

# **Divergent Thinking Process of Prospective Mathematics Teachers: A Case Study of an Open-Ended Problem**

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*Divergent thinking is important for fostering creativity. This study aims to analyze the process of divergent thinking in solving open-ended problems and its cognitive characteristics. The method applied is qualitative. Researchers obtained data with test and interview techniques. Data analysis consists of reduction, display, and conclusion. Based on the level of creativity, three students were taken as informants by purposive sampling. Researchers determine the divergent thinking process from identification, exploration, construction, and evaluation. Students in the very creative, creative, and quite creative categories can write down initial information and the core of the problem. The exploration phase provides information to obtain an initial solution. Students are quite creative not to write down relevant information due to a lack of initial knowledge. Students in the very creative and creative category can create different and new ideas at the construction stage. The main focus at the evaluation stage is examining the completion process and the relevance of the answers to the problem.*

*Keywords: creativity, divergent thinking, open-ended problem, creativity level*

## **INTRODUCTION**

Creativity consists of four aspects: creative individuals, creative people, creative environment, and creative thinking (Batey, 2012; MacKinnon, 2017; Runco, 2004; Wadaani, 2015). The creative thinking process explains an individual's cognitive process in finding new ideas. Wallas' theory describes four creative thinking processes, including preparation, incubation, illumination, and verification (Maharani et al., 2017). Each process has specific characteristics. Creative thinking consists of a set of divergent thinking procedures (An et al., 2016; King, 1982). In this case, divergent thinking supports individuals' creative thinking and creativity.

Divergent thinking refers to a cognitive process of finding new-different ideas to solve various problems with various solutions (Lewis & Lovatt, 2013; Colzato et al., 2013; Wang et al., 2017). Runco and Acar (2012) and Acar and Runco (2015) explain the measures used for divergent thinking, namely fluency, flexibility, and originality. Fluency refers to the capability to make various and correct answers. In this case, the key aspect is the answer. Flexibility also refers to the capability to formulate various solutions and algorithms correctly. The definition of originality is - the capability to create unique and uncommon ideas. In the creativity review, originality refers to the primary aspect of creativity (Acar et al., 2017). Originality refers to an individual capable of creating new, uncommon, and unique ideas. Originality does not refer to novel ideas but new ideas for the students (Briggs & Davis, 2008; Sitorus et al., 2019). In solving mathematics problems, originality includes uncommon answers and solutions. Uncommon answers must be different from other individuals' answers but remain acceptable for the learning process. Cotter et al. (2020) and Silvia et al. (2013) analyzed the positive correlation between frequency and originality. The researchers provided two study cases of low and high fluency characteristics and originality.

Although the indicators are similar to creativity, divergent thinking is not the same as creativity (Runco, 2008). The divergent thinking skill test is highly likely to determine an individual's creativity (Acar & Runco, 2015; Hu & Adey, 2002; Runco et al., 2016). Everyone has different divergent thinking skills due to the given time allotment to think. In this case, the time allotment goes linearly with divergent thinking. Longer provided time allotment led to better divergent thinking (Paek et al., 2021).

Divergent thinking is important for creativity because divergent thinking potentially produces genuine ideas (Runco & Acar, 2012). In mathematics, divergent thinking becomes the potential measurement of creativity (Leikin, 2009; Mann, 2009; Plucker et al., 2011). King (1982) defines the creative thinking process as the cycles of divergent thinking, preparation, and incubation, and the concurrent thinking process, illumination, and verification. Divergent thinking is the initial entry in creating creative ideas. The process of collecting the data and information refers to a preparation stage. The incubation stage is a situation in which students encounter difficulties while trying to solve problems with the maximum capability of the brain (Sitorus & Masrayati, 2016). Huang et al. (2017) explain that divergent thinking skill influences scientific creativity domination.

In the study of creativity, intelligence, and divergent thinking have an important role in creative thinking ((Lee & Therriault, 2013). The level of intelligence can provide data about a person's creativity, including in the process of divergent thinking ((Beaty & Silvia, 2012; Gilhooly et al., 2007). The research by Nuha et al. (2018) and Siswono (2010) define five levels of creativity, namely very creative, creative, quite creative, less creative, and not creative. The characteristics of each level of creativity are identified based on indicators of fluency, flexibility, and originality (Silver, 1997). The third indicator is addressed in a problem called open-ended. One approach that can be used to develop divergent thinking is to be open.

An open-ended problem is defined as a problem that has many different solutions (Becker & Shimada, 1997; Kholil, 2020). There are three types of open-ended problems, namely 1) Problems that have more than one way of different answers. The term used is fluency, 2) Problems with many ways of solving or other algorithms. The term used is flexibility, 3) New problems are obtained from developing previous problems. In this study, the types of problems used are fluency and flexibility. A study by Fatah et al. (2016) explained that the open-ended problem approach could increase student creativity in solving problems. This is in line with the research of Kim et al. (2010), who argued that the open-ended approach could develop creative thinking, explore the knowledge acquired, and find relationships between mathematical concepts.

In this article, the researchers found related literature on divergent thinking. Kwon et al. (2006) provide their findings that open problems have the potential to bring up a variety of relevant solutions and develop divergent thinking skills. This research describes students' answers in solving open-ended questions with the fluency type but does not explain the stages of divergent thinking. Gilhooly et al. (2007) categorized the divergent thinking process into memory strategy, property, strategy utilization, and debunking. Each aspect has a specific definition related to the cognitive process of divergent thinking. Bai et al. (2021) explain the divergent thinking process requires two thinking processes: associative and executive thinking processes. The associative thinking process is the bottom-up subconscious thinking process spontaneously, accurately, and effortlessly (Sowden et al., 2015). On the other hand, the executive thinking process refers

to an up-to-bottom thinking process within the framework of the cognitive process, for example, encountering challenges, recalling the memory inside the brain, and applying some specific strategies (Beaty et al., 2014). Researchers use the term sensitivity as a link between recalling knowledge and spontaneous processes. Sun et al. (2019) explain that the divergent thinking process has some underlying cognitive process principles, such as association, decomposition, and combination.

This study's stages of the divergent thinking process consist of identification, exploration, construction, and evaluation. This stage is what distinguishes it from previous research. This research analyzes students' divergent thinking processes in solving open-ended problems. Follow-up results of this study can be used to develop innovative learning plans based on the characteristics of students' levels of creativity and improve students' different thinking abilities in creative context spaces.

## RESEARCH METHOD

### Research Design

This qualitative research explains in detail the analysis of the stages of students' divergent thinking processes in solving open-ended problems. The characteristics of qualitative research involve in-depth analysis of textual data from interviews, tests, and observations (Smith & Smith, 2018).

