: How do you connect the information you get to solve the problem?*
S25	*: If there is a form of inequality or equation containing an absolute value, we are reviewing it from two cases. From each review, we will obtain a satisfactory value of x.*
P	*: How do you solve the problem?*
S25	*: Problem 1, I started reviewing two cases. However, I tried to observe the problem again because the result was imaginary. I found that the result is satisfactory for all x real numbers.* *Problem 2, I reviewed from |x-2|. I found 4x values from that review, but only two met the review.*
P	*: Does your problem-solving strategy follow similar problem-solving steps?*
S25	*: Yes*
P	*: What steps did you take? Were they correct?*

S25 : Yes
P : Why did you choose this strategy?
S25 : I am looking for a strategy that can find x, and the strategy is easier
P : Have you tried other strategies before?
S25 : Yes, I will try the easier one

Based on the results of the interviews shows that S25 has fulfilled the characteristics of thinking about what is known, thinking about information that is known from the problem. S25 correlated the information he obtained by reviewing each absolute sign. S25 said, "From each review, we will obtain a satisfactory value of x". The S25 can connect known information to solve problems and relate it to previously studied problems. In problem 1, S25 said, "because the result is imaginary, so I tried to check whether there was no x, but I found that the result satisfies all x real numbers".

Based on the presentation of S25's data on questions 1 and 2 in the form of answers to think-aloud transcripts and interview transcripts, it was found that the characteristics of S25's metacognitive awareness were rethinking what was asked, rethinking the information contained in the questions and rethinking the information obtained and being able to link information to complete problem and rethink the strategic steps to solve the problem and think of a different/special way to solve the problem.

Reflective Use

Students (S16) at this level are trying to find solutions that satisfy absolute value inequalities. S16 originally used the definition to describe the absolute value sign. Furthermore, the S16 simplifies each case. S16 can simplify the form of inequalities so that from each inequality, it can be seen that the solutions of the inequalities are all real numbers. Furthermore, S16 is also looking for other ways to solve the problem. S16 analyzes the form of the inequality and can see that all $x \in R$ holds for the inequality. This is shown based on the Figure 6 student's think-aloud transcript.

Based on think-aloud, S16 says that all x values that satisfy the inequality will be determined. S16 tries to find the value of x that satisfies the inequality. This shows that the S16 has thought about what was asked. Furthermore, S16 said that the definition of absolute value would be used. S16 divides the two cases of inequality by using the absolute definition. Then S16 simplifies each of the inequalities in each case so that S16 can find a solution from the simple inequalities. That shows that S16 has thought about the information and connected the information used to solve the problem. S16 also knows what steps he will take in each strategy, and his steps follow absolute value rules. That shows that S16 has thought of strategic steps for solving the problem. Furthermore, S16 looked for a different strategy by analyzing the problems in the questions. S16 analyzes the inequality terms one by one to find that the inequality satisfies all $x \in R$. Based on this, S16 also thought of a different way to solve the problem. S16 also did not forget to check the answer before concluding the inequality solution.

FIGURE 6
TRANSCRIPT OF THINK ALOUD S16 ON REFLECTIVE USE

Translated version:
For the first, we are given a question where we are asked to determine all values of x that satisfy $|x-1|+x^2-3x \geq -\frac{13}{4}$. Now here we will use the definition of absolute value. So, for $|x-1|$ it will be $x-1$ for $x \geq 1$ and will be $-(x-1)$ when $x < 1$, note here x∈R. So here for the first case, we will describe it so that after I simplify it, I write $(x-1)^2 \geq -\frac{5}{4}$. So, from this we can analyze that for any x∈R then $(x-1)^2$ is not negative and will obviously be $\geq -\frac{5}{4}$. Now for the second case which is also similar after simplifying I found that $(x-2)^2 \geq -\frac{1}{4}$.. Where it is similar that for any x∈R then $(x-2)^2$ is not negative. And obviously it will be $\geq -\frac{1}{4}$.
So, after that I also analyzed that $|x-1|+x^2-3x \geq -\frac{13}{4}$, now for x^2-3x, then $x^2 \geq 3x$ for $x = 0, x \geq 3$ with x ∈R and x∈R^-. For $0 < x < 3$ the inequality also holds. So, from here or from the several cases analyzed, we will take the combination for the solution. So here after I checked too, but I didn't include it, then all the x values that satisfy the absolute value inequality in number 1 are x∈R.

