Analytic Geometry Problems Are Completed by Symbolic Visualization of Representation: A Case of a Future Teacher

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One of the most important fundamental mathematical concepts for both students and teachers is mathematical representation. This study uses a qualitative method with a case study approach to examine aspiring teachers use visual and symbolic representations when addressing geometry problems. The participants in this study were 27 pre-service teachers from Indonesian Christian University (UKI) Toraja who were in their fourth semester. The processes of data reduction, data presentation, conclusion-making, and verification were applied to the data. The results of this study demonstrated that the pre-service teachers still lacked as they were provided inaccurate and non-detailed representations in symbolic imagery. Even though the responses are only half correct, the teachers with average academic competence have a semi-detailed symbolic visualization of capturing good visualization skills. Students with strong academic aptitudes have detailed representations in their symbolic representations, as well as great visual and symbolic representation skills. In order to assist students in teaching and studying geometry and resolve difficulties requiring mathematical expressions, pre-service teachers should have strong visual and symbolic representation skills.

Keywords: case study, analytic geometry, visual representation, symbolic representation

INTRODUCTION

In mathematics instruction, the psychological notion of representation is frequently utilized to explain several significant phenomena related to the nature of thought. One of the fundamental skills in mathematics that must be fostered by students or teachers is mathematical representation. There are various definitions of representation among mathematicians. Representation abilities, in the opinion of Hwang, Chen, Dung, and Yang (2007), are essential for solving mathematical problems successfully. The pupils' understanding and application of mathematical topics are communicated through mathematical representation. There are many distinct settings in which representations might take place, including internal (thoughts) and exterior (the real world). In psychology, representation refers to the process of transforming physical real-world objects into abstract ideas or symbols. The interpretation of the thinking process as a complex thinking representation made up of diverse mental elements, such as metaphorical, visual-spatial, and structural information, was shared by Jonassen (2000) and Purnomo et al. (2022).

According to Albert (2001), external representations are those that allow for simple communication with others through the use of writing (written symbols), drawings, geometric sketches, or similarity. The internal representation, on the other hand, paints a picture of how we form our ideas about mathematical objects and procedures. According to Goldin (2002), a configuration can describe or represent something in another form. Furthermore, representation, according to Khalatil and Sherin (2000), is anything students do to exhibit or materialize their work. In contrast, representation, according to Nizaruddin (2014), is an internal abstraction of mathematical concepts or cognitive schemata that students create via experience. The pupil builds their own internal mental network called a schema. Additionally, it was proposed by Sa'dijah et al. (2023) and Luitel (2002) that representation is a method for creating mathematical knowledge. Students can engage in the knowledge production process alone or in groups.

Additionally, Friedlander and Tabach (2001) distinguished four types of representations: verbal, numerical, pictorial, and symbolic. Kartini (2009) states that there are three different types of representations: 1) visual representations (such as pictures, graphs, or tables), 2) symbolic representations (such as mathematical statements and notations, numerical and algebraic symbols), and 3) vocal representations (such as written text or words). The capacity for representation in mathematics, according to Mokhamad & Karunia (2015) and Purnomo et al. (2023), is the capacity to rephrase equations, notations, symbols, tables, images, drawings, diagrams, and other mathematical expressions into various forms. In this study, a representation of images, written words, equations, or mathematical expressions needs to be evaluated as a mathematical representation. In other terms, there are three different types of representation: verbal representation (written text/words), symbolic representation (mathematical statement/mathematical notation, numerical/algebraic symbol), and visual representation (picture, graph diagram, or table).

The data gathered for resolving student success issues will increase if the teacher instructs using visual representation in solving mathematical problems, claim Gursel Guler and Alper Ciltas in their 2011 research article titled "The visual representation usage levels of mathematics teachers and students in solving verbal problems." Researchers such as Goldin (2002), Hitt (2002), Van Nes and De Lange (2007), and others debated the connection between representation research, mathematical visualization, and conceptual understanding. The issue of student visualization has been examined in a lot of studies and some reviews of the literature (Presmeg, 2006). Arcavi (2003) looked at the numerous functions that visualization can perform in the practice and understanding of mathematics. He also added that procedures brought on by visual cues may be studied using visual sequences. Additionally, he demonstrated how pupils can "see" in representations that are connected to their conceptual framework. Additionally, he connects a number of visualization challenges to the absence of adaptable translations across representations.