### Participants and Data Collection

The researchers took the data from sixth-semester mathematics students. The subjects were 35 individuals aged between 18 and 20 years old. The researchers promoted the research during the complex function course with the given topic of limit function. The researchers took three informants with purposive sampling based on certain criteria (Sukestiyarno, 2020). Three informants were taken from the creative, creative, and quite creative categories. In addition, researchers chose informants with excellent communication to facilitate the process of finding the facts comprehensively and writing the answers carefully. The researchers labeled every informant for further data analysis and tabulation. The applied labels were S1, S2, and S3. The applied instruments were open-ended tests and interview guidelines. The researchers designed the test based on the indicators of flexibility and fluency, consisting of two question items. The researchers interviewed the informant to determine the divergent thinking process stage in solving open-ended problems. In the interview process, the researchers used a sound recorder. In this research, all instruments were validated by the experts. The experts screened the published articles in international journals, focused on creativity, and held various scientific seminars at the international level. The obtained results were the validated test and interview guidelines. The instruments were also reliable to use. Figure 1 below presents the open-ended questions used in this study.

**FIGURE 1**  
**OPEN-ENDED PROBLEM QUESTION ITEMS**

Define a function on set  $D$  that has real value. Pay attention to the following questions. Based on the function definition above,

1. Make more than one different function so that each has a limit value.
2. Use more than one different solution to prove that the limit value exists. Can use new functions or functions created in section (1).

Write down your answer in detail.

### Analyzing of Data

Researchers analyzed the data interactively, consisting of three stages: data reduction, data presentation, and conclusion (Glaser et al., 2010). Data reduction includes summarizing the data obtained, selecting the important data, and organizing data into sub-categories. The researchers reduced the data based on the research topic, the divergent thinking process. Then, the researchers presented the data about the divergent

thinking process with tables and figure. After that, the researcher connected the data and explained the findings obtained from students' answers and interviews regarding divergent thinking processes. The divergent thinking process used in this study consists of identification, exploration, construction, and evaluation stages. Table 1 details the cognitive characteristics for each divergent thinking stage.

**TABLE 1**  
**THE CHARACTERISTICS OF THE DIVERGENT THINKING PROCESS**

Stage	Characteristics
Identification	Writing information and the core of the problems
Exploration	Being sensitive to obtaining the initial idea Exploring information and initial idea comprehensively
Construction	Creating solutions or designs conceptually
Evaluation	Rechecking the process and the final results carefully Drawing conclusion

## RESULT

The participants worked on the test for 60 minutes in an online manner. Table 2 shows every informant profile, starting from the academic and communication skills, and student work results in solving open-ended problems. The three informants had average academic skills and excellent communication skills. The researchers took the informants based on the inclusion criteria of this research. Then, the researchers described the divergent thinking process of the students in solving open-ended problems. Here are the profiles of the informants.

**TABLE 2**  
**THE INFORMANT PROFILE'S**

The Informant	Academic Performance	Communication Skills	Test Results			Categories
			Fluency	Flexibility	Originality	
S1	Good	Good	√	√	√	Very Creative
S2	Good	Good	-	√	√	Creative
S3	Good	Good	-	√	-	Quite Creative

### Very Creative Student Category

Subjects could explain the given and the questioned items. The students could write the given information. Figure 2 shows the activity, writing the given information.

**FIGURE 2**  
**IDENTIFICATION STAGE OF S1**

Diberikan fungsi  $f: D \subseteq \mathbb{R} \rightarrow \mathbb{R}$  dan  $c \in D$ . Bilangan real  $L$  adalah limit fungsi  $f$  di titik  $c$  dinotasikan  $\lim_{x \rightarrow c} f(x) = L, \forall x \in D$

**Translation:**  
Given the function  $f: D \subseteq \mathbb{R} \rightarrow \mathbb{R}$  and  $c \in D$ . The real number  $L$  is the limit of the function  $f$  at point  $c$  denoted  $\lim_{x \rightarrow c} f(x) = L, \forall x \in D$ .

The interview also explained the main questions and other information about the problem. The following are the results of the interview.

- R: What is asked in the problem?*  
*S1: There are two things to ask. First, create multiple different functions so that the limit value exists. Second, look for more than one way of solving to show the value limit.*  
*R: What mathematical concepts are related to the main problem?*  
*S1: There are concepts of functions, limits, and algebraic operations.*

In the information exploration stage, the subject digs deeper into the information obtained in the previous stage. The subject successfully collected several mathematical concepts, including types of functions, limit existence theorems, and methods for solving limit functions. This information was obtained from the results of interviews with the subject.

- R: What is the main keyword for the problem above?*  
*S1: There are two main keywords, namely function, and limit.*  
*R: Next, what did you do?*  
*S1: I dug up other related information from the keyword.*  
*R: What did you get?*  
*S1: For the function concept, I found out about the function definition and its types. This corresponds to the main question. I found the theory of the existence of limit values and the solution method for the limit concept.*

In addition to the interview results above, the subject also described the information that could be used to solve the problem. Figure 3 shows the subject rewriting the definition of limit existence through the concept of left and right limits.

**FIGURE 3**  
**EXPLORATION STAGE OF S1**

■ Lim  $f(x)$  ada jika  $\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^+} f(x)$

**Translation:**  
 $\lim_{x \rightarrow 4} f(x)$  exists if  $\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^+} f(x)$ .

Construction stage describes the process of thinking about other ideas in solving open-ended problems. S1 could produce other unusual ideas.

**FIGURE 4**  
**CONSTRUCTION STAGE OF S1**

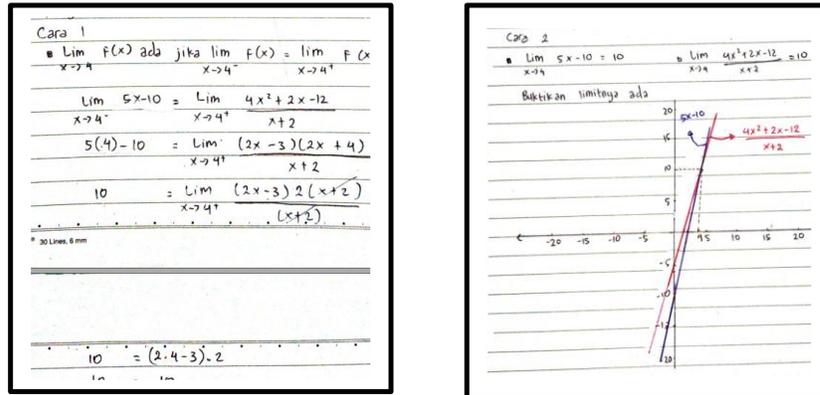


Figure 4 explains the answers to subject 1. It is clear that the subject performs two different functions, namely  $f(x) = 5x - 10$  (linear) and  $f(x) = \frac{4x^2 + 2x - 12}{x + 2}$  (a combination of linear, quadratic, and rational functions). Sections (a) and (b) describe two different settlement methods for determining the limit value. Part (a) the informant applies the left and right limit method. This is in line with the information described in the previous stage. Part (b) shows the informant applying another unusual solution, namely the graphical method. This method is rarely used during classroom learning. Students do not usually do this thinking. The term that best describes this situation is originality thinking. The answers above clearly explain divergent thinking that is unusual or unique. Below are the results of interviews with S1 subjects exploring the construction stage of the completion idea.