Furthermore, S16 was interviewed by the researcher (P) to get confirmation of the written answers and think-aloud he made. The following is an excerpt from an interview with S16 based on solving problem 1.

P *: What information do you know from the questions?*
S16 *: In problem number 1, the information I got from |x-1|+x^2-3x ≥ (-13)/4 is for any value of x in |x-1| and x^2 will always be greater than (-13)/4. For -3x, we have to check and prove again whether any value x will be greater than (-13)/4. So that we get the right answer because, in the question, there is an absolute sign, where the absolute value will be defined in two cases. So, in question number 1, we use absolute value to find all x values that satisfy absolute value inequalities.*
P *: Is the information you get used to solving the problem?*
S16 *: Of course, it is very useful.*
P *: How do you connect the information you get to solve the problem?*
S16 *: From the information I get, I can find out what settlement strategy I will use.*
P *: How do you solve the problem?*
S16 *: Using the absolute value definition. Then I break it down into two cases. Of the two cases, I found an inequality that allowed me to find a solution.*
P *: Does your problem-solving strategy follow similar problem-solving steps?*
S16 *: Yes, it is appropriate.*
P *: What steps did you take? Were they correct?*
S16 *: I am sure it is right. Because I have checked again.*
P *: Why did you choose this strategy?*
S16 *: Because the strategy I made is according to the concept.*
P *: Have you tried other strategies before?*
S16 *: Besides answering using the concept of absolute value definition, I also use an analysis of absolute value and squared value.*
P *: Are you sure about your answer?*
S16 *: Sure. Because I have checked my answers again by substituting the answers I got to the questions given.*

Based on the results of the interviews, it was shown that S16 could draw the information contained in the questions. S16 can also connect the information it has to solve problems and relate it to previous problems. S16 also said that "I believe it is correct. Because I checked again, this proves that S14 has rethought the steps in solving the problem." S16 also said that he had checked and was sure of his work. That shows that S16 rethought the correctness of the answers he got.

Based on the presentation of S16's data in the form of think-aloud transcript answers and interview transcripts, it was found that the characteristics of S16's metacognitive awareness were rethinking what was asked, rethinking the information contained in the problem and rethinking the information it obtained and being able to relate information to solving problems and rethinking steps. -strategic steps to solve the problem, rethink different/special ways to solve, and rethink the correctness of the mathematical answers that have been resolved.

DISCUSSION

Swartz & Perkins (2017) developed a level of awareness that shows a person has metacognitive into four levels, tacit use, aware use, strategic use, and reflective use, that applies to all learning content. In this study, researchers developed Swartz & Perkins' (2017) 4 levels of awareness in solving absolute value problems. Researchers level it by focusing on metacognitive awareness activities by Wilson & Clarke (2002, 2004) and considering Polya's problem-solving stages. The study's results show that out of 125 prospective math teacher students solving absolute value problems, 63 students could not be classified at the Swartz & Perkins level (2017). Therefore, it is necessary to add a new level in determining the characteristics of metacognitive awareness in solving absolute value problems. In the following, explaination further the metacognitive awareness of prospective mathematics teacher students in solving absolute value problems.

TABLE 4
THE LEVEL OF METACOGNITIVE AWARENESS IN PROBLEM SOLVING

Level of Metacognitive Awareness	General Characteristics
Tacit Use	• Students rethink what is being asked
Identify Use	• Students rethink what is being asked • Students rethink the information known from the problem
Semi-aware Use	• Students rethink what is being asked • Students rethink the information known from the problem • Students reconsider known information and can relate the information obtained to solve problems
Aware Use	• Students rethink what is being asked • Students rethink the information known from the problem • Students reconsider known information and can relate the information obtained to solve problems • Students rethink their strategic steps to solve problems
Strategic Use	• Students rethink what is being asked • Students rethink the information known from the problem • Students reconsider known information and can relate the information obtained to solve problems • Students rethink their strategic steps to solve problems • Students rethink different ways of answering given mathematical problems

Reflective Use	• Students rethink what is being asked • Students rethink the information known from the problem • Students reconsider known information and can relate the information obtained to solve problems • Students reconsider the next steps that must be taken to solve the given mathematical problem • Students rethink different ways of answering given mathematical problems. • Students reconsider the correct answers to mathematical problems that have been solved

This study shows six metacognitive awareness levels of prospective mathematics teacher students, as shown in Table 4. The six metacognitive awareness levels shown in Table 4 have met the validity and reliability requirements according to the established criteria. The six levels of metacognitive awareness are tacit use, identify use, semi-aware use, aware use, strategic use, and reflective use. In the description of the characteristics of the level of metacognitive awareness, there are different main characteristics for each hierarchical level.

