The Model of Geometry Learning With Spatial Skills Features: Is It Possible?

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Integrating spatial skills into the model of geometry learning has become a major concern because of its role in solving geometric problems. Therefore, the present study endeavors to design syntax, social systems, reaction principles, support systems, and learning impact. To achieve this goal, a literature review was conducted, involving 58 scholarly articles and 11 literary books to inform the design of the learning model based on key constructs such as learning theory, spatial skills, and model development. The research culminated in articulating a rigorous theoretical rationale and an underlying framework that informs the learning model's conceptualization.

Keywords: learning model, spatial skills

INTRODUCTION

Spatial skills have emerged as a central issue, given their invaluable role in everyday problem-solving. They equip individuals with the ability to navigate and orient themselves in space, perceive and manipulate objects in the mind's eye, and visualize spatial relationships (Wakabayashi & Ishikawa, 2011; Uttal, et al, 2013; Atit, et al, 2020). Spatial skills play a vital role in enhancing an individual's academic learning (Clements, 1999; Yilmaz, 2009; Nagy-Kondor, 2016); for instance, learning multiplication through grids (Clements, 1999), fractions through flat shapes (Clements, 1999), and number lines by visualizing large numbers placed on the right and vice versa (Gunderson, et al, 2012) are all facilitated by spatial skills. Furthermore, spatial skills aid students in solving mathematical problems (NCTM, 2000; Anderson, et al, 2001), with spatial skills supporting 38,16% of performance in solving math word problems (Anderson, et al, 2001). In the future, individuals with strong spatial skills are likely to excel in mathematics (Borriello &

Liben, 2018; Gilligan, et al, 2019; Rittle-Johnson, et al, 2019), and succeed in STEM-related careers (Rittle-Johnson, et al, 2019; Gagnier, & Fisher 2020).

Acquiring spatial skills are imperative for mastering the subject of geometry (Izard, 2020; Carbonell-Carrera et al., 2021). By honing their spatial abilities, individuals become proficient in solving complex spatial problems that involve encoding object characteristics, visualizing or manipulating objects, and arranging shadows based on orientation through appropriate representation (Cohen & Hegarty, 2014). For example, a person with strong spatial skills can skillfully interpret images of shapes, sizes, and positions in a particular orientation (Izard, 2020).

Spatial visualization and spatial orientation are essential and influential components in geometry learning. Spatial visualization is crucial when students are required to identify an object that shares the same characteristics as another object but is situated in a different orientation. Similarly, spatial orientation is fundamental when students need to observe or understand changes in the object's position or orientation. The Indonesian Ministry of Education and Culture's recent mandate for primary education, regulation number 7 of 2022, highlights the need for spatial experiences related to flat and solid shapes, as well as their properties, in explaining the environment. Thus, it is imperative that geometry education effectively harnesses the components of spatial skills to optimize student learning outcomes.

The observations on 375 fourth-grade students from 25 schools in East Java, Indonesia during 2022-2023 revealed several important findings. Firstly, 248 students (66.13%) made errors in stating the properties of cubes and rectangular prisms when placed in a tilted position. Secondly, 334 students (89.07%) concluded that the top face of cubes and rectangular prisms is in the shape of a parallelogram. Thirdly, all students were unable to draw cubes and rectangular prisms from a specific orientation, with 256 students (68.27%) making mistakes in assembling the unit cells that form the nets and 302 students (80.53%) being unable to identify the faces of the cubes that are opposite each other in the nets. These results suggest that the spatial orientation skills of the students have not developed sufficiently.

Observations of geometry learning on the topics of cubes and rectangular prisms in 25 elementary schools in East Java, Indonesia, between 2022-2023 revealed the following: 1) geometry learning was conducted both in group settings and without, yet both approaches were focused on acquiring procedural knowledge, which according to (Silver, 1996), is still geared towards a mastery style of learning; 2) learning experiences in geometry were in the form of visualizations and visualization-orientation, but both focused only on naming shapes and attributes, while according to (Jo & Bednarz, 2009; Jo, et al, 2010) naming shapes and attributes merely represent the characteristics of an object; 3) teachers played the roles of facilitators, informants, and mediators, but they still dominated the learning process and did not provide opportunities for students to interact with media; 4) teachers responded to students' work and assignments, but both only measured skills, 5) support systems provided visual experiences, but did not provide spatial orientation experiences;; 6) geometry learning already involved spatial visualization activities, but spatial orientation activities were not yet evident, such as: a) recognizing the elements of cubes and rectangular prisms from certain orientations, such as when they are placed at an angle; b) recognizing the shapes of the bottom and top side; c) drawing cubes and rectangular prisms from specific orientations; d) viewing cube nets from various perspectives; 7) geometry assignments tended to focus on skill-based learning, and assessments tended to use multiple-choice questions, thus not measuring students' spatial skills. These results conclude that the geometry learning pattern has not yet fully utilized students' spatial orientation abilities.