*R: What ideas did you use to solve the problem?*

*S1: Left and right limits and graphic method.*

*R: Is this idea new to you?*

*S1: Yes. The second solution idea uses the graphical method. This method is challenging because you have to draw the graph clearly and in detail. This idea arose based on experience when taking previous education. I am interested in developing and applying it in more complex functions*

The last applied stage was - concluding and checking the answers. The subject met the requirements based on the answer sheets and the interview results. Here is the evidence of S1 concluding two different problems.

**FIGURE 5**  
**EVALUATION STAGE OF S1**

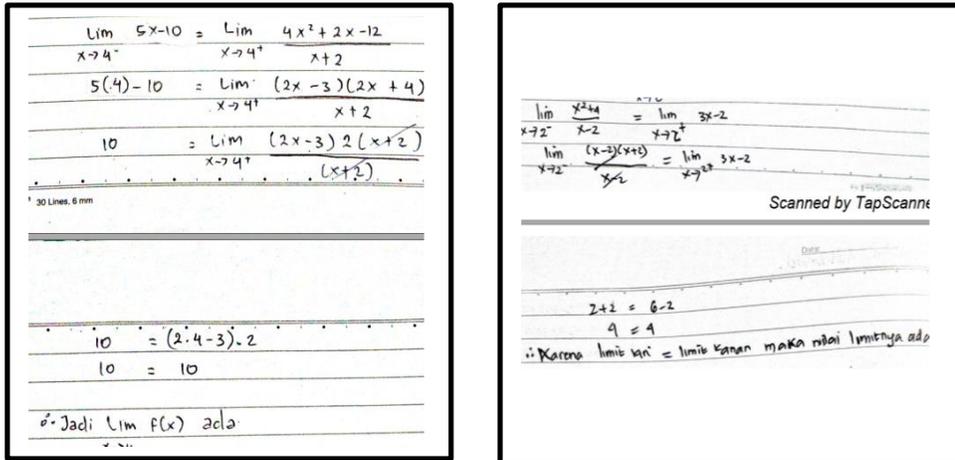


Figure 5 explains that S1 concludes the limit value. At the end of the answer, the subject confirmed that the responses obtained were by the questions. In addition, the subjects also checked the results of the work before it was collected. This information was also explored in the following interviews.

*R: Are you used to giving conclusions?*

*S1: Yes, sir. This confirms that the answer has been completed and is sure it fits the question.*

*R: What do you do next?*

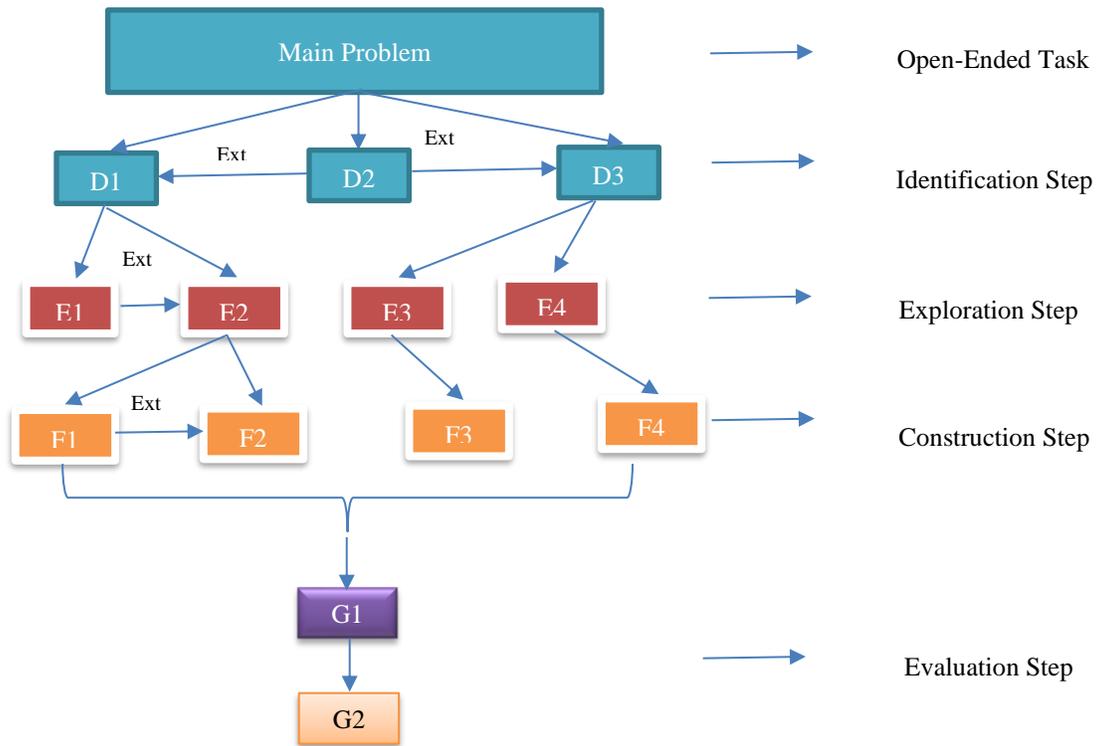
*S1: After working on the questions, I rechecked the answers. Here is the process of correcting the answers. For each test step, I checked the works before submitting the results.*

*R: Are you used to it like that?*

*S1: Yes, sir. Moreover, if there is still time, I will correct it.*

Based on the explanation of the stages of divergent thinking above, it can be concluded that the divergent thinking of S1 subjects in solving problems is shown in Figure 6 and Table 3 below.

**FIGURE 6  
DIVERGENT THINKING PROCESS OF S1**



**TABLE 3  
THE INFORMATIONS OF CODES**

Code	Information
D1	Function theory
D2	Algebra properties
D3	Limit function theory
E1	Limit definition
E2	Types of functions (linear and rational)
E3	The limit existence theorem
E4	Function limit-solving method
F1	Linear function
F2	Mixed functions (linear, quadratic, and rational)
F3	Left limit and right limit theorem
F4	Methods of substitution, factoring, graphs, left limit, and right limit
G1	Give a conclusion
G2	Re-check the results of the work
Ext	Extended

**Creative Student Category**

Subjects looked at the problems given and wrote down the initial information shown in Figure 7.