Students at the level of tacit use have the characteristics of metacognitive awareness, just thinking about what is asked. Subjects at this level need help to think of the information contained in the problem. Thus, the information needed to be obtained. If there is information that can be retrieved, the information obtained is only in the form of a sign in the problem, such as an absolute value sign. At this level, the subject only knows and seeks answers without prior knowledge possessed by students. That is in line with the level of tacit use developed by Swartz & Perkins (2017), where individuals at this level make decisions without thinking about them and apply strategies without special awareness or special knowledge in solving problems. Adinda et al. (2021) stated that one of the reasons students failed to solve absolute value problems was that students needed help to obtain information from the problem. Karaali (2015) states that strong motivation is very helpful in raising one’s metacognitive awareness. Lack of motivation resulted in the subject not trying to find more solutions that had not been obtained.

Students at the identify use level have metacognitive awareness characteristics that have reached the thinking stage about the information contained in the problem. At this level, the subject has thought of information and can obtain information from the problem. However, subjects at this level have yet to be able to use their obtained information to solve problems. The subject at this level already remembers the formula or way to solve it, but he cannot apply it. A student can remember formulas to solve math problems, but he needs to learn how to apply formulas and forget how to manipulate formulas to solve problems. Wilson & Clarke (2004) states that a person’s metacognitive awareness when doing tasks is one of them looking for what can be done from the information we get. Nevertheless, the subject needs help to connect or apply information to solve problems at this level.

Students at the level of Semi-aware use have the characteristics of metacognitive awareness, thinking about what is being asked, thinking about information from the problem and how to relate this information to solving absolute value problems. At this level, the subject can relate the information he has obtained to solve the problem. Purnomo et al. (2017) explained that the component of a person’s metacognitive awareness is when the subject can connect questions with similar problems obtained and solved before. However, subjects at this level have yet to rethink the strategic steps in solving absolute value problems. Subjects at this level do not reconsider that the strategic steps in solving absolute value problems are still wrong or that something still needs to be added. That shows that the subject is still lacking in planning and monitoring the strategies carried out in line with Vrugt & Oort (2008), which states that monitoring strategy refers to one’s awareness of understanding and how to complete tasks. Amram et al. (2019) explained that one of the things that cause the subject to experience difficulties in solving problems is solving problems that involve many technical steps. Adinda et al. (2021) stated that one of the causes of student failure in solving absolute value problems is that students must realize that their work still needs to be corrected.

Students at the aware use level have the characteristics of metacognitive awareness, have thought about what is being asked, thinking about how to connect the information obtained to solve the problem and rethought the stages or steps in solving the problem so that the subject already has a strategy for solving the problem. At this level, the subject already knows the steps in solving the problem, the strategic steps, and whether the questions have been answered. That is in line with the level of aware use developed by Swartz & Perkins (2017), where individuals know what and why they are implementing strategies and are aware of their decisions. In this case, the subject is already thinking about his actions. That is, the subject solves the problem with clear reasons. Even though there are questions that can be searched using specific strategies, subjects at this level have already solved problems by doing procedural things. Even though the subject is looking for a solution procedurally, at this level, the subject can solve the problem by thinking about the right reasons. That follows the opinion of Purnomo et al. (2022) and Sa'dijah et al. (2023) that students are used to doing procedural things, so they need to realize the actual questions.

Students at the strategic use level have the characteristics of metacognitive awareness already thinking about what is being asked, rethinking the information provided in the problem, rethinking how to connect the information obtained to solve the problem, rethinking the stages in solving the problem so that students already have a strategy to solve the problem, and rethinking strategies or other ways of solving problems. Subjects at this level compare the ways they make to solve problems so that they try special or different strategies for solving problems. At this level, the subject initially seeks a procedural solution. However, because the subject experienced problems in solving the problem, the subject took the initiative to find other strategies to find solutions. That shows that this subject is looking for a specific strategy that makes it easier for him to solve the problem, following Akben's opinion (2020), which states that someone with metacognitive awareness must ask himself whether the strategy I have used before is appropriate or not. That is in line with the strategic use level developed by Swartz & Perkins (2017), where individuals have organized their thinking to select strategies (Adinda et al., 2022; Purnomo et al., 2023). Changing cognitive strategies in solving problems is also a regulatory metacognitive activity (Jacobse & Harskamp, 2012; Subanji et al., 2021). Therefore, the subject has rethought different strategies to solve problems in this case.

