Mathematical Comprehension Improvement Patterns to Facilitate Math Problem Solving for Junior High School Students

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This study aims to find a pattern of correcting mathematical understanding errors in solving mathematical problems for junior high school students. The method used in this research is qualitative. Data collection using triangulation technique: observation, test results, and interviews. Interviews were conducted on three subjects for one month at MTs. The instrument used is a mathematical problem-solving test. This study found patterns of improvement in mathematical problems in geometry, namely the area and perimeter of rectangles and lines. This study found that the pattern of mathematical understanding errors can be corrected by applying patterns such as reminding the material of rectangular shapes, parallelograms, algebraic forms, and units of measure, writing down everything that is known in the problem, and making an example of a variable if it cannot be written directly. The pattern of improving mathematical understanding can solve the mathematics problems of junior high school students. Furthermore, this pattern of mathematical understanding errors can be applied by teachers to improve math problems and embed geometric concepts in mathematics learning.

Keywords: mathematical error, model improvement, problem-solving, mathematical understanding

INTRODUCTION

Mathematical understanding is one of the basics of learning mathematics. In the cognitive aspect, understanding must be mastered when learning mathematics. Understanding, according to Bloom, is the ability to capture the meaning of what is learned (Donevska-Todorova, 2015). Understanding is something that takes place in the brain without external action. The types of understanding are; i) conceptual understanding, ii) operational understanding, iii) relational understanding, and iv) mathematical understanding. Understanding is a network resulting from representations related to that mathematical concept (Barmby et al., 2007). Students begin to be able to make connections between mathematical ideas and make generalizations from concepts equipped with basic skills that are already understood at a deep level of understanding (Potter & Kustra, 2012). Mathematical understanding is one of the foundations of learning mathematics. According to Manu (2005), mathematical understanding depends on ideas and pictures rather than words. There is a relationship between students' understanding of mathematics and representation (Güner & Uygun, 2020). Mathematical understanding plays an important and crucial role in learning mathematics (Yang et al., 2021). Research shows that understanding mathematics before formal schooling affects achievement later in life (Cave, 2021). Concept visualization can improve students' mathematical understanding (Yerizon et al., 2021). Teacher-student interaction influences students' mathematical understanding (Ayuwanti et al., 2021). A new mathematical understanding of this framework demonstrates its effectiveness in a much broader setting than previously understood (Avestimehr et al., 2021).

The ability to understand mathematics is one of the important goals in learning, giving an understanding that the materials taught to students are not just rote. With understanding, students can better understand the concept of the subject matter itself. Mathematical understanding is a dynamic, nonlinear, self-replicating continuum and goes through different phases (Güner & Uygun, 2020). This theory is important in mathematics education because it provides deep insight into the meaning of understanding something and understanding mathematics (Lester, 2005). Principle-oriented explanations have been shown to encourage students' mathematical understanding as it integrates conceptual and procedural information to make the solution process real for students (Lachner et al., 2019). In addition, because students' prior knowledge and perceptions are different, their mathematical understanding may differ (George, 2017). Mathematical understanding is important in problem-solving (Siskawati et al., 2022). Mathematics involves problem-solving (Maboya et al., 2020). According to Gr. Voskoglou (2021) problem solving is one of the most important activities of human cognition. One of the most important components of human cognition that has influenced the progress of human society over the centuries is problem-solving (Lester, 2013). Problem-solving is very important in solving mathematical problems because it involves the skills of applying, analyzing, evaluating, and creating (Alhassora et al., 2017).

Problem-solving ability is one of the important competencies in learning mathematics. A person's mathematical ability can facilitate problem-solving. Problem-solving has become a focus of reform in basic mathematics education (Hourigan & Leavy, 2022). Solving mathematical problems is complex and can be cognitively and emotionally challenging (Di Leo & Muis, 2020). Problem-solving involves students applying four processes: reasoning, communication, connection, and representation. One of the abilities of mathematics is to master mathematical concepts (Al-Mutawah et al., 2019). Problem-solving requires some basic conceptual understanding of the situation, both in terms of first-order and second-order symbols (Abramovich, 2015). Unstructured problem-solving consists of five stages: problem representation, solution search, selection of solutions through arguments and justification, monitoring and evaluation, and jointly creating the final solution. From stage one to stage four, students are expected to explore and select information online, learn individually to find solutions, and share information through group interaction. In stage five, students solve problems collaboratively by comparing, analyzing, synthesizing various perspectives, and jointly producing the final product (Kim & Lim, 2019).