The aforementioned identification results require attention. The learning patterns that do not involve spatial skills elements have resulted in insufficient development of students' spatial skills. However, spatial skills have become a crucial need in the curriculum, and therefore, the involvement of spatial skills elements in geometry learning needs to be implemented. One solution is to develop a geometry learning model that incorporates spatial skills as a necessity for students.

Several researchers have developed models for teaching geometry. Ikhsan (2012) developed a geometry learning model based on the Van Hiele theory, consisting of five stages: 1) orientation, 2) group discussion, 3) class discussion, 4) integration, and 5) evaluation. Alim, et al. (2020) proposed a realistic geometry learning model using multimedia with nine stages: 1) orientation to realistic problems, 2) understanding and solving problems horizontally, 3) guidance and stimulation, 4) presenting results, 5) orientation to realistic problems 2, 6) understanding and solving problems vertically, 7) confirmation using interactive media, 8) applying concepts, and 9) evaluation. Nur'aeni (2020) developed a SPADE model based on traditional games with the following stages: 1) singing, 2) playing, 3) analyzing, 4) discussing, and 5) evaluating. However, the three geometry learning models developed so far have not focused on spatial skills.

Howse & Howse (2015) obtained a set of instructional activities aimed at improving preschoolers' spatial reasoning through attribute blocks, which included the following steps: 1) information, 2) directed orientation, 3) explication, 4) free orientation, and 5) integration. The study was limited to two-dimensional shapes. Wulandari (2020) developed a valid, practical, and effective geometry teaching model to enhance the spatial abilities of junior high school students, which involved the following steps: 1) acquiring information, 2) facilitating spatial activities, 3) reinforcing information, and 4) integration. While the imitative model improved the spatial abilities of students as a group, 26 out of 33 (79%) students in the first trial and 22 out of 35% in the second trial showed low spatial orientation skills individually, indicating that the impact of fostering students' interest in viewing things from various perspectives and processing spatial information was not evident for these students. Meanwhile, children's educational experiences must be rich to optimize their cognitive development (Subanji, 2013). Therefore, this model must be adjusted to make it relevant to elementary school students.

The above problem background suggests that a geometry learning model that focuses on spatial skills needs to be developed. Hence, this study aimed to design a syntax, social system, reaction principles, support system, and learning impact supported by theoretical rationale and a foundation for understanding what and how students learn in the context of developing spatial skills in geometry.

LITERATURE REVIEW

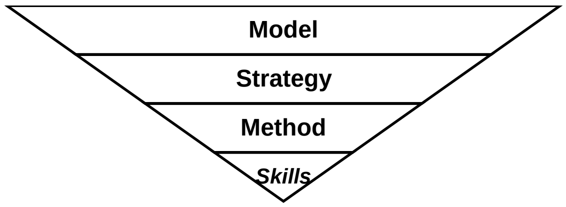
A learning model is a conceptual framework that outlines the systematic procedures for organizing learning activities to achieve learning objectives (Subanji, 2013). The five distinguishing characteristics of one model from another are syntax, social system, response principle, support system, and learning impact. Therefore, a model must be supported by theoretical rationality and a foundational understanding of what and how students learn.

Joyce, et al., (2015) has described four learning models based on different learning processes and knowledge construction: information processing, personal, social interaction, and behavioral. The information processing model is based on information processing theory, which involves acquiring, mastering, and processing information in the students' minds. The personal model is based on individual cognitive theory, where students construct knowledge individually. The social interaction model is based on social cognition theory, which suggests that knowledge construction in students can occur well when supported by social interaction. The behavioral model is based on behaviorist thinking, which emphasizes observable changes in behavior that are expected to be consistent in students.