While some pupils prefer symbolic or abstract representations, others are more interested in visual or real ones. The geometry representation includes both representations. Students that are adept at solving problems typically have the ability to translate and communicate their language (vocals), represent visuals (images, graphics), and manipulate formal representations (sentences, phrases, rules, and formulae). Students with poor problem-solving skills, on the other hand, consistently struggle with translation and representation while solving problems.

Furthermore, more recent studies (Ekmekci and Ozkan, 2021; Brizuela et al., 2020; Hatano and Miyazaki, 2021; Lee et al., 2022) have examined the use of visual and symbolic representations in geometry problem-solving. The efficiency of employing dynamic visual representations when solving geometry problems was examined in Ekmekci and Ozkan's study from 2021. They discovered that pupils who used dynamic visual representations outperformed those who utilized static visual representations in completing geometry problems. The relationship between spatial visualization skills and the use of visual and symbolic representations in geometry problem-solving was examined in research by Brizuela et al. (2020). They discovered that pupils with better spatial visualization skills used both symbolic and visual representations more frequently. In other words, using both representations improved performance on tasks requiring geometry problem-solving. Next, Hatano and Miyazaki's research (2021) looked into how people solve geometry problems by using embodied representations like physical models and gestures. They discovered that embodied representations can help pupils better understand spatial ideas and develop their problem-solving abilities.

In a study conducted in 2022, Lee et al. investigated the use of symbolic and visual representations in group problem-solving for geometry. They discovered that employing visual and symbolic representations to solve problems collaboratively improved communication and collaboration among students as well as performance when compared to individual problem-solving. These studies collectively demonstrate that using visual, symbolic, and other representations can effectively improve students' comprehension of geometry and problem-solving abilities. Incorporating collaborative problem-solving and employing multiple representations may also help children perform better and learn more deeply, according to the study. Examining the "Symbolic Visualization of Representation in Completing Analytic Geometry Problems: A Case of Prospective Teacher Students" piqued researchers' curiosity, per the numerous investigations carried out by numerous specialists.

METHODS

A qualitative methodology was applied in this study (Nowikowski, 2017). The status of natural things is examined through qualitative research, where the researcher is a key tool and the significance findings is prioritized over generalization. Furthermore, a case study was the method of research used. According to Bloomberg and Volpe (2012), case studies are detailed descriptions and in-depth assessments of a phenomenon, social unit, or system that is constrained by space and time. This study's design was carried out to acquire a thorough understanding of the circumstance and the significance of the visualization and symbols used by pupils to solve geometry problems. As a result, this study places more emphasis on the process than the outcomes. In this research, the subject selection method was snowball sampling.

The fourth-semester Toraja UKI Mathematics Education Study Program students served as the study's subjects. Based on the challenges examined, the subject of this work was chosen to be the depiction of visualization and symbolic students in the resolution of analytic flat geometry problems. The research participants included one student with strong academic skills (GPA 3.5), one student with medium academic abilities (3,0 GPA 3,5), and one student with low academic abilities (GPA 3,0), reflecting the wide range of academic talents among students. Research data are gathered by documentation and information from student work. Making geometric images to simplify problems and facilitate their resolution (image representation); creating a problem situation based on data or given representations; writing interpretations of representations; and writing steps to solve mathematical problems with words (word repertoire or written text); creating mathematical equations or models from other representations given; and solving problems by involving math. According to Miles and Huberman (2014)'s data analysis approaches, qualitative data analysis is carried out interactively and continues constantly until finished, resulting in saturated data. As a result, data analysis included data reduction, data presentation, conclusion-making, and verification.

RESULTS

Find the equation of the circle that alludes to the $5x + y = 3$ line at the point $(2, -7)$ and centers on the line $x - 2y = 19$! This field analytic geometry issue is intended to gauge the pre-service teachers' visual representation and symbolic skills.

Students Who Lack In-Depth Representation

Students that perform poorly in school are not specifically shown in visualization symbolism. The following are illustrations of how S2 subjects are represented in Figure 1. It can be seen from the responses to the subject of S1 in Figure 1 that S1 subjects are already capable of generating problematic situations depending on the information or illustration provided. S1 students are able to write the details from the issue, specifically the line $5x + y = 3$, that touches the circle at its point $(2, -7)$ and is centered on it $x - 2y = 19$.