Various intervention programs encourage understanding students' basic concepts (Prediger et al., 2022). Unavoidable basic math concepts include algebra and geometry. Geometry is a very important material in learning mathematics (Hermanto & Santika, 2019). The basic concepts of geometry can solve everyday life problems. Learning mathematics is as important as learning geometry (Deringöl, 2020). Studying geometry is important in mathematics because it allows students to combine everyday life situations with mathematical topics and reach conclusions (Erkek & Işiksal-Bostan, 2015). At the secondary school level, teachers must start preparing students for the more formal study of geometry to follow in secondary school (Training and Technical Assistance, 2004). Generalized forms are made by secondary students when they explore geometric transformations in a dynamic geometric environment (Yao & Manouchehri, 2019).

However, the basic mathematics concepts are not mastered by students, including geometry. This is in line with the understanding of Veloo et al. (2015), which found that 24% of students made conceptual mistakes in understanding mathematical symbols, graphs, and problem-solving. Students think basic geometry concepts are not very important but necessary to solve everyday problems. The problem-solving ability is not only in routine questions but also in the context of daily life (Nitasari et al., 2018). Prerequisite material that is not mastered will make mistakes in solving mathematical problems. Initial mathematical ability is the student's ability to understand the prerequisite material on the material to be taught (Yuliyanto & Turmudi, 2020). So, this study aims to find a model for correcting mathematical misconceptions to facilitate the solving of mathematical problems. This research is important to assist teachers in improving mathematics learning. In addition, this research is a policy for educational authorities regarding the importance of understanding geometry for students.

METHOD

The method used in this research is qualitative. Respondents actively involved in the study were three grade VIII students in the odd semester of 2022/2023. The three subjects were called Subject 1 (S1), Subject 2 (S2), and Subject 3 (S3). The subject selection technique is purposive sampling which is a technique based on special considerations. Selection is based on the consideration that the subject is cooperative to be interviewed and can provide information about the mathematical understanding process. The instrument used is a mathematical problem-solving test on geometry subjects and interview guidelines.

Data were obtained using tests, interviews, and observations. The data analysis technique uses a source triangulation model: data collection, tabulation, data presentation, and inference. The pattern of students' mathematical understanding is based on the pattern of errors found in the study of Siskawati et al. (2022), namely the pattern of errors in mastering basic mathematical concepts, patterns of errors in converting questions into mathematical sentences, patterns of errors in changing units, and patterns of errors in determining relevant information.

RESULTS

TABLE 1
THE PATTERN OF IMPROVEMENT IN MATHEMATICAL UNDERSTANDING

Error Patterns	Improvement Patterns
Patterns of error in the mastery of basic mathematical concepts	Remind a square flat wake
	Remind algebraic forms
	Shows how to operate algebraic addition and subtraction by exemplifying the addition of surrounding objects
	Giving examples of subtraction and multiplication of algebraic forms
	Re-understanding the distributive nature of addition and subtraction
Students cannot change the problem in mathematical form	Write down everything that is known in the question
	Ask to make excuses from questions with variables if they cannot be written directly
Shows the inadequacy of the unit concept	Remind the unit of weight material
	Requesting changing the unit from heavy to light and vice versa

Error Patterns	Improvement Patterns
Demonstrates the inability to identify relevant information	Remind the notion of parallelograms
	Mentioning the pedestal and height of a known parallelogram
	Ask the area of a flat hood parallelogram if the height and base are not the same

Table 1 shows improvement patterns for some error patterns found in previous studies (see Siskawati et al., 2022). The improvement pattern found in this study is to remind the invulnerability of rectangular flat wake material, parallelograms, algebraic shapes, and units of measure, write down everything that is known in the problem, and make excuses from the problem with variables if it cannot be written directly.