**FIGURE 7**  
**IDENTIFICATION STAGE OF S2**

Diberikan fungsi  $f: D \subseteq \mathbb{R} \rightarrow \mathbb{R}$  dan  $c \in D$ . Bilangan real  $L$  adalah limit fungsi  $f$  di titik  $c$  dinotasikan  $\lim_{x \rightarrow c} f(x) = L, \forall x \in D$ .

**Translation:**

Given the function  $f: D \subseteq \mathbb{R} \rightarrow \mathbb{R}$  and  $c \in D$ . The real number  $L$  is the limit of the function  $f$  at point  $c$  denoted  $\lim_{x \rightarrow c} f(x) = L, \forall x \in D$ .

The subject writes the complete limit definition along with the notation. In addition, the subject can also explain other information related to the core problem. The following are the results of interviews that describe the data in question.

*R: Can you explain what is being asked?*

*S2: There are two main questions. First, we are asked to create several different functions, each with a limit value. Second, prove the limit value using another method.*

*R: Mention the mathematical concepts associated with the problem.*

*S2: There are functions, limits, and algebraic properties.*

Based on the mathematical concepts obtained at the above stage, then dig in depth. The information exploration process was carried out through interviews with the following subjects.

*R: What did you get from the identification information above?*

*S2: For the function concept, I obtained the function definition and types of functions. I got the limit existence theorem and several solutions I have used for the limit concept.*

*R: Have you ever obtained this material before?*

*S2: Yes, sir. However, the left and right limits are only sometimes used.*

S2 write down the definitions of the left and right limits as the initial answers to the problems. Informants dig up information that appears at the beginning to obtain a relationship between available data and the essence of the problem. Below is evidence of the activity, revealing information on the topic. Figure 8 explains this.

**FIGURE 8**  
**EXPLORATION STAGE OF S2**

Jika  $\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x)$  maka  $\lim_{x \rightarrow c} f(x)$  ada

**Translation:**

If  $\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x)$  then  $\lim_{x \rightarrow c} f(x)$  exists.

The next stage is to arrange the function and use the solution method to determine the limit value. In the S2 subject's answer sheet, the functions created include linear functions and rational functions. This knowledge is obtained in the previous stage. Rational functions it is a combination of quadratic and linear functions. This explanation is evidenced by Figure 9 below.

**FIGURE 9**  
**CONSTRUCTION STAGE OF S2**

**Solution I**

Cara I.  $\rightarrow \lim_{x \rightarrow 2} \frac{x^2+4}{x-2} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{x-2}$

$= 2+2 = 4 \rightarrow$  Ada nilai limitnya

$\lim_{x \rightarrow 2} 3x-2 = 3(2)-2$

$= 4 \rightarrow$  Ada nilai limitnya

$\lim_{x \rightarrow 2^-} \frac{x^2+4}{x-2} = \lim_{x \rightarrow 2^+} 3x-2$

$\lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x \rightarrow 2^+} 3x-2$

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**Solution II**

$2+2 = 6-2$

$4 = 4$

S2 answered the problem with two different solving algorithms, as shown in Figure 10. The solution in part (c) uses a mixed method: factoring and substitution. Part (d) proves other ideas used by informants, namely the concept of left and right limits. Students rarely think of this method because they must first understand the concept. Divergent thinking fulfills the aspect of originality. Interviews with S2 subjects corroborated this explanation.

R: Describe the function you created to answer the first part of the problem.

S2: I composed two functions: linear and rational functions. Inside the rational function, there are quadratic and linear functions.

R: Explain the solution method used to answer the problem in part 2?

S2: I use the left and right limit rules. In addition, I use factoring and substitution methods.

R: Are there any new ideas for you?

S2: Yes, sir. The left and right limit methods are new to me. I have received the material but need help understanding it. Because of this, I tried to re-implement the problem.

After the subject solves the problem, the final step is to check the results of his work. The corrected part includes the final result according to the question or not and the steps for solving it. In this case, the subject does not write the conclusion like the S1 subject. In interviews with subject S2 explained that it does not always provide conclusions. Several times it needs to make conclusions. Here's an excerpt from the interview.

R: What did you do after you finished working on the questions?

S2: I re-corrected the final result and the steps.

R: Are you usually like that?

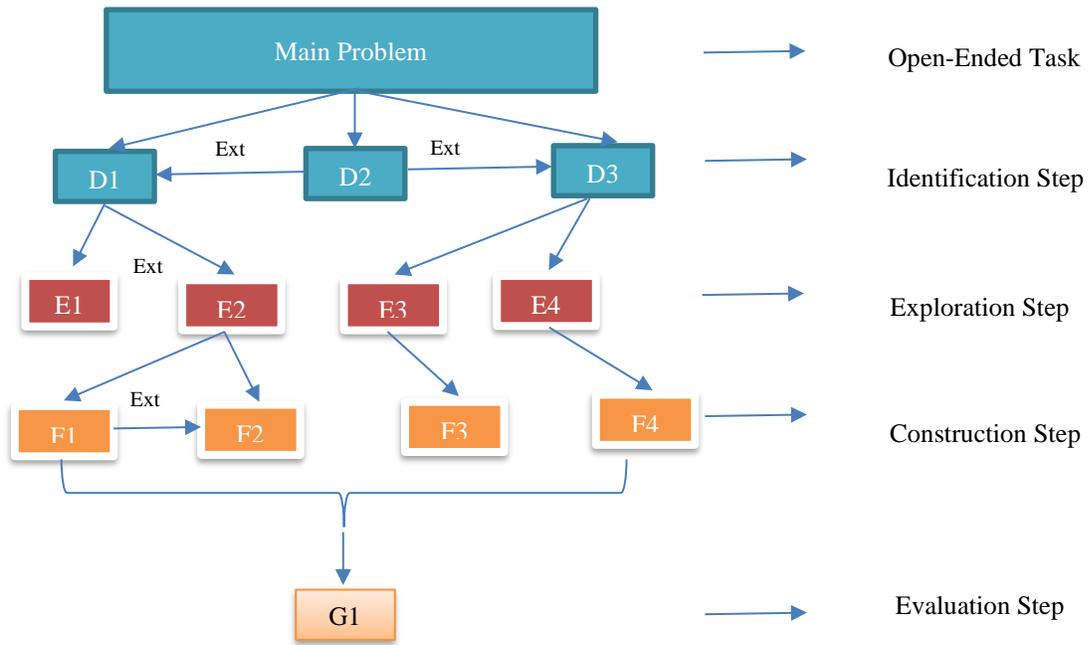
S2: Yes, I always make corrections.

R: Do you always give a conclusion to each answer?

S2: Sometimes, I conclude with my language.

Based on the description of the subject's S2 divergent thinking process, Figure 10 below shows a visualization of divergent thinking processes in solving open-ended problems. It is accompanied by an explanation of the code in Table 4.