The learning model was developed to aid students in acquiring skills, information, and values and developing their thinking skills (Subanji, 2013). This objective can be achieved through the learning process. Therefore, the model is inseparable from the learning process. The learning process is a system that involves strategies, methods, approaches, or models. As described in Figure 1 (Saschatchewan Education, 1988), the levels of the learning process can be delineated.

The term "model" is often used interchangeably with "approach". However, Arends (2004) describes a model as a broader concept than an approach, strategy, or method because it includes: 1) the theoretical framework supporting the model, 2) the foundation of student learning, 3) teacher behavior designed to support the model, and 4) the learning environment that supports learning objectives. According to Arends (2004), a learning model directs a particular approach, including goals, steps, environment, and management. According to Joyce et al, (2015), a learning model guides us in designing learning to achieve learning objectives.

FIGURE 1
THE RELATIONSHIP BETWEEN MODEL, STRATEGY, METHOD, AND SKILLS



The components used to describe the implementation of a learning model include syntax, social system, response principles, and support system (Joyce et al., 2015). Syntax refers to the overall sequence of learning activities. According to Joyce et al, (2015), syntax comprises the phases teachers and students undergo during the learning process. The length of the syntax is influenced by factors such as a) the readiness to learn, b) the desired level of student participation, c) the complexity of skills, and d) the novelty of the learning material (Parta, 2017). The length of syntax also needs to consider the allocation of classroom learning time.

The social system component of a learning model refers to the interactions between students, teachers, and other learners that are expected to occur during the learning process. According to Joyce et al, (2015), the teacher's role is that of a facilitator, consultant, and mediator, while the student is considered the center of activity. As a facilitator, the teacher provides learning resources, encourages students to learn, guides problem-solving, and facilitates skills development. The mediator and consultant roles come into play when there is a conflict or argument between students or concerning the concepts being taught.

The principle of reaction refers to the response created by an individual as a result of an action. According to Parta (2017), the principle of reaction consists of two elements: the teacher's reference in responding to the student's work and the students' reference in responding to the assignments given by the teacher. The principle of reaction will describe to the teacher how to perceive anything done by the student, such as how the teacher sees, teaches, responds to questions, solves problems, provides feedback, or any other action the student takes.

The support system encompasses the necessary infrastructure, tools, and materials required in the learning process (Joyce et al, 2015). According to Arends (2004), the support system refers to the learning approach, including goals, activity stages, environment, and classroom management. In short, the support system represents the conditions necessary to implement the model.

The impact of learning can be designed based on the content underlying the activities or implicitly in the learning environment. The impact of learning will measure the consequences arising from the application of the model. The instructional impact takes the form of direct learning outcomes, resulting from the achievement of the learning environment steered towards a specific goal; the accompanying impact represents the consequences after students experience the learning environment created by implementing the model (Parta, 2017).

RESEARCH METHODOLOGY

The present study employed a library research method, utilizing 58 articles and 11 literature books as data collection tools. The sources were carefully selected to underpin the design of the learning model based on learning theory, spatial skills, and model development. Qualitative analysis in the form of descriptive analysis was conducted to analyze the data. After carefully reading the selected articles and literature books, the data was compiled and analyzed to uncover new findings and draw meaningful conclusions. The use of a library research method provided a solid foundation for developing the learning model, as it allowed for a comprehensive review of the available literature and theoretical concepts in the field.

FINDINGS

Syntax

Syntax is presented in Table 1.