The pattern of improvement in mathematical understanding are:

1. Reminiscent of a flat rectangular shape

FIGURE 1
ERROR USING FORMULA

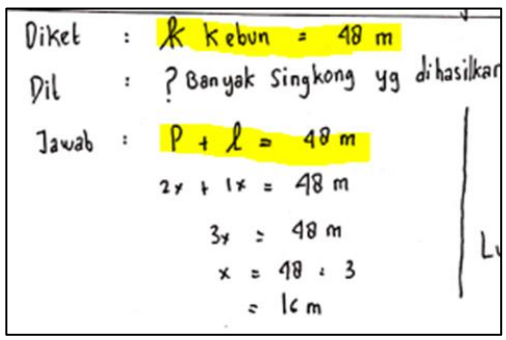
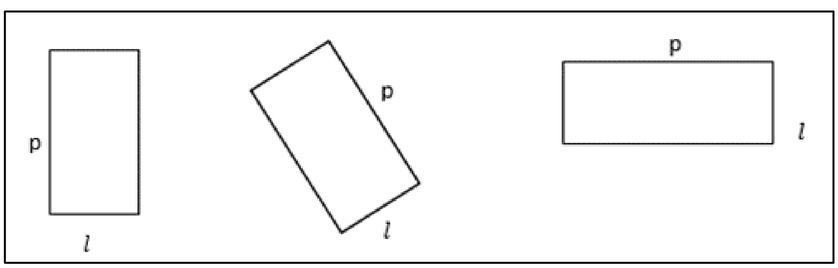


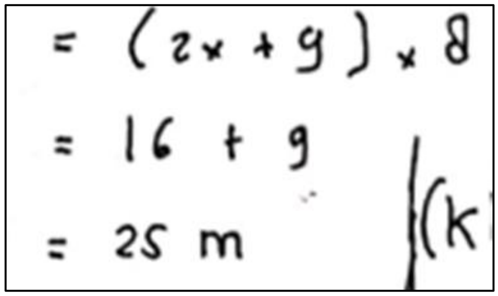
FIGURE 2
DIFFERENT RECTANGLE SHAPES



The mistakes made by students in the questions (Figure 1) were using the wrong formula. Students write the formula for the area of a rectangle equal to 2 multiplied by the sum of the length and width. It should not be the area of the rectangle but what the student meant by the perimeter of the rectangle. The teacher helps students recall what they know about rectangles. To deepen their understanding of rectangles, the teacher asks questions to make students aware of deficiencies or incompleteness in their knowledge and correct the correct definition of rectangles. The question posed is to describe a rectangular shape, draw 3 different rectangles, and determine their length and width. A rectangle is a quadrilateral with two pairs of parallel and equal sides. If L is the area, K is the perimeter, p is the length, and l is the width of a rectangle, then: $L = p \times l$ and $K = 2(p + l)$ or $K = 2p + 2l$. Students understand the elements of length and width in rectangles that have different positions as follow Figure 2.

2. Remind algebraic forms

FIGURE 3
ERROR OPERATING ALGEBRAIC FORMS



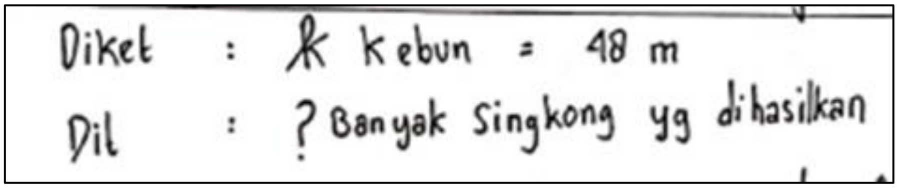
Errors made by students in operating algebraic forms (Figure 3). The teacher helps students recall what they know about algebraic forms. To deepen their understanding of algebra, the teacher asks questions to make students aware of deficiencies or incompleteness in their knowledge and correct their proper algebraic understanding. The questions asked to students are what do you know about algebra? An algebraic form is a mathematical form that contains letters representing unknown numbers in its presentation. A variable is a substitute for a number whose value is not known. Constants are terms of an algebraic form numbered and do not contain variables. Tribes are variables and their coefficients or constants in an algebraic form separated by sum or difference operations.