Students at the reflective use level have the characteristics of metacognitive awareness, have thought about what is being asked, rethought the information provided in the problem, rethought how to connect the information obtained to solve the problem, rethought the stages in solving the problem so that students already have a strategy to solve the problem—rethinking strategies or other ways to solve problems and rethinking the description of answers to problems that have been solved. At this level, the subject always checks the results he has found before concluding. In this case, the subject's metacognitive awareness is full or highest. This is in line with the strategic use level developed by Swartz & Perkins (2017): individuals have pondered their thoughts before, during and after the settlement process. In this case, the subject has reached the stage of realizing and correcting the mistakes made in every step of solving the problem.

CONCLUSION

The results of this study concluded that there are new levels between tacit use and aware use levels at Swartz & Perkins' metacognitive awareness level, namely, identify use and semi-aware use, and there are characteristics of metacognitive awareness in solving absolute value problems. There is six metacognitive awareness of prospective mathematics teacher students in solving absolute value problems: tacit use, identify use, semi-aware use, aware use, strategic use, and reflective use. The determination of the characteristics of each level is based on awareness, regulation and evaluation activities. Nevertheless, what was developed in this research is the awareness component.

The characteristics of students' metacognitive awareness at the levels of tacit use, identify use, semi-aware use, aware use, strategic use, and reflective use are as follows. In tacit use, students can think about what is being asked. Students can rethink the information provided in the problem on identify use, above tacit use. In semi-aware use, above identify use students can rethink how to connect the information obtained to solve problems. In aware use, above semi-aware use, students can rethink their strategic steps to solve problems. In strategic use, students rethink different ways of answering a mathematical problem. In reflective use, students reconsider the correct answers to mathematical problems that have been solved.

This research has produced a valid and reliable metacognitive awareness of prospective mathematics teacher students. The six advanced levels can identify all the characteristics of metacognitive awareness that arise in solving absolute value problems. Metacognitive awareness that has been developed can support the development of mathematical metacognitive awareness levels, the development of instruments to measure mathematical metacognitive awareness, the development of learning models to support mathematical metacognitive awareness, cognitive style characterization of students' mathematical metacognitive awareness levels and tracing the process of mathematical metacognitive awareness.

REFERENCES

Abdellah, R. (2015). Metacognitive Awareness and its Relation to Academic Achievement and Teaching Performance of Pre-service Female Teachers in Ajman University in UAE. *Procedia - Social and Behavioral Sciences*, *174*, 560–567. https://doi.org/10.1016/j.sbspro.2015.01.707

Adinda, A., Ajda, K.H., Purnomo, H., & Zulhamdi. (2022). Mapping Prior Knowledge Middle Students in Solving Arithmatics Problems. *Logaritma: Jurnal Ilmu-ilmu Pendidikan dan Sains*, *10*. Retrieved from http://jurnal.iain-padangsidimpuan.ac.id/index.php/LGR/article/view/6153/pdf

Adinda, A., Purwanto, P., Parta, I.N., & Chandra, T.D. (2021). Investigation Of Students' Metacognitive Awareness Failures About Solving Absolute Value Problems in Mathematics Education. *Eurasian Journal of Educational Research*, (95), 17–35. https://doi.org/10.14689/ejer.2021.95.2

Akben, N. (2020). Effects of the Problem-Posing Approach on Students' Problem Solving Skills and Metacognitive Awareness in Science Education. *Research in Science Education*, *50*(3), 1143–1165. https://doi.org/10.1007/s11165-018-9726-7

Almog, N., & Ilany, B.-S. (2012). Absolute value inequalities: High school students' solutions and misconceptions. *Educational Studies in Mathematics*, *81*(3), 347–364. https://doi.org/10.1007/s10649-012-9404-z

Amram, M., Dagan, M., Satianov, P., & Yoshpe, M. (2019). The absolute value operation as an effective tool for enhancing students' cognitive activities in Calculus courses. *International Journal of Contemporary Mathematical Sciences*, *14*(1), 9–15. https://doi.org/10.12988/ijcms.2019.81235

Aurah, C.M., Keaikitse, S., Isaacs, C., & Fincii, H. (2011). The Role of Metacognition in Everyday Problem Solving Among Primary Students in Kenya. *Problems of Education in the 21st Century*, pp. 9–21.