TABLE 1
SYNTAX

Phase/ Stage	Syntax		Theoretical underpinnings
	Activity	Classroom Implementation	
Introduction	1. Asking students to sit together with their group members 2. Communicating the learning objectives, indicators, and motivation 3. Apperception and review 4. Distributing learning materials (Workbook and Props)	1. The teacher asked students to sit with their group members 2. The teacher communicated the learning objectives, indicators, and motivation 3. The teacher invited students to ask questions, checked for understanding of the material, and reviews assigned tasks. 4. The teacher distributed learning materials, such as LKS and instructional tools	1. Not all individuals were equipped with spatial abilities (Battista, et al, 2020) 2. Spatial ability was more dominant in boys than in girls (Battista, et al, 2020) 3. Spatial ability had an impact on mathematical achievement (Al-Balushi, 2013; Battista, et al, 2020; Oostermeijer, 2014). 4. Motivation theory (Gagne, 1985) 5. Group formation goals (Sutawidjaja & Afgani, 2015; Dahar, 2011) 6. Number of group members (Slavin, 2018) 7. Social interaction (Ramful, et al, 2015) 8. Apperception goals (Puteri, 2018) 9. Role of learning materials (Parta, 2017) 10. Use of media (Rauh, et al, 2005; Newcombe, 2010) 11. Role of media (NCTM, 2000; Fischer, et al, 2014)
Spatial Information Presentation	5. The teacher presented spatial information through the delivery of concepts and varied examples from real-world models, and then transferred to geometric models.	5. The teacher presented spatial information by delivering concepts, varied examples from the real-world model, and bringing it into the geometry model. 6. The teacher presented the terms or attributes of the geometry objects	12. The teacher assisted students in constructing knowledge (Sturz, 2009) 13. Spatial information excavation (Sternberg, 2009; Schunk, 2012) 14. The importance of mastering terms and excavating information (Skemp, 1986)

Phase/ Stage	Syntax		Theoretical underpinnings
	Activity	Classroom Implementation	
	6. The teacher conveyed the terms or attributes of the geometry object that would be discussed. 7. Together with the group members, they connected the acquired terms or attributes to build new knowledge.	that would be discussed. 7. Together with the group members, they linked the terms or attributes they had obtained to build new knowledge	15. Constructivism model (Anderson, 2001) 16. Knowledge acquisition phase (Parta, 2017) 17. Cognitive development (Subanji, 2013)
Group Discussion	8. Working collaboratively to complete spatial activities using provided media according to the instructions in the workbook. 9. Writing down the group's work results on the provided sheet. 10. Representing the group's work results.	8. Collaborating with group members to complete spatial activities using provided media according to instructions in the workbook. 9. Documenting group work results on the provided sheet. 10. Presenting group work results.	18. Spatial activities (Uttal, et al, 2013) 19. Cognitive development (Subanji, 2013) 20. Spatial problem-solving through imagistic approach (Cohen & Hegarty, 2014)
Consolidation	11. Working on consolidation exercises in groups 12. Presenting the group's work results.	11. The teacher instructed students to work on consolidation exercises in groups. 12. The teacher observed students' work in groups and asks questions to check for clarity of understanding. 13. The teacher asked a representative from each group to present their exercise results. 14. The teacher provided independent work questions to students.	21. Group formation goals (Dahar, 2011) 22. Social interaction (Ramful, et al, 2015) 23. Spatial ability domains (Newcombe & Shipley, 2015; Minna, 2001) 24. Imagistic approach (Cohen & Hegarty, 2014) 25. Use of media (Rauh, et al, 2005; Newcombe, 2010) 26. Assignment of independent work (Sternberg, 2009; Schunk, 2012) 27. Storage of information in long-term memory (Taber, 2011; Rafi, 2008) 28. Role of presentation activities (Kukul &

Phase/ Stage	Syntax		Theoretical underpinnings
	Activity	Classroom Implementation	
			Karataş, 2019); Kamal, et al, 2020) 29. Communication skills (Iksan, 2012) 30. Spatial training (Uttal, et al, 2013)
Closing	13. Taking notes of important points during the learning process 14. Delivering independent task assignments to students in the form of questions 15. Conveying the plan for the next activity	15. The teacher asked students to take note of important points during the lesson. 16. The teacher delivered information that was not captured in students' notes. 17. The teacher assigned tasks. 18. The teacher communicated the plan for the next activity	31. Reflection (Taber, 2011; Dahar, 2011) 32. The role of evaluation (McFarland, et al, 2009)

Social System

Social system is presented in Table 2.

TABLE 2
SOCIAL SYSTEM

Phase	Teacher's Role	Student's Role
Introduction	1. Preparing classroom management, including forming groups and distributing learning materials. 2. Presenting objectives, indicators, learning motivation, apperception, and review.	1. Preparing oneself psychologically and cognitively for the learning process.
Spatial Information Presentation	3. Presenting learning information. 4. Presenting terms or attributes of geometric objects. 5. Providing services as an informant, facilitator, and mediator.	2. Processing the information received.
Group Discussion	2. Managing the course of the discussion. 3. Monitoring and ensuring the success of every spatial activity. 4. Monitoring group learning activities. 5. Emphasizing the results of the presentation.	3. Carrying out spatial activities to develop spatial skills. 4. Sharing understanding with group members. 5. Seeking input from the teacher if there are any group obstacles. 6. Formulating group discussion results. Sharing the results of the discussion with the class and draw conclusions.