**FIGURE 10  
DIVERGEN THINKING PROCESS OF S2**



**TABLE 4  
THE INFORMATIONS OF CODES**

Code	Information
D1	Function theory
D2	Algebra properties
D3	Function limit theory
E1	Limit definition
E2	Types of functions (linear and quadratic)
E3	The limit existence theorem
E4	Function limit-solving method
F1	Linear function
F2	Mixed functions (linear, quadratic, and rational)
F3	Left limit and right limit theorem
F4	Substitution method, factoring, left limit, and right limit
G1	Re-check the results of the work
Ext	Extended

**Quite Creative Student Category**

Subject S3 reads the problem carefully. This starts with identifying available information, looking for the main issue, and identifying other related information. The subject S3 needed to be corrected in writing

the limit function notation, as shown in the circle above. The correct spelling should be  $\lim_{x \rightarrow c} f(x) = L$ . Figure 11 below proves the information identification stage carried out by the subject.

**FIGURE 11**  
**IDENTIFICATION STAGE OF S3**

Diberikan fungsi  $f: D \subseteq \mathbb{R} \rightarrow \mathbb{R}$  dan  $c \in D$ . Bilangan real  $L$  adalah limit fungsi  $f$  di titik  $c$  dinotasikan  $\lim_{x \rightarrow c} f(x) = L, \forall x \in D$

**Translation:**

Given the function  $f: D \subseteq \mathbb{R} \rightarrow \mathbb{R}$  and  $c \in D$ . The real number  $L$  is the limit of the function  $f$  at point  $c$  denoted  $\lim_{x \rightarrow c} f(x) = L, \forall x \in D$ .

The results of the following interviews with the subject can explain other related information and the main problem.

*R: What is the main problem?*

*S3: There are two main questions, namely, creating different functions, each of which has a limit. Then prove it using other methods.*

*R: What information is related to the main problem?*

*S3: Functions, limits, and algebraic properties of both.*

Subjects dig up information based on keywords obtained before. Subjects explained the types of functions related to function material and methods of solving function limit values. Subject S3 could not explain the existence of limit values based on the concept of left and right limits; this was different from subjects S1 and S2. These results were obtained from interviews with S3 subjects at the exploration stage.

*R: What did you get from the main keywords obtained in the previous stage?*

*S3: I got the types of functions because one of the questions related to the kinds of functions and methods for determining limit values.*

*R: Have you thought about the left and right limits?*

*S3: No, sir, I don't understand either.*

In the problem-solving construction phase, subject S3 made only one function, namely the rational function with the denominator and quantifier of each quadratic function. In addition, the solution method is also divided by the variable with the highest rank and factoring. There is a part that is different from before. Namely, the point is not an actual number but an infinity ( $\infty$ ). More details will be shown in Figure 12 below.

**FIGURE 12**  
**CONSTRUCTION STAGE OF S3**

$$\rightarrow \text{cara 1 : } \lim_{u \rightarrow \infty} \frac{u^3 - 2u^2 - 3}{u^2 - 2u + 2}$$

$$= \frac{1 - \frac{2}{u} - \frac{3}{u^2}}{1 - \frac{2}{u} + \frac{2}{u^2}}$$

$$= \frac{1 - 0 - 0}{1 - 0 + 0}$$

$$= 1$$

$$\rightarrow \text{cara 2 : } \lim_{u \rightarrow \infty} \frac{(u+1)(u-3)}{(u+1)(u+2)}$$

$$= \lim_{u \rightarrow \infty} \frac{u-3}{u+2}$$

$$= \lim_{u \rightarrow \infty} \frac{\frac{u}{u} - \frac{3}{u}}{\frac{u}{u} + \frac{2}{u}}$$

$$= \frac{1 - \frac{3}{u}}{1 + \frac{2}{u}}$$

$$= 1$$

Solution I
Solution

The interview also explained the same thing as the results on the answer sheet for the S3 subject.

*R: Explain the ideas you used in answering the problem.*

*S3: First, I create a rational function that consists of a quadratic function in each part of the quantifier and denominator. Second, I once used the division by the highest rank method and factoring.*

The last stage is evaluation. At this stage, the S3 subject does not provide conclusions on each answer. Not visible on the subject's answer sheet. However, another cognitive activity carried out by the subject is to re-check the work results before submitting it. The corrected part consists of the final result and the completion steps. Based on the results of the interviews, the subject also explained that the answer was not always correct, meaning that if there were still time to do the work, it would be corrected again. The following is the conversation in the interview with subject S3.

*R: What did you do after you finished working on it?*

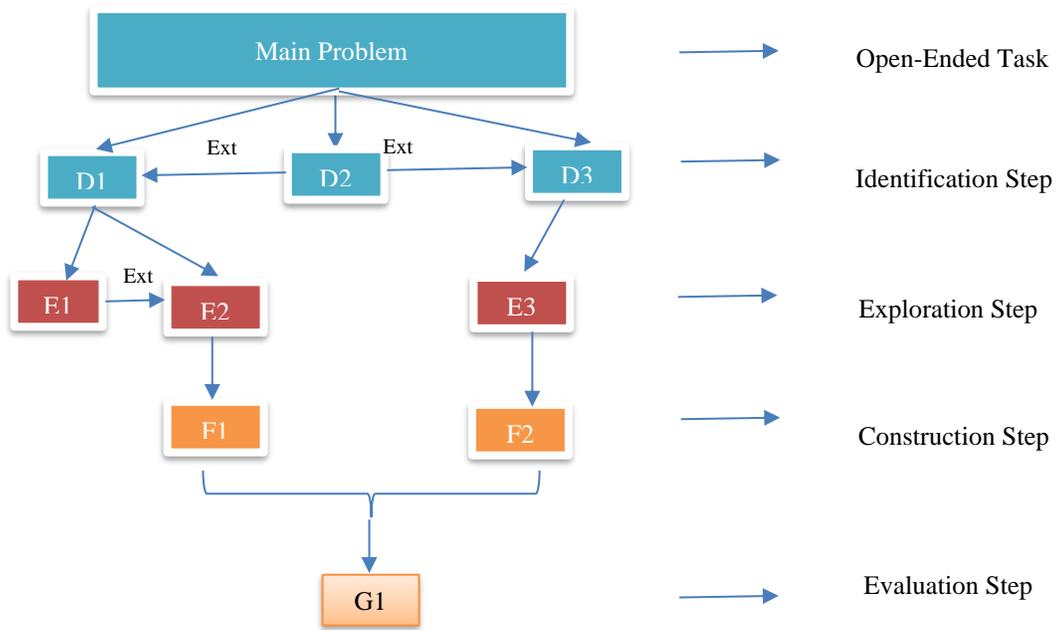
*S3: I corrected the final results and the steps.*

*R: Do you usually do it?*

*S3: No, sir, if there is still time, then I will check again.*

The above descriptions provide an overview of the divergent thinking processes of students who are quite creative in solving open-ended problems. Figure 13 and Table 5 below represent the explanation above.