3. Demonstrate how to operate algebraic addition and subtraction by exemplifying the addition of surrounding objects
Figure 3 also have seen student errors in operating algebra. Developing mathematical understanding often involves carrying out specific activities to solve algebraic problems. The teacher explains algebraic operations with examples using objects around students. The teacher takes a book and 2 pens, then tell the students; what is the amount of one book plus two pens. Students answer three. Questions develop, and the teacher asks three what? Is it three books or three pens? Students are getting confused. The teacher understands students that addition or subtraction in algebra if they are not the same, the results are the same. So, one book plus two pens equal one book plus two pens.
4. Give examples of subtraction and multiplication of algebraic forms
In the mistakes from Figure 3, the teacher gives examples of subtraction and multiplication in algebra using nearby objects. If you take five pencils minus two pencils, what is the result? The student answered correctly, namely, three pencils. Next, for multiplication, what is the result if there are three books multiplied by four? The teacher asked what multiplied means, but the students did not answer. The teacher gives an understanding that multiplication is the same as repeated addition. So three books multiplied by four high schools is three books plus three books plus three books plus three books equals twelve books.
5. Re-understand the distributive property of addition and subtraction
The teacher helps students recall what they know about the distributive property of addition and subtraction. It can be seen in Figure 3 that the student's error in operating $(2x + 9) \times 8 = 16 + 9 = 25$. The student multiplied two by eight by eliminating the x variable, and nine was not multiplied by two. To deepen their understanding of the distributive property of addition or subtraction, the teacher asks questions to make students aware of deficiencies or incompleteness in their knowledge and improve their understanding of the distributive property of appropriate addition and subtraction. The questions asked to students are what do you know about the distributive property of addition and subtraction? The distributive property of

addition and subtraction is a x (b + c) = (a × b) + (a × c) and a x (b – c) = (a × b) – (a × c). Furthermore, the correct problem on distributive operations, in addition, is (2x + 9) × 8 = (2x × 8) + (9 × 8) = 16x + 72.

6. Write down everything that is known about the problem

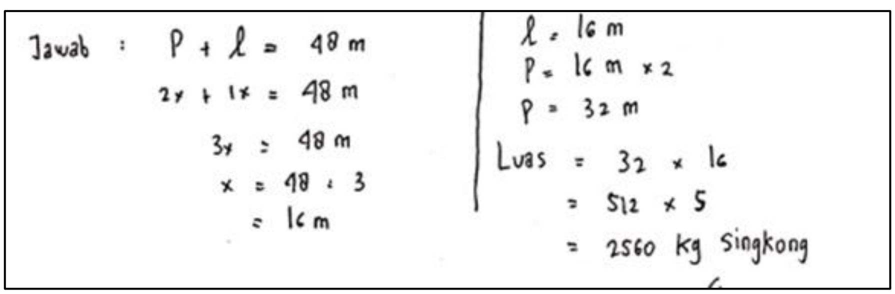
FIGURE 4
INCOMPLETE ERROR WRITES DOWN WHAT IS KNOWN



One of the student's mistakes in solving problem-solving questions was incompletely writing down everything that was known in the problem, as seen in Figure 4. The teacher helps students identify everything that is known in the problem. What should be known about the problem is that a cassava garden is rectangular, the length of the garden is twice the width, the circumference of the garden is forty-eight, and every square meter of the garden produces five kilograms of cassava.

7. Ask to make examples of problems with variables if they cannot be written directly
In solving problem-solving questions, the student did not make a mathematical model, as seen in Figure 5.

FIGURE 5
ERROR WITH NOT CREATING A MATHEMATICAL MODEL



The question states that the length of the garden is twice the width of the garden. The teacher guides students to create a mathematical model from questions that contain variables. Guide students to make a mathematical model by assuming the width is x, and the length is 2x. Next, represent the problem by drawing a rectangle with a length of 2x and a width of x. Seen in solving the problem, students wrote the wrong circumference formula. So the value of x found is wrong. The teacher guides students in correcting mistakes by guiding them to write down what is known in full. Then write down what is being asked. In the completion stage, make a mathematical model, represent the problem, and write down the correct formula.

8. Reminds the unit weight material

FIGURE 6
UNIT CHANGE ERROR

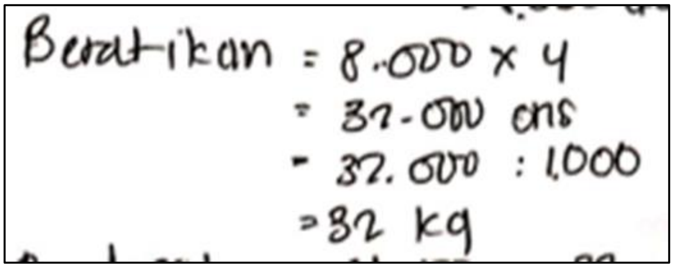


Figure 6 show that students make mistakes in changing units of ounces to kilograms. The teacher helps students recall what they know about units of weight. To deepen their understanding of units of weight, the teacher asks questions to make students aware of deficiencies or incompleteness in their knowledge and corrects units of weight appropriately. The questions asked to students are what do you know about the unit of weight? The weight unit is; tonnes, quintals, kilograms (kg), hectograms (hg), dekagrams (dag), grams (g), decigrams (dg), centigrams (cg), and milligrams (mg). Whereas 1 ton = 1000 kg, 1 quintal = 100 kg, 1 ounce = 0.1 kg = 1 hg, 1 pound = 0.5 kg = 5 ounces.