Bay, E., Cetin, B.B., & Cetin, B. (2012). The Effects of Social Constructivist Approach on the Learners' Problem Solving and Metacognitive Levels. *Journal of Social Sciences*, *8*(3), 343–349.

Boekaerts, M. (1999). Metacognitive experiences and motivational state as aspects of self-awareness: Review and discussion. *European Journal of Psychology of Education*, *14*(4), 571–584. https://doi.org/10.1007/BF03172980

Çiltaş, A., & Tatar, E. (2011). Diagnosing Learning Difficulties Related to the Equation and Inequality that Contain Terms with Absolute Value. *International Online Journal of Educational Sciences*, *3*(2), 461–473. Retrieved from www.iojes.net

Creswell, J.W. (2014). *Research Design: Qualitative, Quantitative and Mixed Methods Approaches, fourth*. Sage Publication Ltd.

Curtis, M.A. (2016). *Solving Absolute Value Equations and Inequalities* (pp. 10–11).

Efklides, A. (2006). Metacognition and affect: What can metacognitive experiences tell us about the learning process? *Educational Research Review*, *1*(1), 3–14. https://doi.org/10.1016/j.edurev.2005.11.001

Elia, I., Özel, S., Gagatsis, A., Panaoura, A., & Özel, Z.E.Y. (2016). Students' mathematical work on absolute value: focusing on conceptions, errors and obstacles. *ZDM - Mathematics Education*, *48*(6), 895–907. https://doi.org/10.1007/s11858-016-0780-1

El-khateeb, M.M.A. (2016). Errors Analysis of Solving Linear Inequalities among the Preparatory Year Students at King Saud University. *Journal of Education and Practice*, *7*(12), 124–133.
Fisher, R. (1998). Teaching thinking and creativity Developing creative minds and creative futures Thinking. *Early Child Development and Care*, *141*, 1–15.
Glaser, B.G., & Strauss, A.L. (1995). *The Discovery of Grounded Theory: Strategies for Qualitative Research*. A Division of Transaction Publisher.
Hassan, N.M., & Rahman, S. (2017). Problem solving skills, metacognitive awareness, and mathematics achievement: A mediation model. *New Educational Review*, *49*(3), 201–212. https://doi.org/10.15804/tner.2017.49.3.16
Hess, J.D., & Bacigalupo, A.C. (2014). Enhancing Management Problem-Solving Processes through the Application of Emotional Intelligence Skills. *Journal of Management Policies and Practices*, *2*(3), 1–17. https://doi.org/10.15640/jmpp.v2n3a1
Jacobse, A.E., & Harskamp, E.G. (2012). Towards efficient measurement of metacognition in mathematical problem solving. *Metacognition and Learning*, *7*(2), 133–149. https://doi.org/10.1007/s11409-012-9088-x
Jaleel, S., & Premachandran. P. (2016). A Study on the Metacognitive Awareness of Secondary School Students. *Universal Journal of Educational Research*, *4*(1), 165–172. https://doi.org/10.13189/ujer.2016.040121
Karaali, G. (2015). Metacognition in the Classroom: Motivation and Self-Awareness of Mathematics Learners. *Primus*, *25*(5), 439–452. https://doi.org/10.1080/10511970.2015.1027837
Magiera, M.T., & Zawojewski, J.S. (2011). Characterizations of Social-Based and Self-Based Contexts Associated with Students' Awareness, Evaluation, and Regulation of Their Thinking During Small-Group Mathematical Modeling. *Journal for Research in Mathematics Education*, *42*(5), 486. https://doi.org/10.5951/jresematheduc.42.5.0486
Misu, L., & Masi, L. (2017). Comparison of metacognition awareness of male and female students based on mathematics ability in Department of Mathematics Education of Halu Oleo University. *International Journal of Education and Research*, *5*(6), 43–50.
Mockford, A. (2008). An Exploratory Descriptive Study. In *Victoria University of Wellington*.
Moleong, L.J. (2012). *Metode Penelitian Penelitian Kualitatif*. Bandung: PT. Remaja Rosdakarya.
Oz, H. (2016, October). The Importance of Personality Traits in Students' Perceptions of Metacognitive Awareness. *Procedia - Social and Behavioral Sciences*, *232*, 655–667. https://doi.org/10.1016/j.sbspro.2016.10.090
Pahayahay, G., & Cisneros-Pahayahay, M.R. (2017). Assessment of students' metacognitive awareness level in college algebra. *Advanced Science Letters*, *23*(2), 1130–1133. https://doi.org/10.1166/asl.2017.7517
Purnomo, D., Nusantara, T., Subanji, S., & Rahardjo, S. (2017). The Characteristic of the Process of Students' Metacognition in Solving Calculus Problems. *International Education Studies*, *10*(5), 13. https://doi.org/10.5539/ies.v10n5p13
Purnomo, H., Sa'dijah, C., Cahyowati, E.T.D., Nurhakiki, R., Anwar, L., Hidayanto, E., & Sisworo, S. (2021, March). Gifted students in solving HOTS mathematical problems. *AIP Conference Proceedings*, *2330*, 1–8. https://doi.org/10.1063/5.0043728
Purnomo, H., Sa'dijah, C., Hidayanto, E., Sisworo, Permadi, H., & Anwar, L. (2022). Development of Instrument Numeracy Skills Test of Minimum Competency Assessment (MCA) in Indonesia. *International Journal of Instruction*, *15(3)*, 635–648. https://doi.org/10.29333/iji.2022.15335a
Purnomo, H., Sa'dijah, C., Permadi, H., Anwar, L., Cahyowati, E.T.D. (2023). Mathematical Creative Processing Abilities of Junior High School Students in Numeracy Tasks. *AIP Conference Proceedings*, *2569*, 040012. https://doi.org/10.1063/5.0113667
Rahman, F. ur, Jumani, N.B., Chaudry, M.A., Chisti, S.ul H., & Abbasi, F. (2010). Impact Of Metacognition Awareness on Performance of Students. *Contemporary Issues in Education Research*, *3*(10).