FIGURE 1
REPRESENTATION OF FORM 1

Version in Translation: The line 5x + y = 3 has a gradient of m = 5 The equation for the line passing through point 2, -7, and having a gradient of m = 5 is y = 5(x – 2) + 7 = 5x – 10 + 7 = 5x – 3 The circle's center is crossed by the lines 5x + y = 3 and x – 2y = 19 5x + y = 3 x – 2y = 19 + 6x + 3y = 21 Consequently, the circle's equation is 6x + 3y - 21 = 0.

Although the circle equation is vague as to what should be decided, the answer to subject of S1 in Figure 1 demonstrates that S1 subjects tried to write questions from the questions presented. The response from an S1 student demonstrates that S1 students do not comprehend the idea of calculating the gradient of a line perpendicular to another line. Additionally, S1 students lack the knowledge necessary to calculate the equation of the line gradient m, pass the point (a, b), and understand how to calculate the radius and center of a circle for two lines that are perpendicular to it. Consequently, the S1 subject does not learn how to solve the issue correctly. Additionally, we see that S1 students were unable to create geometric pictures to help them understand the issue. Considering this scenario, it can be concluded that aspiring teachers should not expect their students to be detailed in their visual and symbolic representations.

Students' Representations Are Moderately Detailed

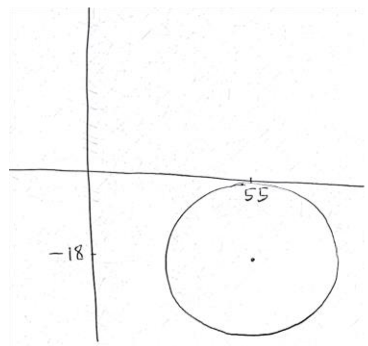
Students with average academic ability may visualize symbols in semi-detailed detail. An illustration of the portrayal of an S2 subject is shown in Figure 2. It has been able to construct a problem situation based on the data or representation provided based on the responses to topic S2 in Figure 2. It is clear from this that the S3 student may draw the line $5x + y = 3$, that intersects the circle at its point $(2, -7)$ and centers on the line $x - 2y = 19$, which is the information found in the issue.

Although the circle equation does not specify what must be decided, it appears that the subject S2 in Figure 2 attempted to compose questions based on the questions given. The response to topic S2 demonstrates that the respondent understands that the gradient of two perpendicular lines must comply with $m_1 \times m_2 = -1$. It is demonstrated that $m = \frac{1}{5}$, for other lines perpendicular to the line $5x + y = 3$, S2 finds a gradient value that is. Because the S2 subject multiplied the two segments incorrectly, the answer to the S2 subject indicates that S2 cannot derive the equation for the line's gradient $m = \frac{1}{5}$. The circle's center point is determined by S2 using the elimination approach, as the topic S2 in the section demonstrates, however the value is off. The final answer will be erroneous due to a mistake the S2 subject made in the earlier phase, which will affect the next phase.

FIGURE 2
REPRESENTATION OF FORM 2

Version in Translation:
Line 5x + y = 3 has a gradient of m = – 5
The perpendicular line's gradient is then m = 1/5
The equation at the position (2, - 7) with a gradient of 1/5 is
y = 1/5 (x – 2) – 7
y = 1/5x – 2/5 – 7
y = 1/5x – 37/5 (multiplied by 5)
y = x – 37
–x + y = –37
Such that
–x + y = –37
<u>x – 2y = 19</u> +
y = –18
– x + y = – 37 times 2 → – 2x + 2y = – 74
x – 2y = 19 → <u>x – 2y = 19</u> +
x = – 55
so that (– 55, – 18) is obtained as the center point.

Version in Translation:
Radius r and the (2, -7) distance from the center to the point, or
$r^2 = (-55 - 2)^2 + (-18+7)^2 = 3461$
next equation
$(x - 2)^2 + (y + 7)^2 = 3461$



According to the solution for S2 in Figure 2, the radius of the circle is calculated using the distance between its center and a given point $(2, -7)$. The S2 response demonstrates how S2 uses mathematical expressions to solve problems. Evidently, subject S2 used a mathematical model to write the final response. It is clear from the subject's final section of responses that he or she attempted to illustrate the problem's solution as a geometric image contained within a circle. The two lines that meet and touch the circle are not mentioned, though. Using this scenario as a model, prospective teachers' students with average academic competence have partially and semi-detailed visual and symbolic representations.