9. Ask to change the unit from heavy to light and vice versa
   Figure 6 also show that the mistakes made by students were wrong in changing units. The teacher helps students recall what they know about units of weight. To deepen their understanding of units of weight, the teacher asks questions to make students aware of deficiencies or incompleteness in the knowledge possessed and corrects the appropriate units of weight. The questions asked students to change the units. How many kilograms of two quintals, how many kilograms three hundred ounces? Then the student answered correctly that two quintals equal two hundred kilograms and three hundred ounces equal thirty kilograms.
10. Remind the meaning of a parallelogram again

FIGURE 7
IMAGE OF A PARALLELOGRAM

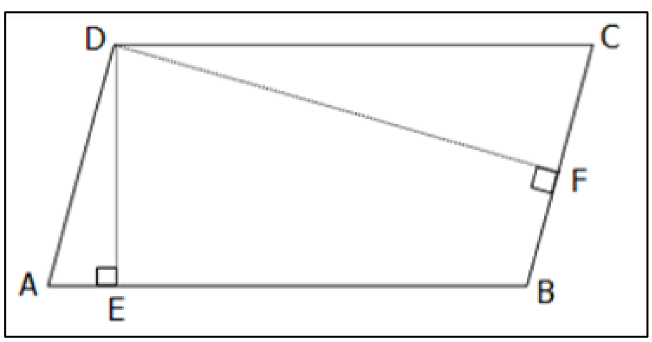
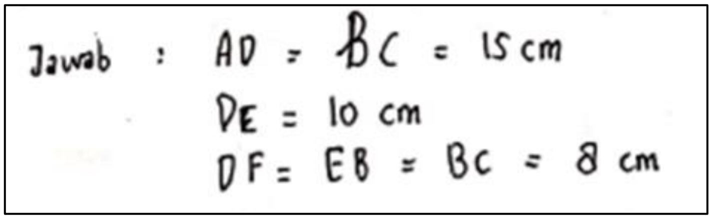


FIGURE 8
ERROR WRITING SIDE LENGTH

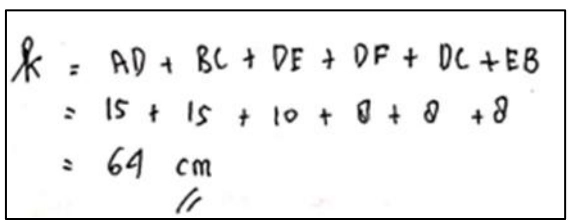


Jawab : AD = BC = 15 cm
DE = 10 cm
DF = EB = BC = 8 cm

To answer question in Figure 7, students make mistakes by writing down the length of DF = EB = BC = 8 cm, as seen in Figure 8. This means that students do not understand correctly about parallelogram. The teacher helps students recall what they know about parallelograms. To deepen their understanding of parallelograms, the teacher asks questions to make students aware of any deficiencies or incompleteness in their knowledge and improve their understanding of parallelograms. The question posed asks students to explain the meaning of a parallelogram. A parallelogram is a quadrilateral with two pairs of parallel sides and equal opposite angles. Suppose L is the area, K is the perimeter, a is the length of the base, l is the width, and t is the height of a parallelogram, then: $L = a \times t$ and K = the sum of all the side lengths.

11. State the base and height of a known parallelogram
Students also seem not to understand parallelograms (seen Figure 9) in solving problems in Figure 7.

FIGURE 9
ERROR NOT UNDERSTANDING PARALLELOGRAM



K = AD + BC + DE + DF + DC + EB
= 15 + 15 + 10 + 8 + 8 + 8
= 64 cm

The teacher helps students recall what they know about parallelograms. To deepen their understanding of parallelograms, the teacher asks questions to make students aware of any deficiencies or incompleteness in their knowledge and improve their understanding of parallelograms. The questions asked students to name the elements in a parallelogram and the formula for determining their area. Based on the picture in the problem, the base of the parallelogram is AB if the height of the parallelogram is DF, and the base is BC if the height is DF. The formula for the area of a parallelogram is AB multiplied by DF or BC multiplied by DF.