Phase	Teacher's Role	Student's Role
		7. Sharing the results of the discussion with the class and draw conclusions.
Consolidation	6. Encouraging students to internalize knowledge through consolidation exercises.	8. Working on reinforcement exercises as a group. 9. Present the results on the classroom whiteboard.
Closing	7. Emphasizing essential aspects. 8. Asking students to reflect on their learning.	10. Creating a summary and take note of important information. 11. Recording plans for the next meeting

Reaction Principle

Reaction principle is presented in Table 3.

TABLE 3
REACTION PRINCIPLE

Phase	Teacher's response	Student's response
Introduction	1. Providing a review of related information or concept 2. Checking on previously assigned tasks 3. Offering assistance with task completion	1. Confirming learning objectives and indicators 2. Inquiring about the material or tasks to be completed
Spatial Information Presentation	4. Checking on the progress of each group 5. Checking on the contribution of each individual 6. Confirming students' questions	3. Observing information in the learning materials 4. Extracting important aspects from the provided information
Group Discussion	7. Observing the flow of discussion, including interaction patterns 8. Recording important points from group work sharing 9. Emphasizing important principles	5. Developing spatial skills through spatial activities 6. Reflecting on prior understanding or knowledge 7. Formulating answers based on discussion results 8. Sharing those answers with the class 9. Finalizing the answers
Consolidation	10. Providing appreciation for student efforts	10. Internalizing knowledge through practice questions or exercises
Closing	11. Designing follow-up activities	11. Assigning developmental tasks

Supports of System

The supporting system included lesson plans, teacher's guidebooks, student workbooks, learning media, and spatial skills tests.

Instructional Impact

The instructional impact was measured by: 1) mastery of the subject matter, 2) students' spatial skills, 3) students' activity in learning, and 4) students' response to spatial skill-based learning models. The accompanying impact included: 1) spatial thinking accuracy, 2) ability to solve spatial problems independently, 3) ability to argue, 4) ability to transfer learning, and 5) interest in viewing objects from a certain perspective.

DISCUSSION AND CONCLUSIONS

Students at primary school level are a group of children aged 7-12 years old. Children learn differently from adults (Squires, 1993). These differences are described by Knowles (Merriam, 2001) as follows: 1) children are recipients of education prepared by the system and their teachers, so what is learned, the materials, methods, and others, all depend on the teachers and the system in place; 2) experiences are shaped by teachers and children rely heavily on them; 3) students are grouped according to class, where teachers prepare the curriculum needs for each class; 4) one-way communication occurs between teachers and students, where the teacher's experience is the main source of learning; and 5) the orientation of learning is on the subject matter that is currently being studied by students as preparation for their future lives.

The geometry learning model, distinguished by its emphasis on spatial skills, had been designed to function within a group dynamic. Its objectives were multifold: firstly, to facilitate interactive engagement, discourse, communication, and the sharing of information among students (Sutawidjaja & Afgani, 2015); secondly, to encourage the exchange and development of ideas between students (Ramful, et al, 2015); thirdly, to enable students to engage in discussions with their peers when confronted with challenging concepts or problems (Dahar, 2011); fourthly, to foster intra- and interpersonal learning processes wherein individuals construct their understanding based on the tasks presented, and express their initial understanding within a group setting to receive constructive feedback (Parta, 2017). To assemble the groups, teachers were required to intervene in the selection process. This was necessary because not all individuals possess the same level of spatial abilities (Hegarty & Waller, 2004); Cohen & Hegarty 2014); male students tend to exhibit more dominance in this domain (Battista et al., 2020). To ensure that students' spatial skills were honed to their full potential, it was crucial to establish academic and gender parity. Consequently, groups of four or five students were formed following this. According to Slavin (2018), who stipulates that the ideal number of members in cooperative learning groups should be four or five.