Students Who Can Depict Ideas in Depth

High-achieving students have comprehensive visualizations that are thoroughly metaphorical. The S3 subject representation shown in Figure 3 is as follows. According to the S3 subject's responses in Figure 3, it is obvous that the S3 subject was able to create a problem situation based on the information or illustration provided. It is clear from this that the S3 student may draw the line $5x + y = 3$, that intersects the circle at its point $(2, -7)$ and centers on the line $x - 2y = 19$, which is the information found in the issue. It appears from the subject's response that S3 students understand the question's intent. It is clear that students in S3 can identify and record the issues that need to be resolved in the issue, specifically figuring out the equation of the circle, which references to the line $5x + y = 3$ at point $(2, -7)$ and centers on the line $x - 2y = 19$.

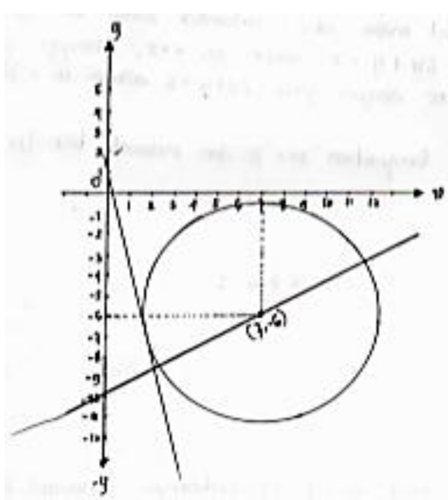
FIGURE 3
REPRESENTATION OF FORM 3

Version in Translation:
Because the circle is touched by the line $5x + y = 3$ at point $(2, -7)$, a right angle will be created if a new line is drawn that runs through the circle's center and crosses it.
The gradient of the line perpendicular to the line $5x + y = 3$ is $m1 = -5$ because the gradient of the line from $5x + y = 3$ is $m = 1/5$
The line running through the point $(2, -7)$ and having a gradient of $m = 1/5$ has the following equation:
$y = 1/5 (x - 2) - 7$ → both sides multiply by 5
$5y = x - 2 - 35$
$5y = x - 37$
$5y = x - 37$ → $-x + 5y = -37$
The circle is intersected by the lines $5x + y = 3$ and $x - 2y = 19$, respectively.
Therefore, the center of the circle is where these two lines connect.

$-x + 5y = -37$ …… Equation (1) $x - 2y = 19$ …… Equation (2)
equations (1) and (2) are eliminated.

$-x + 5y = -37$	
$\underline{x - 2y = 19}$ +	$x - 2y = 19$
$3y = -18$	$x - 2(-6) = 19$
$y = -18/3$	$x - (-12) = 19$
$y = -6$	$x + 12 = 19$
	$x = 7$

Substitute the value of y in place of (2)
the circle's center is located at $(7, -6)$
so that the circle's center is at the point $((2, -7)$ is
the circle's radius, specifically:
$r^2 = (7 - 2)^2 + (-6+7)^2$
$= 5^2 + 1^2$
$= 25 + 1$
$= 26$
The circle in question has the equation $(x - 7)^2 + (y + 6)^2 = 26$.



It is clear from S3 subject's response in Figure 3 that S3 is aware of the information learned from the problem in addition to potential consequences of drawing a new line $5x + y = 3$ that crosses the circle's center and intersects the existing one. While the topical response S3 in the section demonstrates that S3 is already aware that the gradients of two perpendicular lines must comply with $m_1 \times m_2 = -1$. It is established that S3 discovers additional line gradient values that are perpendicular to the line$5x + y = 3$,. The solution to the S3 question in Figure 3 demonstrates that S3 is capable of figuring out the line's equation gradiendt $m = \frac{1}{5}$. While S3 is capable of formulating mathematical equations or models and pinpointing the center of a circle utilizing a combination approach (elimination-substitution). The response from the subject demonstrates how S3 determines the circle radius value from the distance from the circle center to the point $(2, -7)$. S3 may use mathematical expressions to solve problems. Students in S3 can resolve issues by meticulously outlining the steps and utilizing mathematical models.