Sa'dijah, C., Purnomo, H., Abdullah, A.H., Permadi, H., Anwar, L., Tejo, E., . . . Sa'diyah, M. (2023). Students' Numeracy Skills in Solving Numeracy Tasks: Analysis of Students of Junior High Schools. *AIP Conference Proceedings*, *2569*, 040011. https://doi.org/10.1063/5.0113664

Schraw, G., & Dennison, R.S. (1994). Assessing Metacognitive Awareness. *Contemporary Educational Psychology*, *19*, 460–475. https://doi.org/http://dx.doi.org/10.1006/ceps.1994.1033

Sophianingtyas, F., & Sugiarto, B. (2013). Identifikasi Level Metakognitif dalam Memecahkan Masalah Materi Perhitungan Kimia. *UNESA Journal of Chemical Education*, *2*(1), 21–27.

Subanji, Nusantara, T., Rahmatina, D., & Purnomo, H. (2021). The Statistical Creative Framework in Descriptive Statistics Activities Subanji. *International Journal of Instruction*, *14(2)*, 591–608. https://doi.org/10.29333/iji.2021.14233a

Swartz, R., & Perkins, D.O. (2016). Teaching Thinking: Issues and approaches. In *Routledge eBooks*. https://doi.org/10.4324/9781315626468

Vrugt, A., & Oort, F.J. (2008). Metacognition, achievement goals, study strategies and academic achievement: Pathways to achievement. *Metacognition and Learning*, *3*(2), 123–146. https://doi.org/10.1007/s11409-008-9022-4

Wilson, J., & Clarke, D. (2002, April). Monitoring Mathematical Metacognition. *American Education Research Association*, pp. 1–5.

Wilson, J., & Clarke, D. (2004). Towards the modelling of mathematical metacognition. *Mathematics Education Research Journal*, *16*(2), 25–48. https://doi.org/10.1007/BF03217394

Yildirim, S., & Ersözlü, Z.N. (2013). The relationship between students' metacognitive awareness and their solutions to similar types of mathematical problems. *Eurasia Journal of Mathematics, Science and Technology Education*, *9*(4), 411–415. https://doi.org/10.12973/eurasia.2013.946a

Young, A., & Fry, J.D. (2008). Metacognitive awareness and academic achievement in college students. *Journal of the Scholarship of Teaching and Learning*, *8*(5), 1–10. https://doi.org/10.3109/0142159X.2010.487711