The geometry learning model, which prioritizes spatial skills, had been carefully crafted to align with students' developmental stages, thereby optimizing their potential for spatial skill development. The selected framework for this model was the Van Hiele levels of geometric thinking. This choice was based on two principal factors: firstly, the close relationship between geometry and spatial activity, coupled with geometric reasoning (Howse & Howse, 2015), which the Van Hiele levels of geometric thinking provide a robust framework for students to comprehend; secondly, the Van Hiele framework was an effective tool for enhancing geometric abilities (Abdussakir, 2009). The Van Hiele levels of geometric thinking comprised five levels, namely, visualization, analysis, informal deduction, deduction, and rigor (Clements & Battista, 1992; Walle (2004). However, (Škrbec & Tatjana, (2015) had further refined this framework by adding levels 0.5 and 1.5 to capture the nuanced progression of geometric thinking. However, the levels of geometric thinking for elementary school students were still at the levels of 0, 0.5, 1, and 1.5, as described in Table 4.

TABLE 4
LEVELS OF GEOMETRIC THINKING

Degree	Level	Description
0	Visualization	Providing a description of a geometric form based on its visual attributes.
0,5		Identifying a shape through visual observation and analyzing its properties despite any errors.
1	Analysis	Describing a geometric shape based on its inherent properties.
1.5		Analyzing the properties of geometric shapes, comprehending their interrelationships, and accounting for errors.

Degree	Level	Description
2	Informal Deduction	Recognizing the significance of the characteristics and relationships between the characteristics of geometric objects to logically formulate the characteristics of a shape.
3	Deduction	Developing logical reasoning skills and proving theorems through deductive reasoning.
4	Rigor	Constructing and analyzing theorems within various postulate systems.

The Van Hiele framework was characterized by a sequential nature that depended on previous learning (Howse & Howse, 2015). The teacher played a crucial role in providing guidance, motivation, or recognition to ensure that geometric thinking was developed. Therefore, in teaching geometry, teachers needed to: 1) communicate learning objectives, as according to Daryanto (2005), communicating learning objectives would depict the knowledge, skills, and attitudes that students should possess as a result of the learning outcomes; 2) provide an apperception and review, as according to Puteri (2018) an apperception could ensure the readiness and focus of learning, curiosity, investigation of prior knowledge, and abilities of each student; 3) maximize their role in the learning process to ensure the desired outcomes were achieved, as stated by Gagne (1985), providing motivation could cultivate interest and a desire to engage in learning and problem-solving.

The spatial skills-based geometry learning model was designed to suit each student's level of spatial skills. The selected framework was the imagistic stages (Cohen & Hegarty, 2014), as it was believed to improve students' spatial skills. A description of the imagistic stages is provided in Table 5. The imagistic stages framework was sequential, where each level builds upon the previous skills. Thus, the teacher played a vital role in providing guidance, motivation, and recognition to ensure the achievement of spatial skills development.

TABLE 5
IMAGISTIC STAGES

Stages	Description of Imagistic Stages according to (Cohen & Hegarty, 2014)
1	Encoding spatial object characteristics
2	Imagining or manipulating the entire or part of a spatial object
3	Creating appropriate representations

The spatial skills-based geometry learning model utilized student workbooks and pre-distributed media as learning aids. These learning materials were essential in facilitating the implementation of the learning model. The use of media was crucial because it: 1) could enhance spatial skills (Rauh, et al., 2005; Newcombe, 2010) and 2) tangible objects or modeled objects played a significant role in geometry learning (NCTM, 2000; Fischer, 2014). However, the selection of media should considered the characteristics of the target model. At the elementary school, geometry learning was better facilitated with concrete models, pictures, and appropriate software (Howse & Howse, 2015).

According to Budiarto & Artiono (2019), teaching geometry could be done by: 1) introducing real-world models and then translating them into geometric models; 2) starting with the basics, such as points, lines, flat shapes, and three-dimensional figures. In the first approach, students are presented with a whole model and then introduced to its constituent parts, such as a cube, its faces, edges, and vertices. This approach provided opportunities for students to explore, perceive, and observe shapes in their environment or in the world they create through pictures, models, and computers. The activities involved in this approach require visualization, construction, comparison, transformation, and classification of shapes in geometry. According to Lajoie (2018) and Cohen & Hegarty (2014), such activities could be effectively honed if spatial skills were involved. These activities aligned with the constructivist model.