The subject depicts the resolution of the problem in the final section of the replies to the subject as a geometric image, namely two perpendicular lines, one of which alludes to the circle. By switching the result of the solution from the form of symbolic representation to visual representation, S3 subjects further clarify the problem's resolution. High academic ability future teachers have complete, rich visual and symbolic representations based on this scenario.

DISCUSSION

According to this research findings, S1 subjects with low academic ability still have deficiencies because none of the visual representation indications are present in the replies to S1 participants. Geometric pictures help to clarify the solution as one indication that subject S1 does not meet the criteria. As opposed to that, despite the fact that S2 participants' answers are erroneous, S2 subjects with middling academic ability have strong visual and symbolic representation abilities. In addition, S3 subject's capacity for visual and symbolic representation satisfies all signs of such capacity. This indicates that S3 subjects with strong academic aptitudes have outstanding achievements regarding their visual and symbolic representation abilities. The conditions that exist in S1 participants are consistent with the findings of Puji's (2016) research, which demonstrates that student visual representation skills have not significantly improved. The test findings demonstrate find it difficult to that children continue to represent items as images.

Students' capacity for visual and symbolic representation is influenced by a variety of factors in addition to their academic performance, such as their exposure to media during their educational experiences and their background in numerical problem-solving. One way to improve mathematical representation is by employing a learning medium. It is in line with the study done in 2015 by Anggun and Eka. Their study's findings demonstrated that teaching with GeoGebra media improved students' capacity for mathematical representation. Additionally, media is necessary for learning and helps gaining more expertise in solving mathematical problems, both of which help to strengthen prospective students' mathematical representation. According to the study's findings, the capacity for mathematical representation is crucial for aspiring teachers for a variety of reasons, including the ability to create geometric images to clarify problems, facilitate completion, write information based on data already available, write steps to solve mathematical problems, create equations or mathematical models, and solve problems by incorporating mathematical expressions.

In order to solve geometry problems, it is vital to use both symbolic and visual representation. Some researchers' findings (Lesh and Zawojewski, 2018, Fong and Fong, 2019, Singh and Garbett, 2019, Boudreau and Jansen, 2020, Chen, Li, and Li, 2021) were in agreement with this study. The researchers have investigated how these representations are used in problem-solving and how they might be successfully applied to raise student achievement. Lesh and Zawojewski's (2018) research found that using both visual and symbolic representations can help students better comprehend and solve geometrical difficulties. Students who employed visual representations were more likely to have a deeper knowledge of the problem and were better able to locate essential information, according to Fong and Fong's research from 2019. They proposed using visual representations as a method to aid pupils in making sense of challenging issues.

Additionally, according to Singh and Garbett's 2019 research, students could more effectively use mathematical principles to resolve geometry issues when they were provided symbolic representations. The use of visual representations in geometry problem-solving may also be effectively encouraged by digital learning environments, according to a study by Boudreau and Jansen (2020). Last but not least, a study by Chen, Li, and Li (2021) discovered that students who combined the use of visual and symbolic representations with geometry problems were more successful than those who used only one representation. They proposed that pupils may have a more thorough knowledge of mathematical ideas by combining symbolic and visual representations.

CONCLUSION

The ability of the S3 subject to be represented visually and symbolically satisfies all indicators for this ability, according to research and discussion findings. It implies that S3 subjects with strong academic performance have outstanding visual and symbolic representation abilities. Academically talented students can describe themselves symbolically in detail. Even though the responses from S2 respondents are wrong, S2 subjects with medium academic competence show high visual and symbolic representation skills. Additionally, symbolic visualization provides a semi-detailed depiction for kids with middling academic ability. In contrast, due to poor results analysis, the capacity of S1 participants with low academic ability to be mathematically represented is still weak. Furthermore, symbolic visualization still lacks detailed representation for kids with low academic capacity.

Students' capacity for visual and symbolic representation is influenced by a variety of elements in addition to their academic abilities, including the learning media they utilize, their prior experiences, and their practice. In this instance, learning through media is necessary, as is a lot of experience in solving geometry problems, in order to develop strong visual and symbolic representational skills. In order to assist students in creating geometric drawings to clarify solutions, writing information based on existing data, writing steps to solve mathematical problems, creating mathematical equations or models, and solving problems involving mathematical expressions, prospective teachers' students should have strong visual and symbolic representation skills.

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