**FIGURE 13**  
**DIVERGEN THINKING PROCESS OF S3**



**TABLE 5**  
**THE INFORMATIONS OF CODES**

Code	Information
D1	Function theory
D2	Algebra properties
D3	Function limit theory
E1	Function definition
E2	Types of functions (linear)
E3	Function limit solving method (Divide by the variable that has the highest rank)
F1	Mixed functions (quadratic and rational)
F2	The method of dividing the highest power and factoring
G1	Re-check the results of the work
Ext	Extended

**DISCUSSION**

A study by Kwon et al. (2006) explained that the open-ended problem approach could be used to detect divergent thinking to foster creativity. This is because open-ended problems have questions that have many answers or solutions, so they are by the characteristics of divergent thinking. According to Silver (1997), about indicators of fluency and flexibility, open-ended problems provide opportunities for students to create and interpret different solutions based on their experience and knowledge.

Students in the very creative category can write down the information they know, explain the main problems, and provide other conceptual knowledge related to the problem. This cognitive activity is referred to as the problem identification stage. The importance of the problem identification stage is described by Abdulla et al. (2020) that problem identification is an essential component in creative problem-solving. In

line with the research of Nuha et al. (2018) that students who have excellent creative thinking skills can explain the primary information clearly and write down other information related to the main problem. In the exploration stage, the subject rewrote the data obtained in the previous stage in detail. The information written is by the main problem. Sitorus and Masrayati (2016) define the exploratory stage as the preparation stage for an initial process to generate new ideas, including identifying the obtained information and connecting the data to get a possible problem-solving strategy. Based on their knowledge, creative people find it easier to find new ideas than those who are not creative. In other words, students who have excellent creativity will be able to find solutions quickly. This statement intersects with the study of Runco et al. (2006) that prior knowledge that a person has played an essential role in divergent thinking. Subjects are very creative and capable of generating unusual thoughts in solving problems. This thinking is obtained from connecting concepts to find other unique concepts. This result is strengthened by research by Gunawan et al. (2022), who explained that in finding new ideas, very creative students could link information correctly and look for main keywords. This makes it easier for subjects to find unusual or unique ideas. In the final stage, students are creative in providing conclusions and re-examining. An individual with excellent divergent thinking skills could also evaluate his ideas (Grohman et al., 2006).

Creative category, students start solving problems with identification. The identification stage is a process of writing the given information, the focused problem, and the core problem. As with the very creative student category, the creative group could also identify issues well. Identification is the first step in finding lots of ideas. Rubenstein et al. (2020) explain that problem identification is essential in realizing creative ideas. The strategy could support the divergent thinking generation. In the exploratory stage, students can re-explain information related to the main problem. In line with the research of Bridges and Schendan (2019) explain that someone who has high creativity can dig up detailed information and present other related information. The information obtained must be distinct from the experience they have. An et al. (2016) stated that the learning experience significantly affects a person's creative and divergent thinking. Creative people must have a lot of learning experiences so that the information obtained is also more and more. In constructing the idea of completion, the subject creatively uses new or unique ideas. In line with Siswono's research (2010), students with a creative level can find new or unusual ideas. The term used is originality. Students in the creative category also carry out cognitive activities, namely re-examining the completion steps and the final results. In line with Nuha et al. (2018), students in the creative category can reassess in detail, from each completion step to the final result.

Different from the case with students who are quite creative in finding solutions with no new or unique ideas. This is because the knowledge possessed by students needs to be improved. Siswono's research (2010) explained that students in the quite creative category could use several different solving methods but could not make varied answers. Mougnot et al. (2007) applied the term finding facts as the initial step of designing or preparing a creative strategy. The phase consisted of identifying and selecting the information obtained for further problem-based evaluation. It is also different from the very creative and creative category; in the exploration stage, the students in the quite creative category cannot write down various mathematical concepts related to the problem. Students' lack of initial knowledge causes this, so only a little information can be extracted. This is in line with Wijaya et al. (2022), which confirms that prior knowledge strongly influences finding solutions. The higher one's knowledge, the more choices of information that can be used in finding solutions. Less creative students need help constructing the whole idea at the construction stage well. This is caused by student knowledge that is not produced. Aminah et al. (2018) explain that the concept of a new solution results from the collaboration between previously owned knowledge. As with the other categories, students are creative enough to re-examine their solutions for the evaluation stage. Someone with creativity will re-examine the completion process, including in the quite creative category. Relevant to Baer (2013) provided information about evaluation and divergent thinking as the primary aspect of creativity. The evaluation activity refers to the students' sharing the feedback of their generated ideas based on divergent thinking, such as drawing a brief conclusion. Many developed ideas without evaluation led to poor content and meaning.

## CONCLUSION

Potentially open issues lead to different thoughts. Divergent thinking processes in solving open-ended problems include identification, exploration, construction, and evaluation. Students are very creative, creative, and quite creative in writing information and the essence of the problem. Students in the three categories of creativity levels have different backgrounds for the emergence of initial ideas. Knowledge and experience are the main underlying factors. In the constructive stage, students are very creative and creative, able to create different and unusual ideas. The evaluation stage is carried out by carefully re-examining the process and final results before concluding. This research contributed to developing divergent thinking skills as a potential aspect of realizing creativity. The results of divergent thinking apply to classroom learning to elicit students' creativity. The limitation of this research dealt with divergent thinking skills as a creative thinking process. The formulated theory explains that creative thinking skill consists of divergent and convergent thinking, an inseparable order. Convergent thinking refers to the cognitive analysis of information and finding the best ideas based on the problems. Zhu et al. (2019) explain that convergent thinking is directly correlated with the fluency and flexibility aspects of creativity. Further research should describe the convergent thinking process comprehensively as an important cognition as part of creative ideas.