Constructivism was a learning theory that posits learning is a process of independently organizing cognitive structures and building conceptual structures through reflection and abstraction. According to Glasersfeld (1987) constructivism can be viewed from two perspectives: 1) learning as a process of constructing knowledge based on perceptions and conceptions of the world, resulting in different constructions for different learners, and 2) knowledge is related to the environment where children experience or construct it.

The constructivist model includes Beyond the Informational Given (BIG) and Without Informational Given (WIG) (Anderson, 2001). In the BIG model, the teacher provided complete information and examples and engaged students in activities that required them to apply and refine their initial understanding through varied applications or examples (Parta, 2017). The core of this approach was that 1) the teacher provided complete information, 2) students were required to construct understanding, and 3) refine understanding. This meant that students were not limited to written information, but through mental activities, they could understand, build perceptions, investigate possible interrelationships, and so on. In short, students were required to investigate spatial information in-depth.

Conversely, in WIG, the teacher did not provide concepts, so students must find them independently. Students were confronted with phenomena and asked to explain them based on their acquired perceptions. Therefore, students must search and discover independently, while the teacher provides scaffolding in the process. According to Anderson (2001), the pure WIG model was ineffective, especially if the material taught was new and required complex initial abilities and knowledge.

A geometry learning model characterized by spatial skills was designed using the BIG constructivism model, as it aimed to convey geometry material through the presentation of concepts, varied examples, and student engagement in activities that required them to apply their understanding through the application of varied examples. One of the supporting stages was the presentation of spatial information. This model would present spatial information through concepts, and varied examples from real-world models and brought into the geometry model. According to Howse & Howse (2015), presenting spatial information would acquire student reasoning with the physical nature and characteristics of the objects being studied. Thus, students would acquire terms and attributes of the objects being studied. This goal was in line with the opinions of (Sternberg, 2009; Schunk, 2012), that information mining will get terms or attributes according to the object being studied. Furthermore, the teacher and group members associated the acquired terms or attributes to build new knowledge. As Skemp (1986), suggests, mastering terms and information mining is essential in building knowledge.

The teacher's role in obtaining terms and attributes of the learned object was as an informant and facilitator. This role was crucial because the learning material was new to the students. The teacher also facilitated question and answer (dialogue) with the students. The teacher provided assistance and guidance in exploring information about the object through interaction with the students. The goal of the question and the answer was to help students recognize and develop vocabulary (terms or attributes) related to the object being learned. This activity was in line with the opinion of Howse & Howse (2015) that dialogue could lay the foundation for further learning activities.

Students would inevitably encounter a plethora of stimuli, which their sensory registers would select. The selected stimuli would then be forwarded to their short-term memory, with some being immediately responded to. Within short-term memory, the stimuli would be scrutinized once again to determine whether they were important, attention-grabbing, useful, or necessary. Those stimuli that pass the test would then be stored in long-term memory, with some immediately responding to. Conversely, stimuli that fail to meet the criteria would be quickly forgotten. In long-term memory, information is stored permanently and can be accessed anytime. In learning, it was crucial to manage attention-grabbing stimuli so that they were stored in long-term memory (Subanji, 2013).

The model of geometry learning that emphasizes spatial skills was designed to optimize students' cognitive development. According to (Subanji, 2013), an individual's cognitive development was influenced by their environment and social transmission. The effectiveness of an individual's relationship with their environment and social transmission affected their cognitive development stage. According to Piaget, an adaptation process occurs when a person interacts with their environment. This means that when the senses interact with the environment (capturing spatial information from the environment), an individual undergoes an adaptation process. When an individual adapts, they experience the processes of assimilation and accommodation.

Assimilation is integrating new stimuli into an already-formed schema (Subanji, 2013). In the process of assimilation, the structure of the problem was already in line with the individual's thought structure (schema). Therefore, the stimulus could be directly interpreted by the individual, and this meant that there was an integration of the stimulus into the existing schema. When the structure of the problem did not match the schema held by the individual, a process of modifying the old schema or forming a new schema occurred so that the structure of the problem could be integrated into the schema.

Accommodation is integrating new stimuli by forming new schemas to adapt to the received stimuli. In problem-solving, there was a cognitive process related to the imbalance between assimilation and accommodation called disequilibrium. The thinking process in problem-solving would continue until a balance is achieved, called equilibrium. In the problem-solving process, assimilation and accommodation could occur simultaneously.