12. Find the area of a parallelogram if the height and base are not the same
It can be seen from the student's answers (Figure 9) that the student cannot determine the base and height in a parallelogram (Figure 7). The teacher asks for the formula for determining the area of a parallelogram. The area is equal to the base multiplied by the height. The questions are given different heights of parallelograms. The teacher confirms students' understanding of the area of a parallelogram by asking whether the parallelogram ABCD has the same area if the base and height of a parallelogram are different. The student answered differently. The

teacher explains that if the base and height of a parallelogram are the same, then a parallelogram must have the same area.

The teacher gave questions about the meaning of square length, the formula for the area of a square, and the formula for the circumference of a square. A rectangle is a quadrilateral with two pairs of parallel sides and four right angles—rectangle properties. The properties of a rectangle, namely having four sides, with a pair of opposite sides that are equal in length and parallel, the four angles are equal and are right angles (900), the two diagonals are the same length and intersect bisecting the same size, can occupy the frame again with four ways. The formula for the area of a rectangle is length times width. The formula for the perimeter of a rectangle is twice the length plus twice the width.

DISCUSSION

The results of previous studies stated that with proper stimulation and involvement, students could achieve higher statistical literacy levels (Medová et al., 2022). Stimulation is necessary for learners to fully understand the content (Deng & Benckendorff, 2021). In this case, the teacher gives an understanding of algebraic material by asking about the meaning of algebra. The definition of an algebraic form is a mathematical form containing letters representing unknown numbers in its presentation. Algebraic forms can be used to solve problems in everyday life. Ask how many terms are in algebra, coefficients, variables, and constants.

The teacher directs students to certain parts of knowledge that he considers useful in strengthening students' understanding. A Variable is a substitute symbol for a number whose value is unknown. A constant is a term of an algebraic form, a number that does not contain variables. The definition of a coefficient is a constant factor of a term in an algebraic form. Meanwhile, the notion of terms is a variable and its coefficient or constant in an algebraic form separated by a number or difference operation. The findings suggest that to help students understand new mathematical concepts, teachers must first explain the definition of concepts given to students and then teach them how to create specific examples based on existing concepts (Yang et al., 2021).

Knowledge, abilities, and training are forms of understanding that require stimulation (Suparsa et al., 2021). Demonstrating how to operate algebraic addition and subtraction by exemplifying the addition of surrounding objects is a stimulus for students. In the example, if two markers are added to four notebooks, the result is the same: two markers and four notebooks. Levin's research shows that by exemplifying early childhood how algebraic thinking is used in reasoning in various contexts, grade levels, and levels of formality in algebraic instruction (Levin & Walkoe, 2022). The content taught in Patterns, and Algebra provides examples of learning experiences appropriate for students (Wilkie, 2014). Mathematical knowledge is built hierarchically from a basic understanding of addition and subtraction (Chen et al., 2021). The distributive nature of mathematics allows students to experiment in real-time (Schoenle et al., 2017). Rewriting is easier for students to memorize and manipulate and more intuitive in understanding the necessary corrections (Devlin, 2009). Ask to make excuses from questions with variables if they cannot be written directly and provide precise instructions (Ergene & Bostan, 2022).

The pattern of improvement by recalling the unit material of weight is very involved in repeated studies (Zawadzka et al., 2018). Conceptual understanding, such as the understanding of measuring units, is an important pedagogical goal and is prone to misperceptions by students (Sengupta et al., 2017). Presenting the same or related material for repetitive learning can remind learners (students) of previous learning (Putnam et al., 2017). For example, they mention the pedestal and height of a known parallelogram. Teacher-student interaction affects student understanding (Ayuwanti et al., 2021). For example, ask about the area of a flat wake-up parallelogram if the height and base differ. Discussions encourage students to reflect on and talk about their maps and the links they have created (Hammad et al., 2021).

CONCLUSION

This study concludes that the pattern of mathematical understanding errors can be corrected by several improvement patterns found in this study. The pattern of improvement found is to remind the material of flat rectangular shapes, parallelograms, algebraic forms, and units of measure, write down everything that is known in the problem, and make examples of questions with variables if they cannot be written directly. The pattern of correcting mathematical understanding errors can facilitate solving math problems for junior high school students.

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