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## REFERENCES

- Abdulla, A.M., Paek, S.H., Cramond, B., & Runco, M.A. (2020). Problem finding and creativity: A meta-analytic review. *Psychology of Aesthetics, Creativity, and the Arts, 14*(1), 3–14. <https://doi.org/10.1037/aca0000194>
- Acar, S., & Runco, M.A. (2015). Thinking in multiple directions: Hyperspace categories in divergent thinking. *Psychology of Aesthetics, Creativity, and the Arts, 9*(1), 41–53. <https://doi.org/10.1037/a0038501>
- Acar, S., Burnett, C., & Cabra, J.F. (2017). Ingredients of Creativity: Originality and More. *Creativity Research Journal, 29*(2), 133–144. <https://doi.org/10.1080/10400419.2017.1302776>
- Aminah, M., Kusumah, Y.S., Suryadi, D., & Sumarmo, U. (2018). The Effect of Metacognitive Teaching and Mathematical Prior Knowledge on Mathematical Logical Thinking Ability and Self-Regulated Learning. *International Journal of Instruction, 11*(3), 46–62. <https://doi.org/10.12973/iji.2018.1134a>
- An, D., Song, Y., & Carr, M. (2016). A comparison of two models of creativity: Divergent thinking and creative expert performance. *Personality and Individual Differences, 90*, 78–84. <https://doi.org/10.1016/j.paid.2015.10.040>
- Baer, J. (2013). Teaching for Creativity: Domains and Divergent Thinking, Intrinsic Motivation, and Evaluation. In *Teaching Creatively and Teaching Creativity* (pp. 175–181). Springer New York. [https://doi.org/10.1007/978-1-4614-5185-3\\_13](https://doi.org/10.1007/978-1-4614-5185-3_13)
- Bai, H., Mulder, H., Moerbeek, M., Kroesbergen, E.H., & Leseman, P.P.M. (2021). Divergent thinking in four-year-old children: An analysis of thinking processes in performing the Alternative Uses Task. *Thinking Skills and Creativity, 40*, 100814. <https://doi.org/10.1016/j.tsc.2021.100814>
- Batey, M. (2012). The Measurement of Creativity: From Definitional Consensus to the Introduction of a New Heuristic Framework. *Creativity Research Journal, 24*(1), 55–65. <https://doi.org/10.1080/10400419.2012.649181>

- Beaty, R.E., & Silvia, P.J. (2012). Why do ideas get more creative across time? An executive interpretation of the serial order effect in divergent thinking tasks. *Psychology of Aesthetics, Creativity, and the Arts*, 6(4), 309. <https://doi.org/doi.org/10.1037/a0029171>
- Beaty, R.E., Silvia, P.J., Nusbaum, E.C., Jauk, E., & Benedek, M. (2014). The roles of associative and executive processes in creative cognition. *Memory & Cognition*, 42(7), 1186–1197. <https://doi.org/10.3758/s13421-014-0428-8>
- Becker, J.P., & Shimada, S. (1997). *The Open-Ended Approach: A New Proposal for Teaching Mathematics*. National Council of Teachers of Mathematics.
- Bridges, D., & Schendan, H.E. (2019). Sensitive individuals are more creative. *Personality and Individual Differences*, 142, 186–195. <https://doi.org/10.1016/j.paid.2018.09.015>
- Briggs, M., & Davis, S. (2008). *Creative teaching: Mathematics in the early years and primary classroom*. Routledge.
- Colzato, L.S., Szapora, A., Pannekoek, J.N., & Hommel, B. (2013). The impact of physical exercise on convergent and divergent thinking. *Frontiers in Human Neuroscience*, 7. <https://doi.org/10.3389/fnhum.2013.00824>
- Cotter, K.N., Ivcevic, Z., & Moeller, J. (2020). Person-oriented profiles of originality and fluency in divergent thinking responses. *Journal of Research in Personality*, 86, 103941. <https://doi.org/10.1016/j.jrp.2020.103941>
- Fatah, A., Suryadi, D., Sabandar, J., & Turmudi, T. (2016). Open-ended approach: An effort in cultivating students' mathematical creative thinking ability and self-esteem in mathematics. *Journal on Mathematics Education*, 7(1). <https://doi.org/10.22342/jme.7.1.2813.9-18>
- Gilhooly, K.J., Fioratou, E., Anthony, S.H., & Wynn, V. (2007). Divergent thinking: Strategies and executive involvement in generating novel uses for familiar objects. *British Journal of Psychology*, 98(4), 611–625. <https://doi.org/10.1111/j.2044-8295.2007.tb00467.x>
- Glaser, B.G., Astr, T., Gynnild, A., Jamieson, L., Taylor, P.J., Sci, M., & Gibson, B. (2010). The Grounded theory review. *Qualitative Health Research*, 9(2).
- Grohman, M., Wodniecka, Z., & Khusak, M. (2006). Divergent Thinking and Evaluation Skills: Do They Always Go Together? *The Journal of Creative Behavior*, 40(2), 125–145. <https://doi.org/10.1002/j.2162-6057.2006.tb01269.x>
- Gunawan, G., Kartono, K., Wardono, W., & Kharisudin, I. (2022). Analysis of Mathematical Creative Thinking Skill: In Terms of Self Confidence. *International Journal of Instruction*, 15(4), 1011–1034. <https://doi.org/10.29333/iji.2022.15454a>
- Hu, W., & Adey, P. (2002). A scientific creativity test for secondary school students. *International Journal of Science Education*, 24(4), 389–403. <https://doi.org/10.1080/09500690110098912>
- Huang, P.-S., Peng, S.-L., Chen, H.-C., Tseng, L.-C., & Hsu, L.-C. (2017). The relative influences of domain knowledge and domain-general divergent thinking on scientific creativity and mathematical creativity. *Thinking Skills and Creativity*, 25, 1–9. <https://doi.org/10.1016/j.tsc.2017.06.001>
- Kholil, M. (2020). Students' creative thinking skills in solving mathematical logic problem with open-ended approaches. *Journal of Physics: Conference Series*, 1465(1), 012044. <https://doi.org/10.1088/1742-6596/1465/1/012044>
- Kim, S.J., Kwon, Y.M., & Bae, J.S. (2010). The Effects of Open-ended Problems on Mathematical Creativity and Brain Function. *Journal of Elementary Mathematics Education in Korea*, 14(3), 723–744.
- King, R.G. (1982). A General Systems Model of the Creative Process. *Gifted International*, 1(1), 17–43. <https://doi.org/10.1080/15332276.1982.11672662>
- Kwon, O.N., Park, J.H., & Park, J.S. (2006). Cultivating divergent thinking in mathematics through an open-ended approach. *Asia Pacific Education Review*, 7(1), 51–61. <https://doi.org/10.1007/BF03036784>
- Lee, C.S., & Therriault, D.J. (2013). The cognitive underpinnings of creative thought: A latent variable analysis exploring the roles of intelligence and working memory in three creative thinking processes. *Intelligence*, 41(5), 306–320. <https://doi.org/10.1016/j.intell.2013.04.008>