Cognitive development refers to moving from an existing balance to a new one that has been acquired (Howse & Howse, 2015). One of the factors that influenced an individual's cognitive development was their environment and social transmission. The effectiveness of an individual's relationship with their environment and social life affects their cognitive development stage. Teachers play a crucial role in ensuring a child's cognitive development progressed to its maximum potential. The teacher's role included providing educational experiences for students. To ensure that an individual's cognitive development proceeded maximally, their educational experiences must be enriched (Howse & Howse, 2015).

Integrating new stimuli through assimilation or accommodation into cognitive development involves educational experiences. According to Howse & Howse (2015), cognitive development must be enriched with experiences. This was consistent with the view of Uttal (2013) that spatial skills could develop well from experience. Therefore, providing educational experiences can maximize cognitive development.

A geometry learning model characterized by spatial skills was designed by involving students in activities that imagine and manipulate spatial objects through educational experiences in group discussions. Students and their group members solved spatial activities with the help of media according to the instructions in the workbook. The spatial activities involved visualization and spatial orientation activities according to the needs of elementary school levels. The learning process occurred at intra and interpersonal levels. In intra-personal, individuals built personal understanding through (1) personal comprehension, (2) tacit pre-understanding, and (3) personal belief. The interpersonal stage was the implementation or testing of knowledge in the environment. Individuals build understanding based on the task at hand. Then, through group interaction, individuals expressed, narrated, or presented their group work to the class to receive an assessment.

It was essential for students to be trained in presenting the results of their group discussions in front of the class. The teacher might ask one representative from each group to deliver their findings, as this activity helped to hone the students' communication skills. Effective communication skills were crucial during presentations, as Iksan (2012) stated. The Q&A session during the presentation allowed students to share new ideas and solutions, communicate their discussion outcomes, and construct new understandings by addressing previously overlooked questions, alternative assumptions, new problem-solving strategies, and conclusions. This claim was supported by Kukul & Karataş (2019) and Kamal, et al, (2020), which suggested that presentations were crucial for boosting students' confidence in their performance. The teacher's role was facilitating dialogues that enabled students to explain their understandings using appropriate language.

The coverage of geometry material in elementary schools required intrinsic dynamic and extrinsic static tasks. Task classification selection was in line with Newcombe & Shipley (2015), that giving tasks to optimize spatial skills involves intrinsic and extrinsic domains that contributed to the success of STEM, and Minna (2001) that static and dynamic domains positively impacted mathematics. Intrinsic dynamic tasks included object processing tasks through physical or mental transformations, while extrinsic static tasks involved processing the relationships between objects without physical or mental transformations (Uttal, 2013; Buckley, et al., 2018; Xie, et al., 2020).

A geometry learning model characterized by spatial skills was designed to promote the development of students' spatial skills through group and independent problem-solving. Providing problems was crucial, as it reinforced memory (Schunk, 2012). Through memory reinforcement, information can be stored in long-term memory (Taber, 2011; Rafi, 2008). The problem-solving exercises were completed in groups. In the group setting, when students encounter difficulties, they can discuss them with their peers (Dahar, 2011). The teacher observed the students' work in the group and asked questions to check the students' mastery of the given tasks. Next, a group representative presents their opinions in front of the class.

The independent practice items were provided after the completion of the reinforcement exercise presentation. The purpose was to provide repeated practice. According to Uttal (2013), repeated practice yielded better results. However, classroom activities were limited by time. Therefore, this activity was completed at home with an ideal timeframe of one week.

The final component of the spatial skills-focused geometry learning model encouraged students to take note of and communicate important insights gained during discussion and presentation activities. The teacher played a crucial role in conveying any missed or overlooked points to ensure students had acquired new knowledge and ideas and corrected any misconceptions relevant to problem-solving. The teacher also evaluated the learning outcomes to ensure that the knowledge and spatial skills had been internalized, and the learning situation had become sustainable. This aligned with the view that evaluation stimulated students to reflect on their work more accurately and improve their learning (McFarland, 2009).

The results of this research design are presented in a book outlining the geometry learning model characterized by spatial skills. Subsequently, the book model needs to be validated to obtain an assessment from experts. To support the validation of the book model, it is necessary to develop an assessment instrument in the form of a validation sheet for the book model.

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