- Leikin, R. (2009). Exploring Mathematical Creativity Using Multiple Solution Tasks. In *Creativity in Mathematics and the Education of Gifted Students* (pp. 129–145). BRILL.  
[https://doi.org/10.1163/9789087909352\\_010](https://doi.org/10.1163/9789087909352_010)
- Lewis, C., & Lovatt, P.J. (2013). Breaking away from set patterns of thinking: Improvisation and divergent thinking. *Thinking Skills and Creativity*, 9, 46–58.  
<https://doi.org/10.1016/j.tsc.2013.03.001>
- MacKinnon, D.W. (2017). IPAR's Contribution to the Conceptualization and Study of Creativity. In D.W. MacKinnon (Ed.), *Perspectives in Creativity* (pp. 60–89). Routledge.  
<https://doi.org/10.4324/9781315126265-3>
- Maharani, H.R., Sukestiyarno, S., & Waluya, B. (2017). Creative Thinking Process Based on Wallas Model in Solving Mathematics Problem. *International Journal on Emerging Mathematics Education*, 1(2), 177. <https://doi.org/10.12928/ijeme.v1i2.5783>
- Mann, E.L. (2009). The Search for Mathematical Creativity: Identifying Creative Potential in Middle School Students. *Creativity Research Journal*, 21(4), 338–348.  
<https://doi.org/10.1080/10400410903297402>
- Mougenot, C., Bouchard, C., & Aoussat, A. (2007). Creativity in design—How designers gather information in the “Preparation” phase. *IASDR*, pp. 11–15.
- Nuha, M.A., Waluya, S.B., & Junaedi, I. (2018). Mathematical Creative Process Wallas Model in Students Problem Posing with Lesson Study Approach. *International Journal of Instruction*, 11(2), 527–538. <https://doi.org/10.12973/iji.2018.11236a>
- Paek, S.H., Abdulla Alabbasi, A.M., Acar, S., & Runco, M.A. (2021). Is more time better for divergent thinking? A meta-analysis of the time-on-task effect on divergent thinking. *Thinking Skills and Creativity*, 41, 100894. <https://doi.org/10.1016/j.tsc.2021.100894>
- Plucker, J.A., Qian, M., & Wang, S. (2011). Is Originality in the Eye of the Beholder? Comparison of Scoring Techniques in the Assessment of Divergent Thinking. *The Journal of Creative Behavior*, 45(1), 1–22. <https://doi.org/10.1002/j.2162-6057.2011.tb01081.x>
- Rubenstein, L.D., Callan, G.L., Speirs Neumeister, K., Ridgley, L.M., & Hernández Finch, M. (2020). How problem identification strategies influence creativity outcomes. *Contemporary Educational Psychology*, 60, 101840. <https://doi.org/10.1016/j.cedpsych.2020.101840>
- Runco, M.A. (2004). Creativity. *Annual Review of Psychology*, 55, 657–687.
- Runco, M.A. (2008). Commentary: Divergent thinking is not synonymous with creativity. *Psychology of Aesthetics, Creativity, and the Arts*, 2(2), 93–96. <https://doi.org/10.1037/1931-3896.2.2.93>
- Runco, M.A., & Acar, S. (2012). Divergent Thinking as an Indicator of Creative Potential. *Creativity Research Journal*, 24(1), 66–75. <https://doi.org/10.1080/10400419.2012.652929>
- Runco, M.A., Abdulla, A.M., Paek, S.H., Al-Jasim, F.A., & Alsuwaidi, H.N. (2016). Which Test of Divergent Thinking Is Best? *Creativity. Theories – Research - Applications*, 3(1), 4–18.  
<https://doi.org/10.1515/ctra-2016-0001>
- Runco, M.A., Dow, G., & Smith, W.R. (2006). Information, Experience, and Divergent Thinking: An Empirical Test. *Creativity Research Journal*, 18(3), 269–277.  
[https://doi.org/10.1207/s15326934crj1803\\_4](https://doi.org/10.1207/s15326934crj1803_4)
- Silver, E.A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *Zentralblatt Für Didaktik Der Mathematik*, 29(3), 75–80.  
<https://doi.org/10.1007/s11858-997-0003-x>
- Silvia, P.J., Beaty, R.E., & Nusbaum, E.C. (2013). Verbal fluency and creativity: General and specific contributions of broad retrieval ability (Gr) factors to divergent thinking. *Intelligence*, 41(5), 328–340. <https://doi.org/10.1016/j.intell.2013.05.004>
- Siswono, T.Y.E. (2010). Leveling Students' Creative Thinking in Solving and Posing Mathematical Problem. *Indonesian Mathematical Society Journal on Mathematics Education*, 1(1), 17–40.  
<https://doi.org/https://doi.org/10.22342/jme.1.1.794.17-40>
- Sitorus, J., & Masrayati. (2016). Students' creative thinking process stages: Implementation of realistic mathematics education. *Thinking Skills and Creativity*, 22, 111–120.  
<https://doi.org/10.1016/j.tsc.2016.09.007>

- Sitorus, J., Anas, N., & Waruhu, E. (2019). Creative thinking ability and cognitive knowledge: Big Five personality. *Research and Evaluation in Education*, 5(2), 85–94. <https://doi.org/10.21831/reid.v5i2.22848>
- Smith, R., & Smith, L. (2018). Qualitative methods. In *Research methods in human rights* (pp. 70–93). Routledge.
- Sowden, P.T., Pringle, A., & Gabora, L. (2015). The shifting sands of creative thinking: Connections to dual-process theory. *Thinking & Reasoning*, 21(1), 40–60. <https://doi.org/10.1080/13546783.2014.885464>
- Sukestiyarno, Y.L. (2020). *Metode Penelitian Pendidikan*. UNNES Press.
- Sun, M., Wang, M., & Wegerif, R. (2019). Using computer-based cognitive mapping to improve students' divergent thinking for creativity development. *British Journal of Educational Technology*, 50(5), 2217–2233. <https://doi.org/10.1111/bjet.12825>
- Wadaani, M.R. (2015). Teaching for Creativity as Human Development toward Self-Actualization: The Essence of Authentic Learning and Optimal Growth for All Students. *Creative Education*, 6(7), 669–679. <https://doi.org/10.4236/ce.2015.67067>
- Wang, M., Hao, N., Ku, Y., Grabner, R.H., & Fink, A. (2017). Neural correlates of serial order effect in verbal divergent thinking. *Neuropsychologia*, 99, 92–100. <https://doi.org/10.1016/j.neuropsychologia.2017.03.001>
- Wijaya, A.P., Nusantara, T., & Hidayanto, E. (2023). How Are Students' Prior Knowledge Differentiate Analytical Thinking Process in Identifying the Convergence of Real Number Sequences? *International Journal of Instruction*, 16(1), 206–218. <https://doi.org/10.29333/iji.2023.16112a>
- Zhu, W., Shang, S., Jiang, W., Pei, M., & Su, Y. (2019). Convergent Thinking Moderates the Relationship between Divergent Thinking and Scientific Creativity. *Creativity Research Journal*, 31(3), 320–328. <https://doi.org/10.1080/10400419.2019.1641685>